Monetary Policy Shocks and the Link between Exchange Rate and Relocation of Firms

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Abstract

This paper extends the three country model of Corsetti et al. (2000) to include international relocation of firms, and explores the effects of monetary expansion by each country on the exchange rates, international relocation of firms, and relative consumption levels. The paper shows a new international transmission mechanism: the exchange rate depreciation induced by monetary expansion in one of the three countries causes foreign firms to relocate to that country, and consequently raises the relative consumption level of that country. In contrast, the monetary expansion can be detrimental to other countries.

1. Introduction

In the new open economy macroeconomics (NOEM) literature, a number of studies have focused on how the macroeconomic activity of each country and the exchange rate are influenced by monetary policy shocks under monopolistic distortions and price rigidities (Obstfeld and Rogoff, 1995 and 2002; Betts and Devereux, 2000a and 2000b; Hau, 2000; Bergin and Feenstra, 2001; Corsetti and Pesenti, 2001 and 2005; Cavallo and Ghironi, 2002; Devereux and Engel, 2002; Kollmann, 2001 and 2002; Smets and Wouthers, 2002; Johdo 2010, 2013a, 2013b, 2015a, 2016). The benchmark model of Obstfeld and Rogoff (1995) shows that a domestic monetary expansion raises foreign and domestic output and welfare through the first-order effect of increasing world consumption.

Since the publication of the Redux model of Obstfeld and Rogoff (1995), most models in the NOEM literature have assumed that firms are immobile across countries. Although it is feasible to explore the effects of monetary policy shocks in this framework, recent empirical evidence suggests that exchange rates affect the production locations and foreign direct investments of firms (Cushman, 1985 and 1988; Froot and Stein, 1991; Campa, 1993; Klein and Rosengren, 1994; Goldberg and Kolstad, 1995; Blonigen, 1997; Goldberg and Klein, 1998; Bénassy-quéré et al, 2001; Chakrabarti and Scholnick, 2002; Farrell et al., 2004).[1)]

So far little attention has been paid in the NOEM literature to the relationship between the international relocation of firms and macroeconomic variables, including consumption and exchange rates. The exception is the work of Johdo (2015b), who presents a NOEM model with international relocation of firms. In this study, Johdo (2015b) contrasts a two-country NOEM model without international relocation with a NOEM model with international relocation, and succeeds in showing explicitly the effects of a country's monetary expansion on exchange rate and consumption of each country, leading to firm relocation from the other country.[2)] However, because Johdo (2015b) begins with the assumption of a two-country economy, the literature cannot consider how allowing for a third country affects the impact of monetary expansion on international relocation of firms and other macroeconomic variables. Recently, multinational firms have very actively invested across national borders: American, Japanese and China's multinational firms are increasingly making their way into each other's markets. It is, therefore, appropriate that a multi-country model be adopted to examine how allowing for international relocation of firms affects the impact of monetary expansion on consumption and exchange rate.

Given this motivation, this paper investigates the impacts of monetary shocks on the international distribution of firms, exchange rate, and consumption by generalizing the three-country model of Corsetti et al. (2000) to include international relocation of firms. In particular, a novel feature of our model is that its model does not assume a fixed and exogenously given international distribution of firms. Instead, the model allows imperfectly competitive firms to respond to monetary shocks as in the work of Johdo (2015b).[3)] This implies that our model generates an additional international transmission effect that operates through the international relocation of firms, which has been overlooked by the work of Corsetti et al. (2000).

In this analysis, we show that exchange rate depreciation induced by a monetary expansion in one of three countries causes firms whose production sites are abroad to relocate to that country, which raises the relative consumption of that country. This is because the relocation increases labor demand in that country, which in turn raises labor income. In contrast, the monetary expansion can be detrimental to other countries, in terms of relative consumption level. This is because other countries experience the opposite response to the firm relocation and exchange rate adjustment.

The remainder of this paper is structured as follows. Section 2 outlines the features of the model. Section 3 describes the equilibrium. In Sections 4 and 5, following the methods and techniques of Obstfeld and Rogoff (1995), we examine the impacts of monetary policy shocks on international distribution of firms across the three countries, exchange rate, and relative consumption level. The final section summarizes the findings and concludes the paper.

2. The model

In this section, we construct a perfect-foresight, three-country model with international relocation of firms. This model is similar to the one presented by Johdo (2015b), in which there are two countries in the world economy: home country and foreign country. However, in our model, the economy consists of three countries and they are denoted by A, B and C, respectively. The size of the world population is normalized to unity, and households in countries A and B inhabit the intervals $[0, 1/3]$ and $(1/3, 2/3]$, respectively, and those in country C inhabit the interval $(2/3, 1]$. Therefore, the shares of households in A, B, and C are 1/3, 1/3, and 1/3, respectively. There is monopolistic competition in the markets for goods and labor, whereas the markets for money and international bonds are perfectly competitive. On the production side, monopolistically competitive producers exist continuously in the range $[0, 1]$, each of which produces a single differentiated product that is freely tradable. In this model, country A consists of those producers in the interval $[0, n_t]$, country B consists of those producers in the interval $[n_t, v_t]$, and the remaining $[v_t, 1]$ producers are in country C, where n_t and v_t are endogenous variables. Finally, we assume that firms are mobile internationally but their owners are not.

2.1. Households

The intertemporal objective function of representative household x in country h at time t, with $h = A, B, C$, is:

$$U^h{}_t(x) = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} (\log C^h{}_\tau(x) + \chi \log(M^h{}_\tau(x)/P^h{}_\tau) - (\kappa/2)(\ell^{sh}{}_\tau(x))^2), \qquad (1)$$

where E_t represents the mathematical expectation conditional on the information set available to household i at time t; β is a constant subjective discount factor $(0<\beta<1)$; $C^h{}_t(x)$ is the consumption index that is defined later; $M^h{}_t(x)/P^h{}_t$ is real money holdings, where $M^h{}_t(x)$ denotes nominal money balances held at the beginning of period $t+1$, and $P^h{}_t$ is the consumption price index of country h; and $\ell^{sh}{}_t(x)$ is the amount of labor supplied by household x. In this context, we assume that there is an international risk-free nominal bond market in which all households in the world can lend and borrow at the same interest rate and that nominal international bonds are denominated in terms of the country C's currency. At each point in time, households receive returns on risk-free nominal bonds, earn wage income by supplying labor, and receive profits from all firms equally. Therefore, a typical household in country h faces the following budget constraint:

$$\begin{aligned}\varepsilon^h{}_t B^h{}_{t+1}(x)+M^h{}_t(x) = {} & (1+i_t)\varepsilon^h{}_t B^h{}_t(x)+M^h{}_{t-1}(x)+W^h{}_t(x)\ell^{sh}{}_t(x)-P^h{}_t C^h{}_t(x)\\ & -P^h{}_t\tau^h{}_t+(\varepsilon^h{}_t/\varepsilon^A{}_t)\int_0^{n_t}\Pi_t^A(z)dz+(\varepsilon^h{}_t/\varepsilon^B{}_t)\int_{n_t}^{v_t}\Pi_t^B(z)dz\\ & +\varepsilon^h{}_t\int_{v_t}^1\Pi_t^C(z)dz,\end{aligned} \tag{2}$$

where $\varepsilon^h{}_t$ denotes the nominal exchange rate, defined as country h's currency per unit of country C's currency (so that $\varepsilon^C{}_t=1$); $B^h{}_{t+1}(x)$ denotes the nominal bond denominated in the country C's currency held by country h's agent x in period $t+1$; i_t denotes the nominal yield on the bond in terms of the country C's currency; $W^h{}_t(x)\ell^{sh}{}_t(x)$ is nominal labor income, where $W^h{}_t(x)$ denotes the nominal wage rate of labor supplied by household x in period t; $\int_0^{n_t}\Pi_t^A(z)dz$, $\int_{n_t}^{v_t}\Pi_t^B(z)dz$, and $\int_{v_t}^1\Pi_t^C(z)dz$ represent the total nominal profit flows of firms located in countries A, B, and C, respectively; $P^h{}_t C^h{}_t(x)$ represents nominal consumption expenditure; and $\tau^h{}_t$ denotes real lump-sum transfers from the government in period t. Note that all variables in (2) are measured in per capita terms. In the government sector, we assume that government spending is zero and that all seignorage revenues derived from printing the national currency are rebated to the public in the form of lump-sum transfers. Hence, the government budget constraint in country h is $0=s^h\tau^h{}_t+[(M^h{}_t-M^h{}_{t-1})/P^h{}_t]$, where $M^h{}_t$ is aggregate money supply and $s^h=1/3$ denotes the population share of country h in the world population.

Here, we assume that any monopolistically competitive firm that operates in every country employs the same production technology. In country A, firm $z\in[0, n_t]$ hires a continuum of differentiated labor inputs domestically and produces a unique product in a single location according to the CES production function:

$$y_{At}(z) = ((1/3)^{1/\phi}\int_0^{1/3}\ell_{At}(z,x)^{(\phi-1)/\phi}dx)^{\phi/(\phi-1)}, \tag{3}$$

where $y_{At}(z)$ denotes the production of firm z in period t; $\ell_{At}(z,x)$ is firm z's input of labor from household x in period t; and $\phi>1$ is the elasticity of input substitution. Given the firm's cost minimization problem, firm z's optimal demand function for labor x is as follows:

$$\ell_{At}(z,x) = s_A{}^{-1}(W_t{}^A(z)/W_t{}^A)^{-\phi}y_{At}(z), \tag{4}$$

where $W_t^A \equiv ((1/3)^{-1}\int_0^{1/3} W_t^A(x)^{(1-\phi)}dx)^{1/(1-\phi)}$ is a price index for labor input. Similarly, the other countries' firms have an optimal demand function for labor x that is analogous to equation (4).

2. 1. 1. Definition of consumption basket

The consumption basket of household x living in country h at period t is:

$$C_t^h(x) = \left[\int_0^{n_t} c_{At}^h(z,x)^{(\theta-1)/\theta}dz + \int_{n_t}^{v_t} c_{Bt}^h(z,x)^{(\theta-1)/\theta}dz + \int_{v_t}^{1} c_{Ct}^h(z,x)^{(\theta-1)/\theta}dz\right]^{\theta/(\theta-1)}, \tag{5}$$

where $\theta > 1$ is the elasticity of substitution among varieties produced within each country; and $c^h{}_{jt}(z,x)$ denotes consumption by household x located in country h of the good produced by firm z located in country j.[4] From (5), the consumption-based price index is defined as:

$$P^h{}_t = \left[\int_0^{n_t} (P_{At}^h(z))^{1-\theta}dz + \int_{n_t}^{v_t} (P_{Bt}^h(z))^{1-\theta}dz + \int_{v_t}^{1} (P_{Ct}^h(z))^{1-\theta}dz\right]^{1/(1-\theta)},$$

where $P^h{}_{jt}(z)$ is the price in country h of the good produced by firm z in country j, $j = A, B, C$.

