Cascading Effects of Government Spending

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Abstract

In this paper, we extend a new open economy macroeconomic model to a three-country framework including relocation of firms and explore the effects of government spending in each country on relative consumption and exchange rates. The feature of this model is that firms in one country can only relocate to the adjacent country, but not the distant country. This relocation structure leads to cascading effects on three countries through the interaction between exchange rate movements and relocation of firms. This allows us to show that a government spending rise in country A induces relocation of firms from country B to country A, which in turn induces relocation of firms from country C to country B. The main conclusion is that a government spending rise in a country always depreciates that country's currency, causes firms located abroad to relocate to that country and consequently decreases that country's relative consumption. In contrast, its government spending can be beneficial for other countries.

1. Introduction

The theoretical relationship between fiscal policy shocks and aggregate economic activity has been studied extensively in the new open economy macroeconomics (NOEM) literature, e.g., the works of Obstfeld and Rogoff (1995, 1996), Betts and Devereux (2000), Caselli (2001), Corsetti and Pesenti (2001), Cavallo and Ghironi (2002), Ganelli (2003, 2005, 2008, 2010), Lombardo and Sutherland (2004), Chu (2005), Evers (2006), Johdo (2013), Ganelli and Tervala (2010), and Di Giorgio et al (2015, 2018).[1] This literature has focused on how the macroeconomic activity of each country and the exchange rate are influenced by unanticipated fiscal shocks under monopolistic distortions and price rigidities.

Since the publication of the works of Obstfeld and Rogoff (1995, 1996), most NOEM models have assumed that firms are immobile across countries. Although it is feasible to explore the effects of fiscal policy shocks in this framework under the assumption of a fixed international distribution of firms, recent empirical evidence (e.g., Cushman 1985, 1988; Froot and Stein 1991; Campa 1993; Klein and Rosengren 1994; Goldberg and Kolstad 1995; Blonigen 1997; Goldberg and Klein 1998; Kosteletou and Liargovas 2000; Bénassy-quéré et al 2001; Chakrabarti and Scholnick 2002; Farrell et al 2004; Schmidt and Broll, 2009; Takagi and Shi 2011) suggests that exchange rates affect the production locations of firms. In addition, the international shift of production location of firms aiming for higher profits has been expanding rapidly between emerging countries (e.g., China, India, Brazil, etc.) and developed countries (e.g., the United States, Japan, South Korea, etc.).

Despite of the large body of empirical literature on the relationship between exchange rate movements and international relocation of firms, in the theoretical literature on the NOEM, there has been little study of how allowing for international relocation of firms affects the macroeconomic impacts of fiscal policy shocks. One exception is Johdo (2019c), who attempts to present a new NOEM model with international relocation of firms and succeeds in showing explicitly the effects of one country's government spending on consumption of the two countries and the exchange rate.[2] However, because Johdo (2019c) begins with the assumption of a two-country economy, the literature cannot consider how allowing for a third country affects the impacts of a government spending shock on international relocation and other macroeconomic variables, including consumption and the exchange rate.[3]

Given this motivation, this paper investigates the impacts of government spending shocks on the international distribution of firms, the exchange rate, and consumption by extending the two-country framework of Johdo (2019c) to a three-country model. Other related papers that use a similar framework as Johdo (2019c) include Cavallari (2010), Russ (2007) and Zhao and Xing (2006). Based on the NOEM model, Cavallari (2010) determines the number of firms by using free entry conditions where firms can adjust their production location by comparing their prospective profits and entry costs. This relocation structure, however, implies that it is impossible to study how the degree of firm mobility (or the flexibility of international relocation) influences the macroeconomic effects of fiscal shocks, because the number of multinational firms is assumed to be adjusted instantaneously at any point in time.[4] In addition, an issue now arises in her sticky price model: whether the number of firms will always be determined more quickly than the adjustment of nominal prices. This is because in her model the number of firms is determined instantaneously by the free entry conditions in the short run for which nominal rigidities exist. Furthermore, although Cavallari (2010) succeeds in deriving the equilibrium solutions for the nominal exchange rate, firm entry, employment and consumption, the general equilibrium impacts on these macroeconomic variables of monetary shocks rely heavily on numerical simulations. On the other hand, Russ (2007) introduces potential sources of firm heterogeneity that explain why some firms become multinational into an open-economy macroeconomic model and investigates the relationship between exchange rate volatility and horizontal FDI. Russ (2007) shows that unpredictable exchange rate fluctuations could either encourage or discourage FDI, depending on the source of the volatility. Although Russ (2007) shows the effect of home real shocks on the entry of foreign multinationals, she does not consider the general equilibrium effects on the nominal exchange rate and consumption of the real shocks. In addition, Russ (2007) determines the number of firms by using free entry conditions (zero-profit conditions). This implies that it is impossible to study how the degree of firm mobility influences the macroeconomic effects of real shocks analytically. Finally, Zhao and Xing (2006) theoretically analyze the relationship between exchange rates and production locations in the context of a static three-country model. In particular, in this study, they model the production allocation choices of a multinational firm in a three-country context, and investigate the relationship between exchange rates and welfare of each country. However, although they show the effects of exchange rate shocks on productions, employments and profit of the multinational firm under a partial equilibrium system, they do not explore the general equilibrium effects of real shocks such as government spending on exchange rates, consumptions and welfare.