2. 1. 2. Household decisions

In the first stage, households maximize the consumption index $C^h{}_t(x)$ subject to a given level of expenditure by optimally allocating differentiated goods produced in the three countries $c^h{}_{jt}(z,x)$, $j = A, B, C$. From this problem, we obtain the following private demand functions:

$$C^h{}_{jt}(z,x) = (P^h{}_{jt}(z)/P^h{}_t)^{-\theta}C^h{}_t(x), \quad j = A, B, C. \tag{6}$$

Summing the above demand functions and equating the resulting equation to the product of firm z located in country j yield the following market-clearing condition for any product z produced in country j:

$$y_{jt}(z) = (P^A{}_{jt}(z)/P^A{}_t)^{-\theta}C^A{}_t + (P^B{}_{jt}(z)/P^B{}_t)^{-\theta}C^B{}_t + (P^C{}_{jt}(z)/P^C{}_t)^{-\theta}C^C{}_t,$$
$$j = A, B, C, \tag{7}$$

where $C^A{}_t = \int_0^{1/3} C_t^A(x)dx$, $C^B{}_t = \int_{1/3}^{2/3} C_t^B(x)dx$, and $C^C{}_t = \int_{2/3}^{1} C_t^C(x)dx$. From the law of one price and the purchasing power parity (hereafter, PPP) arising from symmetric preferences, (7) is rewritten as:

$$y_{jt}(z) = (P^{j}{}_{jt}(z)/P^{j}{}_{t})^{-\theta}C^{w}{}_{t}, \quad j = A, B, C, \tag{8}$$

where $C^{w}{}_{t} \equiv C^{A}{}_{t} + C^{B}{}_{t} + C^{C}{}_{t}$. In the second stage, households maximize (1) subject to (2). The first-order conditions for this problem with respect to $B^{h}{}_{t+1}(x)$ and $M^{h}{}_{t}(x)$ can be written as:

$$1/C^{h}{}_{t}(x) = \beta(1+i_{t+1})E_{t}[(P^{h}{}_{t}/\varepsilon^{h}{}_{t})/(P^{h}{}_{t+1}/\varepsilon^{h}{}_{t+1})C^{h}{}_{t+1}(x)], \tag{9}$$

$$M^{h}{}_{t}(x)/P_{t} = \chi C^{h}{}_{t}(x)[(1+i_{t+1})E_{t}\varepsilon^{h}{}_{t+1}/((1+i_{t+1})E_{t}\varepsilon^{h}{}_{t+1} - \varepsilon^{h}{}_{t})], \tag{10}$$

where i_{t+1} is the nominal interest rate for country C's currency loans between periods t and $t+1$, defined as usual by $1+i_{t+1} = (1+r_{t+1})E_{t}[(P^{C}{}_{t+1}/P^{C}{}_{t})]$, and where r_{t+1} denotes the real interest rate. Equation (9) is the Euler equation for consumption, and (10) is the money demand equation.

Following the work of Corsetti and Pesenti (2001), we introduce nominal rigidities into the model in the form of one-period wage contracts under which nominal wages in period t are predetermined at the end of period $t-1$. In monopolistic labor markets, each household provides a single variety of labor input to a continuum of domestic firms. Hence, in country A, the equilibrium labor-market conditions can be expressed as $\ell_{t}^{sA}(x) = \int_{0}^{n_t} \ell_{At}(z, x)dz$, $x \in [0, 1/3]$, where the left-hand side represents the amount of labor supplied by household x, and the right-hand side represents firms' total demand for labor x. By taking W_{t}^{A}, P_{t}^{A}, and n_{t} as a given, then by substituting $\ell_{t}^{sA}(x) = \int_{0}^{n_t} \ell_{At}(z, x)dz$ and equation (4) into the budget constraint given by (2), and finally by maximizing the lifetime utility given by (1) with respect to the nominal wage rate $W_{t}^{A}(x)$, we obtain the following first-order condition for the optimal nominal wage rate, $W_{t}^{A}(x)$:

$$\phi(W_{t}^{A}(x)/P_{t}^{A})^{-1}E_{t-1}[\kappa \ell_{t}^{sA}(x)^{2}] = (\phi-1)E_{t-1}[(\ell_{t}^{sA}(x)/C^{A}{}_{t})]. \tag{11}$$

The labor suppliers of countries B and C have analogous optimal wage conditions.

2. 2. Firm's decision

Since country A-located firm z hires labor domestically, given $W^{A}{}_{t}$, $P^{A}{}_{At}$, and $P^{A}{}_{t}$, n_{t}, (3), subject to (8), country A-located firm z faces the following profit-maximization problem:

$$\max_{P^{A}_{At}(z)} \Pi_{At}(z) = P^{A}{}_{At}(z)y_{At}(z) - \int_{0}^{1/3} W^{A}_{tA}(z)\ell_{At}(z, x)dx = (P^{A}{}_{At}(z) - W^{A}{}_{t})y_{At}(z),$$

$$\text{subject to } y_{At}(z) = (P^A{}_{At}(z)/P^A{}_t)^{-\theta} C^W{}_t.$$

Given the above, the price mark-up is chosen according to:

$$P^A{}_{At}(z) = (\theta/(\theta-1))\, W^A{}_t. \tag{12}$$

Since $W^A{}_t$ is given, (12) yields $P^A{}_{At}(z) = P^A{}_{At}, z \in [0, n_t]$. These relationships imply that each firm located in country A supplies the same quantity of goods. Similarly, other firms located in different country have the price mark-up that is analogous to equation (12). Denoting the maximized real profit flows of country j-located firms by $\Pi_{jt}(x)/P^j{}_t$, and by substituting (8) and (12) into Π_j, we obtain

$$\Pi_{jt}(z)/P^j{}_t = (1/\theta)(P^j{}_{jt}(z)/P^j{}_t)^{1-\theta} C^w{}_t, \quad j = A, B, C. \tag{13}$$

2. 3.　Relocation behavior

Following the formulation of Johdo (2015b), we assume that all firms are not allowed to relocate instantaneously even if there is the profit gap.[5] At each point in time, this adjustment mechanism of relocation between countries A and B is formulated as follows:

$$n_t - n_{t-1} = \gamma[\Pi_{At}(z)/P_t^A - \Pi_{Bt}(z)/P_t^B] = \gamma[\Pi_{At}(z)/P_t^A - (\varepsilon^A{}_t/\varepsilon^B{}_t)\Pi_{Bt}(z)/P_t^A]. \tag{14}$$

Analogously, the adjustment mechanism for relocation between countries B and C is formulated as follows:

$$v_t - v_{t-1} = \gamma[\Pi_{Bt}(z)/P_t^B - \Pi_{Ct}(z)/P_t^C] = \gamma[\Pi_{Bt}(z)/P_t^B - \varepsilon^B{}_t \Pi_{Ct}(z)/P_t^B], \tag{15}$$

where $\gamma (0 \leq \gamma < \infty)$ is a constant positive parameter that determine the degree of firm mobility between two countries: a larger value of γ implies higher firm mobility between countries. As we explain in Section 4, this relocation adjustment makes our international transmission mechanism differ crucially from those in models in which the international allocation of firms is fixed.

2. 4.　Market conditions

The equilibrium condition for the integrated international bond market is given by:

$$\int_0^{1/3} B_t^A(x)dx + \int_{1/3}^{2/3} B_t^B(x)dx + \int_{2/3}^{1} B_t^C(x)dx = 0. \tag{16}$$

Money markets are always assumed to be clear in all countries. Hence, the equilibrium conditions are given by $M^{A}{}_{t}=\int_{0}^{1/3} M_t^A(x)dx$, $M^{B}{}_{t}=\int_{1/3}^{2/3} M_t^B(x)dx$, and $M^{C}{}_{t}=\int_{2/3}^{1} M_t^C(x)dx$.

3. Steady state values

In this section, we derive the solution for a symmetric steady state in which all variables are constant, the initial net foreign assets are zero ($B^{h}{}_{0}=0$), $h=A, B, C$. Then, we denote the steady-state values by using the subscript ss. In the symmetric steady state, given the Euler equation for consumption (equation (9)), the constant real interest rate is given by:

$$r_{ss} = (1-\beta)/\beta \equiv \delta, \tag{17}$$

where δ is the rate of time preference. Because symmetry, which implies $C^h_{ss}=C^w_{ss}$, holds, the steady-state spatial allocations of firms are:

$$n_{ss} = 1/3, \tag{18}$$

$$v_{ss} = 2/3. \tag{19}$$

The steady state output levels are:

$$y_{hss} = \ell^{sh}{}_{ss} = C^{h}{}_{ss} = C^{w}{}_{ss} = ((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}, \quad h = A, B, C. \tag{20}$$

Substituting $C^{w}{}_{ss}$ from equation (20) into equation (13) yields the following steady-state levels of real profit flows of country j-located firms, which have equal values:

$$\Pi_{jss}/P^{j}{}_{ss} = (1/\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}, \quad j = A, B, C. \tag{21}$$

From equations (10), (17), and (20), the real balances of country h agents are identical in the steady state:

$$M^{h}{}_{ss}/P^{h}{}_{ss} = \chi((1+\delta)/\delta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}, \quad h = A, B, C. \tag{22}$$

From these money demand equations, and given PPP, the steady-state nominal exchange rates are determined by the ratio of $M^{h}{}_{ss}$ to $M^{C}{}_{ss}$; i.e., $\varepsilon^{h}{}_{ss}=M^{h}{}_{ss}/M^{C}{}_{ss}$, $h=A, B, C$.

4. A log-linearized analysis

To examine the macroeconomic effects of unanticipated permanent monetary policy shocks, we solve a log-linear approximation of the system around the initial, zero-shock steady state with $B^{h}{}_{ss,0}=0$, $h=A, B, C$, as described in the previous section. For any variable X, we use

$\widehat{X}$ to denote short-run percentage deviations from the initial steady-state value, i.e., $\widehat{X}=dX_1/X_{ss,0}$, where $X_{ss,0}$ is the initial, zero-shock steady-state value, and the subscript 1 denotes the period in which the shock has occurred. These short-run percentage deviations are consistent with the length of nominal wage contracts. Thus, nominal wages and goods prices can be determined as $\widehat{W}^h=\widehat{P}_j^j(z)=0$, $h, j=A, B, C$, in the short-run log-linearized equations. In addition, we use $\overline{X}$ to denote long-run percentage deviations from the initial steady-state value, i.e., $\overline{X}=dX_2/X_{ss,0}=dX_{ss}/X_{ss,0}$, which is consistent with flexible nominal wages. $X_2=X_{ss}$ because a new steady state is reached at period 2.

By log-linearizing equations (14) and (15) around the symmetric steady state and setting $\widehat{P}_j^j(z)=0$, $j=A, B, C$, we obtain the following log-linearized expressions for the international distribution of firms:

$$\hat{n} = 3\gamma((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{3/2}(1/\kappa)^{1/2}(\hat{\varepsilon}^A-\hat{\varepsilon}^B), \tag{23}$$

$$\hat{v} = (3\gamma/2)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{3/2}(1/\kappa)^{1/2}\hat{\varepsilon}^B. \tag{24}$$

Equation (23) shows that under given ε^B, v_t and $\Pi_{Ct}(z)/P_t^C$, exchange rate depreciation of country A's currency ($\hat{\varepsilon}^A-\hat{\varepsilon}^B>0$) induces the relocation of firms located in country B toward the country A. This result is consistent with the evidence found in the empirical literature on the relationship between exchange rates and FDI (Cushman, 1988; Caves, 1989; Froot and Stein, 1991; Campa, 1993; Klein and Rosengren, 1994; Blonigen, 1997; Goldberg and Klein, 1998; Baek and Okawa 2001; Bénassy-quéré et al, 2001; Chakrabarti and Scholnick, 2002; Bolling et al, 2007; Udomkerdmongkol et al 2008). Intuitively, with fixed nominal wages, which cause nominal product prices to be sticky because of the mark-up pricing by monopolistic product suppliers, depreciation in country A's currency increases relative production of country A's goods through the 'expenditure-switching effect'; i.e., $\hat{y}^A-\hat{y}^B=\theta(\hat{\varepsilon}^A-\hat{\varepsilon}^B)$. This phenomenon increases the relative profits of country A-located firms, and consequently, firms located in country B relocate to the country A. Equation (23) also shows that nominal exchange rate changes have greater effects the larger is γ. By contrast, when relocation costs are high ($\gamma=0$), nominal exchange rate changes have a negligible effect on the relocation of firms. The intuition behind the impacts of ε^B in equations (23) and (24) on the international relocation of firms across borders can be explained analogously: Under given ε^A and $\Pi_{At}(z)/P_t^A$ exchange rate depreciation of country B's currency ($\hat{\varepsilon}^B>0$) induces the relocation of firms located in country C toward the country B.

5. Monetary policy shocks

Now, we consider the effects of an unanticipated permanent monetary policy shock in each country.

5.1. The case of $\widehat{M}^A=\overline{M}^A>0$, $\widehat{M}^B=\overline{M}^B=\widehat{M}^C=\overline{M}^C=0$

In this subsection, we focus on the impacts of a permanent monetary shock in country A: $\widehat{M}^A=\overline{M}^A>0$. In this case, the closed-form solutions for the seven key variables are as follows:

$$\hat{\varepsilon}^A-\hat{\varepsilon}^B=\left[\frac{\alpha_2 M_2-\alpha_1 M_1}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^A>0, \tag{25}$$

$$\hat{\varepsilon}^B=\left[\frac{\alpha_2 M_1-\alpha_1 M_2}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^A>0, \tag{26}$$

$$\hat{n}=3\gamma\theta_1\left[\frac{\alpha_2 M_2-\alpha_1 M_1}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^A>0, \tag{27}$$

$$\hat{v}=(3\gamma/2)\theta_1\left[\frac{\alpha_2 M_1-\alpha_1 M_2}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^A>0, \tag{28}$$

$$\hat{v}-\hat{n}=(3\gamma/2)\theta_1\left[\frac{\alpha_1(2M_1-M_2)+\alpha_2(M_1-2M_2)}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^A<0, \tag{29}$$

$$\widehat{C}^A-\widehat{C}^B=\widehat{M}^A-(\hat{\varepsilon}^A-\hat{\varepsilon}^B)=\left\{1-\left[\frac{\alpha_2 M_2-\alpha_1 M_1}{(\alpha_2)^2-(\alpha_1)^2}\right]\right\}\widehat{M}^A>0, \tag{30}$$

$$\widehat{C}^A-\widehat{C}^C=(\hat{\varepsilon}^A-\hat{\varepsilon}^B)+\hat{\varepsilon}^B>0, \tag{31}$$

where

$$M_1=\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1>0,$$

$$M_2=-\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]<0,$$

$$\alpha_1=M_1+\tilde{\theta}(\theta-1)+6\gamma\theta_1\tilde{\theta}>0,$$

$$\alpha_2=M_2-3\gamma\theta_1\tilde{\theta}<0,$$

$$\theta_1=((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{3/2}(1/\kappa)^{1/2}>0,$$

$$\tilde{\delta}\equiv(1+\delta)/\delta,\ \tilde{\theta}\equiv(\theta-1)/\theta,\ \tilde{\phi}\equiv(\phi-1)/\phi,\ M_1>M_2,\ (\alpha_1)^2>(\alpha_2)^2.$$

Equations (25) and (26) indicate that an unanticipated monetary expansion in country A

leads to exchange rate depreciation in $\varepsilon^A - \varepsilon^B$ and ε^B. Equation (27) shows that an unanticipated monetary expansion in country A causes country B's firms to relocate to country A. Equation (28) shows that an unanticipated monetary expansion in country A causes country C's firms to relocate to country B. Equation (29) shows that an unanticipated monetary expansion in country A causes the number of country B's firms to decline. Equations (30) and (31) show that the relative consumption levels of country A rise when there is an unanticipated monetary expansion in country A.

The results (25), (26), (27), (28), (29), (30), and (31) can be explained intuitively as follows. An unanticipated monetary expansion in country A requires an instantaneous depreciation of its currency to restore money market equilibrium for a given level of initial relative consumption. Under a given ε^B, this then leads to a reduction in the real price of country A's goods relative to country B's goods because of the depreciation of country A's currency (see equation (25)), which causes world demand to switch from country B's goods to country A's goods (the expenditure-switching effect). Under a given the real profits of firms located in country C, this demand shift increases the relative profits of firms located in country A, which causes firms located in country B to relocate to country A. Hereafter, we shall call this the 'relocation effect' (see equations (27), (28) and (29)). As a result, the relocation raises the relative labor income of country A, which raises the relative consumption of country A (see equation (30)). Furthermore, from the 'relocation effect' of (29), the consumption in country B decreases, which requires a depreciation of country B's currency to restore money market equilibrium. This happens because, given that the demand for real money balances is increasing with consumption, the country B's currency must depreciate and reduce the supply of real money balances in that country (see equation (26)). Therefore, the monetary expansion in country A reduces the real price of country B's goods relative to country C's goods because of the depreciation of country B's currency, which causes world demand to switch from country C's goods to country B's goods. This demand shift increases the relative profits of firms located in country B, which causes firms located in country C to relocate to country B (see equation (28)). As a result, the relocation reduces the relative labor income of country C, which decreases the relative consumption of country C (see equation (31)). In sum, when relocation is included in the model, a permanent monetary shock in country A always benefits country A in terms of relative consumption level, while it can be detrimental not only to country B, but also to country C.

Incidentally, we can see the impacts that the absence of relocation of firms ($\gamma=0$) has on exchange rates and relative consumption levels. Substituting $\gamma=0$ into equations (25) to

(31), we obtain:

$$\hat{\varepsilon}^A - \hat{\varepsilon}^B = \left(\frac{M_1}{\alpha_1}\right)\widehat{M}^A > 0, \quad \hat{\varepsilon}^B = 0, \quad \hat{n} = 0, \quad \hat{v} = 0, \quad \hat{v} - \hat{n} = 0,$$

$$\widehat{C}^A - \widehat{C}^B = \widehat{M}^A - (\hat{\varepsilon}^A - \hat{\varepsilon}^B) = \left(1 + \frac{M_1}{\alpha_1}\right)\widehat{M}^A > 0, \quad \widehat{C}^A - \widehat{C}^C = \left(\frac{M_1}{\alpha_1}\right)\widehat{M}^A > 0,$$

where

$$M_1 = \delta^{-1}\left[\frac{2\theta^2 - \theta + 1}{\theta(\theta+1)}\right] + 1 > 0, \quad M_2 = 0, \quad \alpha_1 = M_1 + \frac{(\theta-1)^2}{\theta} > 0, \quad \alpha_2 = 0.$$

The above equations are the same as those in the model of Corsetti et al. (2000) if the elasticity of substitution between the Center and the Periphery countries in the consumption basket are equal to the elasticity of substitution between the two Periphery consumption indexes in their model.

5. 2. The case of $\widehat{M}^B = \overline{M}^B > 0$, $\widehat{M}^A = \overline{M}^A = \widehat{M}^C = \overline{M}^C = 0$

In this subsection, we focus on the impacts of a permanent monetary shock in country B: $\widehat{M}^B = \overline{M}^B > 0$. In this case, the closed-form solutions for the seven key variables are as follows:

$$\hat{\varepsilon}^A - \hat{\varepsilon}^B = \left[\frac{(\alpha_1+\alpha_2)(M_1-M_2)}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^B < 0, \tag{32}$$

$$\hat{\varepsilon}^B = \left[\frac{(\alpha_1+\alpha_2)(M_2-M_1)}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^B > 0, \tag{33}$$

$$\hat{n} = 3\gamma\theta_1\left[\frac{(\alpha_1+\alpha_2)(M_1-M_2)}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^B < 0, \tag{34}$$

$$\hat{v} = (3\gamma/2)\theta_1\left[\frac{(\alpha_1+\alpha_2)(M_2-M_1)}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^B > 0, \tag{35}$$

$$\hat{v} - \hat{n} = (9\gamma/2)\theta_1\left[\frac{(\alpha_1+\alpha_2)(M_2-M_1)}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^B > 0, \tag{36}$$

$$\widehat{C}^A - \widehat{C}^B = -\widehat{M}^B - (\hat{\varepsilon}^A - \hat{\varepsilon}^B) = -\left\{1 + \frac{(\alpha_1+\alpha_2)(M_1-M_2)}{(\alpha_2)^2-(\alpha_1)^2}\right\}\widehat{M}^B < 0, \tag{37}$$

$$\widehat{C}^B - \widehat{C}^C = \widehat{M}^B - \hat{\varepsilon}^B = \left\{1 - \frac{(\alpha_1+\alpha_2)(M_2-M_1)}{(\alpha_2)^2-(\alpha_1)^2}\right\}\widehat{M}^B > 0. \tag{38}$$

The above results can be explained intuitively as follows. First, an unanticipated monetary expansion in country B requires an instantaneous depreciation of its currency to restore money market equilibrium for a given level of initial relative consumptions. Under a given ε^A, this then leads to a reduction in the real price of country B's goods relative to both country

A's and country C's goods because of the depreciation of country B's currency (see equations (32) and (35)), which causes world demand to switch from both country A's and country C's goods to country B's goods (the expenditure-switching effect). These demand shifts increase the relative profits of firms located in country B, which causes firms located in countries A and C to relocate to country B (the relocation effect; see equations (34), (35) and (36)). As a result, the relocation increases the labor income of county B and decreases the labor incomes of countries A and C, which raises the relative consumptions of country B (see equation (37) and (38)). In sum, when relocation is included in the model, a permanent monetary shock in country B always benefits country B in terms of relative consumption level, while it can be detrimental both to country A and to country C.

5.3. The case of $\widehat{M}^C=\overline{M}^C>0$, $\widehat{M}^A=\overline{M}^A=\widehat{M}^B=\overline{M}^B=0$

In this subsection, we focus on the impacts of a permanent monetary shock in country C: $\widehat{M}^C=\overline{M}^C>0$. In this case, the closed-form solutions for the seven key variables are as follows:

$$\hat{\varepsilon}^A-\hat{\varepsilon}^B = -\left[\frac{\alpha_2 M_1-\alpha_1 M_2}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^C < 0, \tag{39}$$

$$\hat{\varepsilon}^B = -\left[\frac{\alpha_2 M_2-\alpha_1 M_1}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^C < 0, \tag{40}$$

$$\hat{n} = -3\gamma\theta_1\left[\frac{\alpha_2 M_1-\alpha_1 M_2}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^C < 0, \tag{41}$$

$$\hat{v} = -(3\gamma/2)\theta_1\left[\frac{\alpha_2 M_2-\alpha_1 M_1}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^C < 0, \tag{42}$$

$$\hat{v}-\hat{n} = -(3\gamma/2)\theta_1\left[\frac{\alpha_2(M_2-2M_1)+\alpha_1(2M_2-M_1)}{(\alpha_2)^2-(\alpha_1)^2}\right]\widehat{M}^C < 0, \tag{43}$$

$$\widehat{C}^B-\widehat{C}^C = -\widehat{M}^C-\hat{\varepsilon}^B = -\left\{1-\frac{\alpha_2 M_2-\alpha_1 M_1}{(\alpha_2)^2-(\alpha_1)^2}\right\}\widehat{M}^C < 0, \tag{44}$$

$$\widehat{C}^A-\widehat{C}^C = -(\hat{\varepsilon}^A-\hat{\varepsilon}^B)-\widehat{M}^C-\hat{\varepsilon}^B = -\left\{1-\frac{(\alpha_1-\alpha_2)(M_1+M_2)}{(\alpha_2)^2-(\alpha_1)^2}\right\}\widehat{M}^C < 0. \tag{45}$$

The above results can be explained intuitively as follows. First, an unanticipated monetary expansion in country C requires an instantaneous depreciation of country C's currency to restore money market equilibrium for a given level of initial relative consumptions ($\hat{\varepsilon}^A=\hat{\varepsilon}^B<0$, see equation (47)). In this stage, country$A$'s currency relative to B's remains unchanged because $\hat{\varepsilon}^A-\hat{\varepsilon}^B=0$. Therefore, the monetary expansion in country C reduces the real price of country C's goods relative to country B's goods because of the depreciation of country C's currency, which causes world demand to switch from country B's goods to country C's goods (the expenditure-switching effect). Under a given the real profits of firms located in country A, this demand shift increases the relative profits of firms located in country C, which causes firms located in country B to relocate to country C (the relocation effect; see equation (42)). The relocation then raises the labor income of country C and decreases the labor income of country B, which raises the relative consumption of country C (see equations (44)). Because of these effects, country B's currency must depreciate to restore equilibrium in the market for real balances. This depreciation of country B's currency weakens the initial appreciation of its currency, and consequently the relative change in country A's currency is negative ($\hat{\varepsilon}^A - \hat{\varepsilon}^B < 0$, see equation (39)). Furthermore, this then leads to a reduction in the real price of country B's goods relative to country A's goods, which causes world demand to switch from country A's goods to country B's goods. Under a given the real profits of firms located in country C, this demand shift increases the relative profits of firms located in country B, which causes firms located in country A to relocate to country B (see equation (41)). The relocation then decreases the labor income of country A, which decreases the relative consumption of country A (see equations (45)). In sum, when relocation is included in the model, a permanent monetary shock in country C always benefits country C in terms of relative consumption level, while it can be detrimental not only to country B, but also to country A.

6. Conclusion

In this paper, we extend the *Redux* model to include relocation of firms among three countries and consider the question of how allowing for the relocation of firms among three countries affects the responses of consumption and exchange rate to monetary policy shocks. The findings indicate that when relocation matters, a permanent monetary shock in one of the three countries always benefits that country, while it can be detrimental to the other two countries, in terms of relative consumption.

Appendix A

Consumption basket, price indices and the purchasing power parity

The consumption basket of household x living in country A is

$$C_t^A(x) = \left(\int_0^{n_t} c_A^A(z,x)^{(\theta-1)/\theta}dz + \int_{n_t}^{v_t} c_B^A(z,x)^{(\theta-1)/\theta}dz + \int_{v_t}^{1} c_C^A(z,x)^{(\theta-1)/\theta}dz\right)^{\theta/\theta-1}, \quad \text{(A. 1)}$$

where $\theta > 1$ is the elasticity of substitution among varieties produced in the same country, and $c^A{}_{jt}(z,x)$ denotes consumption by household x located in country A of the brand produced by firm z located in country j.

From (2), the consumption-based price indexes are:

$$P_t^A = \left(\int_0^{n_t} P_{At}^A(z)^{1-\theta}dz + \int_{n_t}^{v_t} P_{Bt}^A(z)^{1-\theta}dz + \int_{v_t}^{1} P_{Ct}^A(z)^{1-\theta}dz\right)^{1/(1-\theta)}, \quad \text{(A. 2)}$$

where $P^A{}_j(z)$ is the price in the country A of the good produced by firm z in country j, and P^A is the price index of the country A. Under the law of one price, we obtain

$$P_t^A{}_A(z) = (\varepsilon^A/\varepsilon^B)P_t^B{}_A(z) = \varepsilon^A P_t^C{}_A(z), \quad \text{(A. 3)}$$

$$P_t^B{}_B(z) = (\varepsilon^B/\varepsilon^A)P_t^A{}_B(z) = \varepsilon^B P_t^C{}_B(z), \quad \text{(A. 4)}$$

$$P_t^C{}_C(z) = (1/\varepsilon^A)P_t^A{}_C(z) = (1/\varepsilon^B)P_t^B{}_C(z). \quad \text{(A. 5)}$$

Because the price of the same good is equalized across the three countries by the law of one price, $P_t{}^A = (\varepsilon^A/\varepsilon^B)P_t{}^B = \varepsilon^A P_t{}^C$ must be consistent with (A. 2).

Household's decisions in the first stage

In the first stage, households solve the following problem:

$Max_{C_{jt}^A(z,x)}$ (A. 1),

$$\text{s.t. } E^A{}_t(x) = \int_0^{n_t} P_{At}^A(z)C^A{}_{At}(z,x)dz + \int_{n_t}^{v_t} P_{Bt}^A(z)C^A{}_{Bt}(z,x)dz + \int_{v_t}^{1} P_{Ct}^A(z)C^A{}_{Ct}(z,x)dz. \quad \text{(A. 6)}$$

We then obtain the following private demand functions by household x located in country A of the brand produced by firm z located in country $j (j=A, B, C)$:

$$C^A{}_{At}(z,x) = (P^A{}_{At}(z)/P^A{}_t)^{-\theta}C^A{}_t(x), \quad \text{(A. 7)}$$

$$C^A{}_{Bt}(z,x) = (P^A{}_{Bt}(z)/P^A{}_t)^{-\theta}C^A{}_t(x), \quad \text{(A. 8)}$$

$$C^A{}_{Ct}(z,x) = (P^A{}_{Ct}(z)/P^A{}_t)^{-\theta}C^A{}_t(x). \quad \text{(A. 9)}$$

Similarly, the private demand functions by household x located in country B of the brand produced by firm z located in country $j (j=A, B, C)$:

$$C^B{}_{At}(z,x) = (P^B{}_{At}(z)/P^B{}_t)^{-\theta}C^B{}_t(x), \quad \text{(A. 10)}$$

$$C^B{}_{Bt}(z,x) = (P^B{}_{Bt}(z)/P^B{}_t)^{-\theta}C^B{}_t(x), \quad \text{(A. 11)}$$

$$C^{B}{}_{Ct}(z,x) = (P^{B}{}_{Ct}(z)/P^{B}{}_{t})^{-\theta}C^{B}{}_{t}(x). \qquad \text{(A. 12)}$$

Similarly, the private demand functions by household x located in country C of the brand produced by firm z located in country $j\,(j=A,B,C)$:

$$C^{C}{}_{At}(z,x) = (P^{C}{}_{At}(z)/P^{C}{}_{t})^{-\theta}C^{C}{}_{t}(x), \qquad \text{(A. 13)}$$

$$C^{C}{}_{Bt}(z,x) = (P^{C}{}_{Bt}(z)/P^{C}{}_{t})^{-\theta}C^{C}{}_{t}(x), \qquad \text{(A. 14)}$$

$$C^{C}{}_{Ct}(z,x) = (P^{C}{}_{Ct}(z)/P^{C}{}_{t})^{-\theta}C^{C}{}_{t}(x). \qquad \text{(A. 15)}$$

Summing the private demand functions (A. 7), (A. 10), and (A. 13), yields the total world demand for the product of firm z located in the country A

$$y_{At}(z) = (P^{A}{}_{At}(z)/P^{A}{}_{t})^{-\theta}C^{A}{}_{t}+(P^{B}{}_{A_t}(z)/P^{B}{}_{t})^{-\theta}C^{B}{}_{t}+(P^{C}{}_{At}(z)/P^{C}{}_{t})^{-\theta}C^{C}{}_{t}, \qquad \text{(A. 16)}$$

where $C^{A}{}_{t}=\int_{0}^{s_A}C^{A}_{t}(x)dx$, $C^{B}{}_{t}=\int_{s_A}^{s_B}C^{B}_{t}(x)dx$ and $C^{C}{}_{t}=\int_{s_B}^{1}C^{C}_{t}(x)dx$. From the law of one price and the purchasing power parity, (A. 16) is rewritten as

$$y_{At}(z) = (P^{A}{}_{At}(z)/P^{A}{}_{t})^{-\theta}C^{w}{}_{t}, \qquad \text{(A. 17)}$$

where $C^{w}{}_{t}\equiv\int_{0}^{s_A}C^{A}_{t}(x)dx+\int_{s_A}^{s_B}C^{B}_{t}(x)dx+\int_{s_B}^{1}C^{C}_{t}(x)dx$ is aggregate per capita world consumption. Similarly, the total world demand for the product of firm z located in the country B and C are

$$y_{Bt}(z) = (P^{B}{}_{Bt}(z)/P^{B}{}_{t})^{-\theta}C^{w}{}_{t}, \qquad \text{(A. 18)}$$

$$y_{Ct}(z) = (P^{C}{}_{Ct}(z)/P^{C}{}_{t})^{-\theta}C^{w}{}_{t}. \qquad \text{(A. 19)}$$

Firm's decision

Since the home-located firm j hires labor domestically, given W_t, P_{ht}, P_t, n_t and s_w, and subject to (A. 17), firm z located in the country A faces the following profit-maximization problem:

$$\max_{P^{A}_{At}(z)} \Pi_{At}(z) = P^{A}{}_{At}(z)y_{At}(z)-\int_{0}^{1/3}W^{A}_{tA}(z)\ell_{At}(z,x)dx$$

$$= (P^{A}{}_{At}(z)-W^{A}{}_{t})y_{At}(z), \qquad \text{(A. 20)}$$

subject to $y_{At}(z)=(P^{A}{}_{At}(z)/P^{A}{}_{t})^{-\theta}C^{w}{}_{t}$.

Given the above, the price mark-up is chosen according to:

$$P^{A}{}_{At}(z) = (\theta/(\theta-1))W^{A}{}_{t}. \qquad \text{(A. 21)}$$

Since W is given, (A. 21) yields $P^A{}_{At}(z)=P^A{}_{At}$, $z\in[0, n_t]$. Similarly, the price mark-ups of foreign-located firms are identical, since $P^B{}_{Bt}(z)=P^B{}_{Bt}$, $z\in[n_t, v_t]$, $P^C{}_{Ct}(z)=P^C{}_{Ct}$, $z\in[v_t,1]$. Dropping the firm index because of symmetry and denoting the maximized real profit flows of country j-located firms by $\Pi_{jt}(x)/P^j{}_t$, $j=A, B, C$, and substituting (A. 17) and (A. 21) into Π_{jt} of (A. 20) yields:

$$\Pi_{At}(z)/P^A{}_t = (1/\theta)(P^A{}_{At}(z)/P^A{}_t)^{1\text{-}\theta}C^w{}_t, \quad (A.22)$$

$$\Pi_{Bt}(z)/P^B{}_t = (1/\theta)(P^B{}_{Bt}(z)/P^B{}_t)^{1\text{-}\theta}C^w{}_t, \quad (A.23)$$

$$\Pi_{Ct}(z)/P^C{}_t = (1/\theta)(P^C{}_{Ct}(z)/P^C{}_t)^{1\text{-}\theta}C^w{}_t. \quad (A.24)$$

Equilibrium values

In the symmetric steady state, given the Euler equation for consumption (equation (9)), the constant real interest rate is given by

$$r_{ss} = (1-\beta)/\beta \equiv \delta. \quad (A.25)$$

In the steady state, because consumption must equal real income, which comprises labor and profit income, we have

$$C^A{}_{ss} = (W^A{}_{ss}/P^A{}_{ss})\ell^{sA}{}_{ss}+(1/P^A{}_{ss})[n_{ss}\Pi_{Ass}+(v_{ss}-n_{ss})(\varepsilon^A{}_{ss}/\varepsilon^B{}_{ss})\Pi_{Bss}+(1-v_{ss})\varepsilon^A{}_{ss}\Pi_{Css}], \quad (A.26)$$

$$C^B{}_{ss} = (W^B{}_{ss}/P^B{}_{ss})\ell^{sB}{}_{ss}+(1/P^B{}_{ss})[(\varepsilon^B{}_{ss}/\varepsilon^A{}_{ss})n_{ss}\Pi_{Ass}+(v_{ss}-n_{ss})\Pi_{Bss}+(1-v_{ss})\varepsilon^B{}_{ss}\Pi_{Css}], \quad (A.27)$$

$$C^C{}_{ss} = (W^C{}_{ss}/P^C{}_{ss})\ell^{sC}{}_{ss}+(1/P^C{}_{ss})[(1/\varepsilon^A{}_{ss})n_{ss}\Pi_{Ass}+(1/\varepsilon^B{}_{ss})(v_{ss}-n_{ss})\Pi_{Bss}+(1-v_{ss})\Pi_{Css}]. \quad (A.28)$$

In the symmetric steady state, we obtain the following steady-state real prices:

$$P_{ss}{}^A{}_A(z)/P_{ss}{}^A = P_{ss}{}^B{}_B(z)/P_{ss}{}^B = P_{ss}{}^C{}_C(z)/P_{ss}{}^C = 1. \quad (A.29)$$

From $P^A{}_{At}(z)=(\theta/(\theta-1))W^A{}_t$, $P^B{}_{Bt}(z)=(\theta/(\theta-1))W^B{}_t$, and $P^C{}_{Ct}(z)=(\theta/(\theta-1))W^C{}_t$, and (A. 29), steady-state real wages are

$$W_{ss}{}^A/P_{ss}{}^A = W_{ss}{}^B/P_{ss}{}^B = W_{ss}{}^C/P_{ss}{}^C = (\theta-1)/\theta. \quad (A.30)$$

Substituting (A. 29) into (A. 17), (A. 18) and (A. 19), respectively, yields

$$y_{Ass} = C^w{}_{ss}, y_{Bss} = C^w{}_{ss}, y_{Css} = C^w{}_{ss}. \quad (A.31)$$

Substituting (A. 29) into (A. 22), (A. 23) and (A. 24), respectively, yields

$$\Pi_{Ass}/P_{ss}{}^{A} = \Pi_{Bss}/P_{ss}{}^{B} = \Pi_{Css}/P_{ss}{}^{C} = (1/\theta)C^{w}{}_{ss}. \tag{A. 32}$$

Substituting (A. 30) and (A. 32) into (A. 26), (A. 27) and (A. 28), respectively, and considering $P_{ss}{}^{B}/P_{ss}{}^{A}=\varepsilon^{B}{}_{ss}/\varepsilon^{A}{}_{ss}$, $P_{ss}{}^{C}/P_{ss}{}^{B}=1/\varepsilon^{B}{}_{ss}$, and $P_{ss}{}^{C}/P_{ss}{}^{A}=1/\varepsilon^{A}{}_{ss}$, yields

$$C^{A}{}_{ss} = ((\theta-1)/\theta)\ell^{sA}{}_{ss}+(1/\theta)C^{w}{}_{ss}, \tag{A. 33}$$

$$C^{B}{}_{ss} = ((\theta-1)/\theta)\ell^{sB}{}_{ss}+(1/\theta)C^{w}{}_{ss}, \tag{A. 34}$$

$$C^{C}{}_{ss} = ((\theta-1)/\theta)\ell^{sC}{}_{ss}+(1/\theta)C^{w}{}_{ss}. \tag{A. 35}$$

Because symmetry, which implies $C^{A}{}_{ss}=C^{B}{}_{ss}=C^{C}{}_{ss}=C^{w}{}_{ss}$, holds, we obtain

$$\ell^{sA}{}_{ss} = \ell^{sB}{}_{ss} = \ell^{sC}{}_{ss} = C^{w}{}_{ss}. \tag{A. 36}$$

In the steady state, the labor-market clearing conditions in country A, B and C reduce to $\ell^{sA}{}_{ss} = n_{ss}\ell_{At}(z,x)$, $\ell^{sB}{}_{ss} = (v_{ss}-n_{ss})\ell_{Bt}(z,x)$, and $\ell^{sC}{}_{ss} = (1-v_{ss})\ell_{Css}(z,x)$, respectively. Substituting the labor demand function in country A, $\ell_{Ass}(z,x)=s_{A}{}^{-1}y_{Ass}(z)=3C^{w}{}_{ss}$, into $\ell^{sA}{}_{ss}=n_{ss}\ell_{Ass}(z,x)$ and considering $\ell^{sA}{}_{ss}=C^{w}{}_{ss}$ yields

$$n_{ss} = 1/3. \tag{A. 37}$$

Similarly, substituting the labor demand function in country B, $\ell_{Bss}(z,x)=s_{B}{}^{-1}y_{Bss}(z)=4C^{w}{}_{ss}$, into $\ell^{sB}{}_{ss}=(v_{ss}-n_{ss})\ell_{Bss}(z,x)$ and considering $\ell^{sB}{}_{ss}=C^{w}{}_{ss}$ and $n_{ss}=1/3$ yields

$$v_{ss} = 2/3. \tag{A. 38}$$

Hence, from (A. 31), (A. 33), (A. 34), (A. 35) and (A. 36), we obtain

$$\begin{aligned}\ell^{sA}{}_{ss} &= \ell^{sB}{}_{ss} = \ell^{sC}{}_{ss} = C^{A}{}_{ss} = C^{B}{}_{ss} = C^{C}{}_{ss} = C^{w}{}_{ss} = y_{Ass}(z) = y_{Bss}(z) = y_{Css}(z)\\ &= ((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}.\end{aligned} \tag{A. 39}$$

Substituting $C^{w}{}_{ss}$ from equation (A. 39) into equation (A. 32) yields the following steady-state levels of real profit for country A-, B-, and C-located firms, which are equal:

$$\Pi_{Ass}/P_{ss}{}^{A} = \Pi_{Bss}/P_{ss}{}^{B} = \Pi_{Css}/P_{ss}{}^{C} = (1/\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}. \tag{A. 40}$$

Appendix B

Long-run equilibrium conditions

The long-run equilibrium conditions of this model are derived. By log-linearizing the model around the initial, zero-shock symmetric steady state with $B_{ss,0}=0$, we obtain the following equations to characterize the long-run equilibrium of the system:

$$\overline{P}^A = \overline{M}^A - \overline{C}^A, \quad \overline{P}^B = \overline{M}^B - \overline{C}^B, \quad \overline{P}^C = \overline{M}^C - \overline{C}^C, \tag{B. 1}$$

$$\overline{C}^A = \delta\overline{B}^A + ((\theta-1)/\theta)(\overline{W}^A - \overline{P}^A + \overline{\ell}^{As}) + (1/3\theta)[\overline{\Pi}^A + \overline{\Pi}^B + \overline{\Pi}^C + 2\overline{\varepsilon}^A - \overline{\varepsilon}^B] - (1/\theta)\overline{P}^A, \tag{B. 2}$$

$$\overline{C}^B = \delta\overline{B}^B + ((\theta-1)/\theta)(\overline{W}^B - \overline{P}^B + \overline{\ell}^{Bs}) + (1/3\theta)[\overline{\Pi}^A + \overline{\Pi}^B + \overline{\Pi}^C - \overline{\varepsilon}^A + 2\overline{\varepsilon}^B] - (1/\theta)\overline{P}^B, \tag{B. 3}$$

$$\overline{C}^C = \delta\overline{B}^C + ((\theta-1)/\theta)(\overline{W}^C - \overline{P}^C + \overline{\ell}^{Cs}) + (1/3\theta)[\overline{\Pi}^A + \overline{\Pi}^B + \overline{\Pi}^C - \overline{\varepsilon}^A - \overline{\varepsilon}^B] - (1/\theta)\overline{P}^C, \tag{B. 4}$$

$$\overline{y}^A = \theta(\overline{P}^A - \overline{P}^A_A) + \overline{C}^W, \quad \overline{y}^B = \theta(\overline{P}^B - \overline{P}^B_B) + \overline{C}^W, \quad \overline{y}^C = \theta(\overline{P}^C - \overline{P}^C_C) + \overline{C}^W, \tag{B. 5}$$

$$\overline{C}^W \equiv (1/3)\overline{C}^A + (1/3)\overline{C}^B + (1/3)\overline{C}^C = (1/3)\overline{y}^A + (1/3)\overline{y}^B + (1/3)\overline{y}^C \equiv \overline{y}^W, \tag{B. 6}$$

$$\overline{n} = (3\gamma/\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}[\overline{\Pi}^A - \overline{\Pi}^B - \overline{\varepsilon}^A + \overline{\varepsilon}^B], \tag{B. 7}$$

$$\overline{v} = (3\gamma/2\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}[\overline{\Pi}^B - \overline{\Pi}^C - \overline{\varepsilon}^B], \tag{B. 8}$$

$$\overline{\Pi}^A = (1-\theta)\overline{P}^A_A + \theta\overline{P}^A + \overline{C}^W, \tag{B. 9}$$

$$\overline{\Pi}^B = (1-\theta)\overline{P}^B_B + \theta\overline{P}^B + \overline{C}^W, \tag{B. 10}$$

$$\overline{\Pi}^C = (1-\theta)\overline{P}^C_C + \theta\overline{P}^C + \overline{C}^W, \tag{B. 11}$$

$$\overline{y}^A = \overline{\ell}^{Ad}, \quad \overline{y}^B = \overline{\ell}^{Bd}, \quad \overline{y}^C = \overline{\ell}^{Cd}, \tag{B. 12}$$

$$\overline{\ell}^{As} = \overline{n} + \overline{\ell}^{Ad}, \quad \overline{\ell}^{Bs} = 2\overline{v} - \overline{n} + \overline{\ell}^{Bd}, \quad \overline{\ell}^{Cs} = -2\overline{v} + \overline{\ell}^{cd}, \tag{B. 13}$$

$$\overline{P}^A_A = \overline{W}^A, \quad \overline{P}^B_B = \overline{W}^B, \quad \overline{P}^C_C = \overline{W}^C, \tag{B. 14}$$

$$\overline{P}^A - \overline{P}^B = \overline{\varepsilon}^A - \overline{\varepsilon}^B, \quad \overline{P}^B - \overline{P}^C = \overline{\varepsilon}^B, \quad \overline{P}^A - \overline{P}^C = \overline{\varepsilon}^A \tag{B. 15}$$

$$\overline{\ell}^{As} = \overline{W}^A - \overline{P}^A - \overline{C}^A, \quad \overline{\ell}^{Bs} = \overline{W}^B - \overline{P}^B - \overline{C}^B, \quad \overline{\ell}^{Cs} = \overline{W}^C - \overline{P}^C - \overline{C}^C, \tag{B. 16}$$

where $\overline{B} \equiv dB_{t+1}/C^W_{ss,0}$, in which $C^W{}_{ss,0}$ is the initial value of world consumption. The equations in (B. 1) correspond to the money-demand equations. Equations (B. 2), (B. 3) and (B. 4) represent the long-run change in incomes (returns on real bonds, real labor incomes, and real profit incomes), which are equal to the long-run changes in consumption in each country. The equations in (B. 5) represent the world demand schedules for home and foreign products. Equation (B. 6) is the world goods-market equilibrium condition. Equation (B. 7) and (B. 8) are the dynamic relocation equation. The equations in (B. 9, 10, 11) are the nominal profit equations for firms. The equations in (B. 12) represent the production technology, and those in (B. 13) represent the long-run labor-market clearing conditions for both countries. The equations in (B. 14) represent the optimal pricing equations for firms in each country. Equation (B. 15) is the purchasing power parity equation. The equations in

(B. 16) represent the first-order conditions for optimal wage setting.

Subtracting (B. 3) from (B. 2) yields the long-run response of relative per capita consumption levels,

$$\overline{C}^A-\overline{C}^B = (\delta/P^C)(\overline{B}^A-\overline{B}^B)+((\theta-1)/\theta)(\overline{\ell}^A-\overline{\ell}^B)+((\theta-1)/\theta)(\overline{W}^A-\overline{W}^B-\overline{P}^A+\overline{P}^B), \tag{B.17}$$

Subtracting (B. 4) from (B. 3) yields the long-run response of relative per capita consumption levels,

$$\overline{C}^B-\overline{C}^C = (\delta/P^C)(\overline{B}^B-\overline{B}^C)+((\theta-1)/\theta)(\overline{\ell}^B-\overline{\ell}^C)+((\theta-1)/\theta)(\overline{W}^B-\overline{W}^C-\overline{P}^B+\overline{P}^C), \tag{B.18}$$

Substituting (B. 9), (B. 10), (B. 11), (B. 14), and (B. 15) into equations (B. 7) and (B. 8), respectively, yields

$$\overline{n} = 3\gamma\theta_1[\overline{\varepsilon}^A-\overline{\varepsilon}^B-(\overline{W}^A-\overline{W}^B)], \tag{B.19}$$

$$\overline{v} = (3\gamma/2)\theta_1[\overline{\varepsilon}^B-(\overline{W}^B-\overline{W}^C)]. \tag{B.20}$$

From equations (B. 5), (B. 12), (B. 13), (B. 14), and (B. 15), we obtain

$$\overline{\ell}^A-\overline{\ell}^B = 2(\overline{n}-\overline{v})+\theta[\overline{\varepsilon}^A-\overline{\varepsilon}^B-(\overline{W}^A-\overline{W}^B)], \tag{B.21}$$

$$\overline{\ell}^B-\overline{\ell}^C = 4\overline{v}-\overline{n}+\theta[\overline{\varepsilon}^B-(\overline{W}^B-\overline{W}^C)]. \tag{B.22}$$

From equations (B. 15) and (B. 16), we obtain

$$\overline{\ell}^A-\overline{\ell}^B+\overline{C}^A-\overline{C}^B = \overline{W}^A-\overline{W}^B-(\overline{\varepsilon}^A-\overline{\varepsilon}^B), \tag{B.23}$$

$$\overline{\ell}^B-\overline{\ell}^C+\overline{C}^B-\overline{C}^C = \overline{W}^B-\overline{W}^C-\overline{\varepsilon}^B. \tag{B.24}$$

From (B. 15), (B. 17) and (B. 18),

$$\overline{C}^A-\overline{C}^B = (\delta/P^C)(\overline{B}^A-\overline{B}^B)+((\theta-1)/\theta)(\overline{\ell}^A-\overline{\ell}^B)+((\theta-1)/\theta)(\overline{W}^A-\overline{W}^B-(\overline{\varepsilon}^A+\overline{\varepsilon}^B)), \tag{B.25}$$

$$\overline{C}^B-\overline{C}^C = (\delta/P^C)(\overline{B}^B-\overline{B}^C)+((\theta-1)/\theta)(\overline{\ell}^B-\overline{\ell}^C)+((\theta-1)/\theta)(\overline{W}^B-\overline{W}^c-\overline{\varepsilon}^B) \tag{B.26}$$

Substituting (B. 23) into (B. 25) yields

$$\overline{C}^A-\overline{C}^B = (\delta/P^C)(\overline{B}^A-\overline{B}^B)+((\theta-1)/\theta)(\overline{\ell}^A-\overline{\ell}^B)+((\theta-1)/\theta)(\overline{\ell}^A-\overline{\ell}^B+\overline{C}^A-\overline{C}^B) \tag{B.27}$$

Substituting (B. 24) into (B. 26) yields

$$\overline{C}^B - \overline{C}^C = (\delta/P^C)(\overline{B}^B - \overline{B}^C) + ((\theta-1)/\theta)(\overline{\ell}^B - \overline{\ell}^C) + ((\theta-1)/\theta)(\overline{\ell}^B - \overline{\ell}^C + \overline{C}^B - \overline{C}^C) \tag{B. 28}$$

Substituting (B. 23) into (B. 19) yields

$$\overline{n} = -3\gamma\theta_1[\overline{\ell}^A - \overline{\ell}^B + \overline{C}^A - \overline{C}^B]. \tag{B. 29}$$

Substituting (B. 24) into (B. 20) yields

$$\overline{v} = -(3\gamma/2)\theta_1|\overline{\ell}^B - \overline{\ell}^C + \overline{C}^B - \overline{C}^C]. \tag{B. 30}$$

Substituting (B. 23), (B. 29), and (B. 30) into (B. 21) yields

$$(1+6\gamma\theta_1+\theta)(\overline{\ell}^{As} - \overline{\ell}^{Bs}) = -(6\gamma\theta_1+\theta)(\overline{C}^A - \overline{C}^B) + 3\gamma\theta_1[\overline{\ell}^B - \overline{\ell}^C + \overline{C}^B - \overline{C}^C]. \tag{B. 31}$$

Substituting (B. 24), (B. 29), and (B. 30) into (B. 22) yields

$$\overline{\ell}^B - \overline{\ell}^C = -\left[\frac{6\gamma\theta_1+\theta}{1+6\gamma\theta_1+\theta}\right](\overline{C}^B - \overline{C}^C) + \left[\frac{3\gamma\theta_1}{1+6\gamma\theta_1+\theta}\right](\overline{\ell}^A - \overline{\ell}^B + \overline{C}^A - \overline{C}^B). \tag{B. 32}$$

Substituting (B. 32) into (B. 31) yields

$$\begin{aligned}&\overline{\ell}^A - \overline{\ell}^B \\ &= -\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta) - 9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2 - 9\gamma^2\theta_1^2}\right](\overline{C}^A - \overline{C}^B) + \left[\frac{3\gamma\theta_1}{(1+6\gamma\theta_1+\theta)^2 - 9\gamma^2\theta_1^2}\right](\overline{C}^B - \overline{C}^c).\end{aligned} \tag{B. 33}$$

Substituting (B. 33) into (B. 27) yields

$$\begin{aligned}&\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta) - 9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2 - 9\gamma^2\theta_1^2}\right] - \tilde{\theta}\right\}(\overline{C}^A - \overline{C}^B) \\ &= \frac{\delta}{P^C}(\overline{B}^A - \overline{B}^B) + \left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2 - 9\gamma^2\theta_1^2}\right](\overline{C}^B - \overline{C}^C)\end{aligned} \tag{B. 34}$$

Substituting (B. 32) and (B. 33) into (B. 28) yields

$$\begin{aligned}&\left\{1+2\tilde{\theta}\left[\frac{6\gamma\theta_1+\theta}{1+6\gamma\theta_1+\theta}\right] - \tilde{\theta} - 2\tilde{\theta}\left[\frac{3\gamma\theta_1}{1+6\gamma\theta_1+\theta}\right]\left[\frac{3\gamma\theta_1}{(1+6\gamma\theta_1+\theta)^2 - 9\gamma^2\theta_1^2}\right]\right\}(\overline{C}^B - \overline{C}^C) \\ &= \frac{\delta}{P^c}(\overline{B}^B - \overline{B}^C) + \left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2 - 9\gamma^2\theta_1^2}\right](\overline{C}^A - \overline{C}^B)\end{aligned} \tag{B. 35}$$

Short-run equilibrium conditions

The short-run equilibrium conditions of this model are derived. By log-linearizing the model around the initial, zero-shock symmetric steady state with $B_{ss,0}=0$, we obtain the following equations to characterize the short-run equilibrium of the system:

$$\overline{C}^A = \widehat{C}^A+(\delta/(1+\delta))\overline{r}+\overline{\varepsilon}^A-\widehat{\varepsilon}^A, \quad \text{(B. 36)}$$

$$\overline{C}^B = \widehat{C}^B+(\delta/(1+\delta))\overline{r}+\overline{\varepsilon}^B-\widehat{\varepsilon}^B, \quad \text{(B. 37)}$$

$$\overline{C}^C = \widehat{C}^C+(\delta/(1+\delta))\overline{r}, \quad \text{(B. 38)}$$

$$\widehat{M}^A-\widehat{P}^A = \widehat{C}^A-\overline{r}/(1+\delta)-(\overline{P}^A-\widehat{P}^A)/\delta-\overline{\varepsilon}^A/\delta+\widehat{\varepsilon}^A/\delta, \quad \text{(B. 39)}$$

$$\widehat{M}^B-\widehat{P}^B = \widehat{C}^B-\overline{r}/(1+\delta)-(\overline{P}^B-\widehat{P}^B)/\delta-\overline{\varepsilon}^B/\delta+\widehat{\varepsilon}^B/\delta, \quad \text{(B. 40)}$$

$$\widehat{M}^C-\widehat{P}^C = \widehat{C}^C-\overline{r}/(1+\delta)-(\overline{P}^C-\widehat{P}^C)/\delta, \quad \text{(B. 41)}$$

$$\overline{B}^A/P^C = -(\theta-1/\theta)\widehat{P}^A+((\theta-1)/\theta)(\widehat{n}+\widehat{\ell}^{Ad})$$
$$+(1/3\theta)[\widehat{\Pi}^A+\widehat{\Pi}^B+\widehat{\Pi}^C+2\widehat{\varepsilon}^A-\widehat{\varepsilon}^B-3\widehat{P}^A]-\widehat{C}^A, \quad \text{(B. 42)}$$

$$\overline{B}^B/P^C = -(\theta-1/\theta)\widehat{P}^B+((\theta-1)/\theta)(2\widehat{v}-\widehat{n}+\widehat{\ell}^{Bd})$$
$$+(1/3\theta)[\widehat{\Pi}^A+\widehat{\Pi}^B+\widehat{\Pi}^C-\widehat{\varepsilon}^A+2\widehat{\varepsilon}^B-3\widehat{P}^B]-\widehat{C}^B, \quad \text{(B. 43)}$$

$$\overline{B}^C/P^C = -(\theta-1/\theta)\widehat{P}^C+((\theta-1)/\theta)(-2\widehat{v}+\widehat{\ell}^{Cd})$$
$$+(1/3\theta)[\widehat{\Pi}^A+\widehat{\Pi}^B+\widehat{\Pi}^C-\widehat{\varepsilon}^A-\widehat{\varepsilon}^B-3\widehat{P}^C]-\widehat{C}^C, \quad \text{(B. 44)}$$

$$\widehat{y}^A = \theta\widehat{P}^A+\widehat{C}^W, \quad \widehat{y}^B = \theta\widehat{P}^B+\widehat{C}^W, \quad \widehat{y}^C = \theta\widehat{P}^C+\widehat{C}^W, \quad \text{(B. 45)}$$

$$\widehat{y}^A = \widehat{\ell}^{Ad}, \quad \widehat{y}^B = \widehat{\ell}^{Bd}, \quad \widehat{y}^C = \widehat{\ell}^{Cd}, \quad \text{(B. 46)}$$

$$\widehat{\Pi}^A = \theta\widehat{P}^A+\widehat{C}^W, \quad \widehat{\Pi}^B = \theta\widehat{P}^B+\widehat{C}^W, \quad \widehat{\Pi}^C = \theta\widehat{P}^C+\widehat{C}^W, \quad \text{(B. 47)}$$

$$\widehat{n} = (3\gamma/\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}[\widehat{\Pi}^A-\widehat{\Pi}^B-\widehat{\varepsilon}^A+\widehat{\varepsilon}^B], \quad \text{(B. 48)}$$

$$\widehat{v} = (3\gamma/2\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}[\widehat{\Pi}^B-\widehat{\Pi}^C-\widehat{\varepsilon}^B], \quad \text{(B. 49)}$$

$$\widehat{C}^W \equiv (1/3)\widehat{C}^A+(1/3)\widehat{C}^B+(1/3)\widehat{C}^C = (1/3)\widehat{y}^A+(1/3)\widehat{y}^B+(1/3)\widehat{y}^C \equiv \widehat{y}^W, \quad \text{(B. 50)}$$

$$\widehat{P}^A = (2/3)\widehat{\varepsilon}^A-(1/3)\widehat{\varepsilon}^B, \quad \widehat{P}^B = -(1/3)\widehat{\varepsilon}^A+(2/3)\widehat{\varepsilon}^B, \quad \widehat{P}^C = -(1/3)\widehat{\varepsilon}^A-(1/3)\widehat{\varepsilon}^B, \quad \text{(B. 51)}$$

$$\widehat{\ell}^{As} = \widehat{n}+\widehat{\ell}^{Ad}, \quad \widehat{\ell}^{Bs} = 2\widehat{v}-\widehat{n}+\widehat{\ell}^{Bd}, \quad \widehat{\ell}^{Cs} = -2\widehat{v}+\widehat{\ell}^{Cd}, \quad \text{(B. 52)}$$

where we set nominal wages and prices of goods as $\widehat{W}^h=\widehat{P}^j_j(z)=0$, $h, j=A, B, C$, for the above short-run log-linearized equations. The equations in (B. 36, 37, 38) are the Euler equations. The equations in (B. 39, 40, 41) describe equilibrium in the money markets in the short run. The equations in (B. 42, 43, 44) are linearized short-run current account equations. The equations in (B. 45) represent the world demand schedules for representative country j products ($j=A, B, C$). Equation (B. 46) is the production function. The equations in (B. 47) are the nominal profit equations for representative country j firms ($j=A, B, C$). Equation (B. 48) and (B. 49) are the dynamic relocation equation. Equation (B. 50) is the world goods-market equilibrium condition. Equation (B. 51) is the price index equation in the short run.

The equations in (B. 52) represent the short-run labor-market clearing conditions for both countries. Subtracting (B. 43) from (B. 42) yields

$$(\overline{B}^A-\overline{B}^B)/P^C = -((\theta-1)/\theta)(\widehat{P}^A-\widehat{P}^B)+2((\theta-1)/\theta)(\widehat{n}-\widehat{v})+((\theta-1)/\theta)(\widehat{\ell}^{Ad}-\widehat{\ell}^{Bd}) + (1/\theta)(\widehat{\varepsilon}^A-\widehat{\varepsilon}^B-\widehat{P}^A+\widehat{P}^B)-(\widehat{C}^A-\widehat{C}^B) \quad \text{(B. 53)}$$

Substituting (B. 47) and (B. 51) into (B. 48) and (B. 49), respectively, yields

$$\widehat{n} = 3\gamma((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{3/2}(1/\kappa)^{1/2}(\widehat{\varepsilon}^A-\widehat{\varepsilon}^B), \quad \text{(B. 54)}$$

$$\widehat{v} = (3\gamma/2)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{3/2}(1/\kappa)^{1/2}\widehat{\varepsilon}^B. \quad \text{(B. 55)}$$

From equations (B. 45), (B. 46), and (B. 51), we obtain the relative labor demand

$$\widehat{\ell}^{Ad}-\widehat{\ell}^{Bd} = \theta(\widehat{P}^A-\widehat{P}^B) = \theta(\widehat{\varepsilon}^A-\widehat{\varepsilon}^B). \quad \text{(B. 56)}$$

By subtracting (B. 55) from (B. 54), we obtain

$$\widehat{n}-\widehat{v} = 3\gamma\theta_1(\widehat{\varepsilon}^A-\widehat{\varepsilon}^B)-(3/2)\gamma\theta_1\widehat{\varepsilon}^B. \quad \text{(B. 57)}$$

The derivation of $\widehat{\varepsilon}^h$, $\widehat{C}^h-\widehat{C}^{h^*}$, $\widehat{n}$ *and* $\widehat{v}$ $(h=A, B, C)$

Substituting (B. 51), (B. 56), and (B. 57) into (B. 53) yields

$$(\overline{B}^A-\overline{B}^B)/P^C = \tilde{\theta}(\theta-1)(\widehat{\varepsilon}^A-\widehat{\varepsilon}^B)+2\tilde{\theta}[3\gamma\theta_1(\widehat{\varepsilon}^A-\widehat{\varepsilon}^B)-(3/2)\gamma\theta_1\widehat{\varepsilon}^B]-(\widehat{C}^A-\widehat{C}^B). \quad \text{(B. 58)}$$

From (B. 36), (B. 37), and (B. 38)

$$\overline{C}^A-\overline{C}^B = \widehat{C}^A-\widehat{C}^B, \quad \text{(B. 59)}$$

$$\overline{C}^B-\overline{C}^C = \widehat{C}^B-\widehat{C}^C. \quad \text{(B. 60)}$$

Substituting (B. 59) and (B. 60) into (B. 34) yields

$$\frac{1}{P^C}(\overline{B}^A-\overline{B}^B) = \delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}(\widehat{C}^A-\widehat{C}^B) - \delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\widehat{C}^B-\widehat{C}^C) \quad \text{(B. 61)}$$

Substituting (B. 61) into (B. 58) yields

$$\left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1\right\}(\widehat{C}^A-\widehat{C}^B) - \delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\widehat{C}^B-\widehat{C}^C)$$

$$= \tilde{\theta}(\theta-1)(\hat{\varepsilon}^A-\hat{\varepsilon}^B)+2\tilde{\theta}[3\gamma\theta_1(\hat{\varepsilon}^A-\hat{\varepsilon}^B)-(3/2)\gamma\theta_1\hat{\varepsilon}^B]. \tag{B. 62}$$

From (B. 39), (B. 40), (B. 41), (B. 51), (B. 59) and (B. 60),

$$\hat{\varepsilon}^A-\hat{\varepsilon}^B = \widehat{M}^A-\widehat{M}^B-(\widehat{C}^A-\widehat{C}^B), \tag{B. 63}$$

$$\hat{\varepsilon}^B = \widehat{M}^B-\widehat{M}^C-(\widehat{C}^B-\widehat{C}^C) \tag{B. 64}$$

From (B. 62), (B. 63) and (B. 64), we obtain

$$\left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1\right\}(\widehat{M}^A-\widehat{M}^B)$$

$$-\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\widehat{M}^B-\widehat{M}^C)$$

$$= \left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1+\tilde{\theta}(\theta-1)+6\gamma\theta_1\tilde{\theta}\right\}(\hat{\varepsilon}^A-\hat{\varepsilon}^B)$$

$$-\left\{\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]+3\gamma\theta_1\tilde{\theta}\right\}\hat{\varepsilon}^B \tag{B. 65}$$

(B. 65) can be rewritten as

$$M_1(\widehat{M}^A-\widehat{M}^B)+M_2(\widehat{M}^B-\widehat{M}^C) = \alpha_1(\hat{\varepsilon}^A-\hat{\varepsilon}^B)+\beta_1\hat{\varepsilon}^B, \tag{B. 66}$$

where

$$M_1 = \delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1 > 0, \tag{B. 67}$$

$$M_2 = -\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right] < 0, \tag{B. 68}$$

$$\alpha_1 = M_1+\tilde{\theta}(\theta-1)+6\gamma\theta_1\tilde{\theta}, \tag{B. 69}$$

$$\beta_1 = M_2-3\gamma\theta_1\tilde{\theta}. \tag{B. 70}$$

Subtracting (B. 44) from (B. 43) and considering (B. 45), (B. 46), (B. 51), (B. 54) and (B. 55) yields

$$(\overline{B}^B-\overline{B}^C)/P^C = 6\gamma\theta_1\tilde{\theta}\hat{\varepsilon}^B-3\gamma\theta_1\tilde{\theta}(\hat{\varepsilon}^A-\hat{\varepsilon}^B)+\tilde{\theta}(\theta-1)\hat{\varepsilon}^B-(\widehat{C}^B-\widehat{C}^C). \tag{B. 71}$$

Substituting (B. 59) and (B. 60) into (B. 35) yields

$$\frac{1}{P^C}(\overline{B}^B-\overline{B}^C) = \delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{6\gamma\theta_1+\theta}{1+6\gamma\theta_1+\theta}\right]\right.$$

$$\left.-\tilde{\theta}-2\tilde{\theta}\left[\frac{3\gamma\theta_1}{1+6\gamma\theta_1+\theta}\right]\left[\frac{3\gamma\theta_1}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]\right\}(\widehat{C}^B-\widehat{C}^C)$$

$$-\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\widehat{C}^A-\widehat{C}^B) \tag{B. 72}$$

Substituting (B. 72) into (B. 71) yields

$$\left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{6\gamma\theta_1+\theta}{1+6\gamma\theta_1+\theta}\right]-\tilde{\theta}-2\tilde{\theta}\left[\frac{3\gamma\theta_1}{1+6\gamma\theta_1+\theta}\right]\left[\frac{3\gamma\theta_1}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]\right\}+1\right\}(\widehat{C}^B-\widehat{C}^C)$$

$$-\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\widehat{C}^A-\widehat{C}^B) = 6\gamma\theta_1\tilde{\theta}\tilde{\varepsilon}^B-3\gamma\theta_1\tilde{\theta}(\tilde{\varepsilon}^A-\tilde{\varepsilon}^B)+\tilde{\theta}(\theta-1)\tilde{\varepsilon}^B \tag{B. 73}$$

From (B. 63), (B. 64) and (B. 73), we obtain

$$-\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\widehat{M}^A-\widehat{M}^B)+$$

$$\left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{6\gamma\theta_1+\theta}{1+6\gamma\theta_1+\theta}\right]-\tilde{\theta}-2\tilde{\theta}\left[\frac{3\gamma\theta_1}{1+6\gamma\theta_1+\theta}\right]\left[\frac{3\gamma\theta_1}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]\right\}+1\right\}(\widehat{M}^B-\widehat{M}^C)$$

$$= -\left\{\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]+3\gamma\theta_1\tilde{\theta}\right\}(\tilde{\varepsilon}^A-\tilde{\varepsilon}^B)+$$

$$\left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{6\gamma\theta_1+\theta}{1+6\gamma\theta_1+\theta}\right]-\tilde{\theta}-2\tilde{\theta}\left[\frac{3\gamma\theta_1}{1+6\gamma\theta_1+\theta}\right]\left[\frac{3\gamma\theta_1}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]\right\}\right.$$

$$\left.+1+6\gamma\theta_1\tilde{\theta}+\tilde{\theta}(\theta-1)\right\}\tilde{\varepsilon}^B \tag{B. 74}$$

(B. 74) can be rewritten as

$$M_3(\widehat{M}^A-\widehat{M}^B)+M_4(\widehat{M}^B-\widehat{M}^C) = \alpha_2(\tilde{\varepsilon}^A-\tilde{\varepsilon}^B)+\beta_2\tilde{\varepsilon}^B, \tag{B. 75}$$

where

$$M_4 = M_1 = \delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1 > 0, \tag{B. 76}$$

$$M_3 = M_2 = -\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right] < 0, \tag{B. 77}$$

$$\alpha_2 = M_3-3\gamma\theta_1\tilde{\theta} = \beta_1, \tag{B. 78}$$

$$\beta_2 = M_4+\tilde{\theta}(\theta-1)+6\gamma\theta_1\tilde{\theta} = M_1+\tilde{\theta}(\theta-1)+6\gamma\theta_1\tilde{\theta} = \alpha_1 \tag{B. 79}$$

From (B. 66) and (B. 75), we obtain

$$\tilde{\varepsilon}^A-\tilde{\varepsilon}^B = \left[\frac{\beta_1M_3-\beta_2M_1}{\alpha_2\beta_1-\alpha_1\beta_2}\right](\widehat{M}^A-\widehat{M}^B)+\left[\frac{\beta_1M_4-\beta_2M_2}{\alpha_2\beta_1-\alpha_1\beta_2}\right](\widehat{M}^B-\widehat{M}^C), \tag{B. 80}$$

$$\tilde{\varepsilon}^B = -\left[\frac{\alpha_2M_1-\alpha_1M_3}{\alpha_2\beta_1-\alpha_1\beta_2}\right](\widehat{M}^A-\widehat{M}^B)+\left[\frac{\alpha_2M_2-\alpha_1M_4}{\alpha_2\beta_1-\alpha_1\beta_2}\right](\widehat{M}^B-\widehat{M}^C). \tag{B. 81}$$

From $M_1=M_4$, $M_2=M_3$, $\alpha_1=\beta_2$, *and* $\alpha_2=\beta_1$, (B. 80) and (B. 81) can be rewritten as

$$\hat{\varepsilon}^A-\hat{\varepsilon}^B=\left[\frac{\alpha_2 M_2-\alpha_1 M_1}{(\alpha_2)^2-(\alpha_1)^2}\right](\widehat{M}^A-\widehat{M}^B)+\left[\frac{\alpha_2 M_1-\alpha_1 M_2}{(\alpha_2)^2-(\alpha_1)^2}\right](\widehat{M}^B-\widehat{M}^C), \qquad \text{(B. 82)}$$

$$\hat{\varepsilon}^B=\left[\frac{\alpha_2 M_1-\alpha_1 M_2}{(\alpha_2)^2-(\alpha_1)^2}\right](\widehat{M}^A-\widehat{M}^B)+\left[\frac{\alpha_2 M_2-\alpha_1 M_1}{(\alpha_2)^2-(\alpha_1)^2}\right](\widehat{M}^B-\widehat{M}^C). \qquad \text{(B. 83)}$$

The relative consumption changes are

$$\widehat{C}^A-\widehat{C}^B=\widehat{M}^A-\widehat{M}^B-(\hat{\varepsilon}^A-\hat{\varepsilon}^B), \qquad \text{(B. 84)}$$

$$\widehat{C}^B-\widehat{C}^C=\widehat{M}^B-\widehat{M}^C-\hat{\varepsilon}^B. \qquad \text{(B. 85)}$$

From (B. 54) and (B. 55), the log-linearized expressions for the international distribution of firms are

$$\hat{n}=3\gamma((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{3/2}(1/\kappa)^{1/2}(\hat{\varepsilon}^A-\hat{\varepsilon}^B), \qquad \text{(B. 86)}$$

$$\hat{v}=(3\gamma/2)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{3/2}(1/\kappa)^{1/2}\hat{\varepsilon}^B. \qquad \text{(B. 87)}$$

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Notes

1) For a survey of the literature examining determinants of foreign direct investment, see Blonigen (2005).
2) Johdo (2019) also investigates the macroeconomic effects of a tariff based on the model of Johdo (2015b).
3) Empirical evidence finds that the number of firms is reacting to monetary policy shocks (see e. g., Bergin and Corsetti, 2006).
4) In Corsetti et al. (2000), the elasticity of substitution between the Center and the Periphery countries in the consumption basket are different from the elasticity of substitution between the two Periphery consumption indexes. The analysis of this paper differs from that of Corsetti et al. (2000) in that we assume the two types of elasticity are equal, for simplicity.
5) By using a closed economy model with firm entry, Bergin and Corsetti (2006) show numerically that there is some persistence in the effect of monetary expansion on entry of firms.

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