This paper also aims at addressing the above-mentioned issues. For this purpose, we propose a NOEM model that rest on the following five assumptions. First, unlike the model of Cavallari (2010), we assume that each agent in the world consumes a basket composed of tradable goods only and therefore exclude nontradable goods from consumers' preference as in the canonical model of Obstfeld and Rogoff (1995).[5] This allows us to show that government spending shocks not only have a short-run impact under nominal rigidities, but also have a long-run impact through changes in net foreign assets. Therefore, unlike the NOEM model of Cavallari (2010) and the static model of Zhao and Xing (2006), we can analyze the interaction between the short-run dynamics and the long-run dynamics and how this interaction affects the consumer's dynamic behavior.[6] Second, the international relocation of firms is addressed in a framework where some firms can relocate to another country when there is a difference in real profit flows between two countries. This allows us to consider the implications of the flexibility of firm relocation across countries in response to government spending shocks. In addition, the assumption of sluggish adjustment of firm relocation allows us to resolve the issue arising in Cavallari (2010) that the number of foreign multinational firms is determined more quickly than the adjustment of nominal prices. This is because in our model nominal prices are flexible in the long run, but the sluggish adjustment of firm relocation remains even in the long-run equilibrium. Third, unlike the model of Russ (2007) where firms are heterogeneous in terms of productivity, monopolistically competitive firms are assumed to be homogeneous and each product is freely traded across two countries (no transport costs). Fourth, as in the canonical model of Obstfeld and Rogoff (1995), we maintain the assumption of complete pass-through.[7] Fifth, there is no equity bias in the portfolio structure. Instead, the ownership of the shares of all firms is exogenously fixed across three countries. Under this assumption, as is the study of Johdo (2019c), even if there is international relocation of firms, it is possible to consider the influence of government spending shocks without taking compulsory asset redistribution into consideration.[8] As stated above, there are many crucial differences between our model and the works of Cavallari (2010), Russ (2007) and Zhao and Xing (2006). The important point to note is that our paper uses minor modifications of the original Obstfeld and Rogoff (1995) model. Therefore, this paper contributes to the NOEM literature by providing an analytically tractable framework for the analysis of the macroeconomic consequences of government spending shocks even if there is international relocation of firms across three countries. From this analysis, we can show cascading effects of government spending shocks through firm relocation among three countries, and it is found that a permanent government spending shock in one of the three countries always depreciates its currency, causes firms located abroad to relocate to the country and consequently decreases the country's relative consumption in spite of the inflows of foreign firms. In contrast, the government spending shock can be beneficial for other foreign countries through the cascading effects.

The remainder of this paper is structured as follows. Section 2 outlines the features of the model. Section 3 describes the equilibrium. In Sections 4 and 5, we examine the impacts of government spending shocks on the distribution of firms across the three countries, the exchange rate, and consumption. The final section summarizes the findings and concludes.

2. The model

In this section, we construct a perfect-foresight, three-country NOEM model with international relocation of firms and government consumption spending. The three countries are denoted by A, B, and C, respectively. The size of the world population is normalized to unity, and households in countries A and B inhabit the intervals $[0, 1/3]$ and $(1/3, 2/3]$, respectively, and those in country C inhabit the interval $(2/3, 1]$. Therefore, the shares of households in A, B, and C are 1/3, 1/3, and 1/3, respectively. The markets for goods and labor have a monopolistic competition, whereas the markets for money and international bonds are perfectly competitive. On the production side, monopolistically competitive producers exist continuously in the range $[0, 1]$, each of which produces a single differentiated product that is freely tradable. This implies that productive activity cannot be carried out in more than one location. In this model, country A consists of those producers in the interval $[0, m_t]$, country B consists of those producers in the interval $[m_t, n_t]$, and the remaining $[n_t, 1]$ producers are in country C, where m_t and n_t are endogenous variables. Finally, we assume that firms are mobile internationally but their owners are not. Therefore, all profit flows from firms are distributed to their immobile owners according to their share of holdings.

2. 1.　Households

The intertemporal objective function of representative household x in country h at time t, with $h=A, B, C$, is:

$$U^h{}_t(x) = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} (\log C^h{}_\tau(x) + \chi \log(M^h{}_\tau(x)/P^h{}_\tau) - (\kappa/2)(\ell^{sh}{}_\tau(x))^2), \tag{1}$$

where E_t represents the mathematical expectation conditional on the information set made available to household i at time t; β is a constant subjective discount factor $(0<\beta<1)$; $C^h{}_t(x)$ is the consumption index that is defined later; $M^h{}_t(x)/P^h{}_t$ represents real money holdings, where $M^h{}_t(x)$ denotes nominal money balances held at the beginning of period $t+1$, and $P^h{}_t$ is the consumption price index of country h; and $\ell^{sh}{}_t(x)$ is the amount of labor supplied by household x. We assume that there is an international risk-free nominal bond market in which representative households in countries A, B, and C can lend and borrow at the same interest rate and that nominal bonds are denominated in the country C's currency. At each point in time, a typical domestic household faces the following budget constraint:

$$\varepsilon^h{}_t B^h{}_{t+1}(x) + M^h{}_t(x) = (1+i_t)\varepsilon^h{}_t B^h{}_t(x) + M^h{}_{t-1}(x) + W^h{}_t(x)\ell^{sh}{}_t(x) - P^h{}_t C^h{}_t(x)$$

$$- P^h{}_t \tau^h{}_t + \Big((\varepsilon^h{}_t/\varepsilon^A{}_t) \int_0^{m_t} \Pi^A_t(z)\,dz + (\varepsilon^h{}_t/\varepsilon^B{}_t) \int_{m_t}^{n_t} \Pi^B_t(z)\,dz$$

$$+\varepsilon^h{}_t\int_{n_t}^{1}\Pi_t^C(z)dz\Big) \tag{2}$$

where $\varepsilon^h{}_t$ denotes the nominal exchange rate, defined as country h's currency per unit of country C's currency (so that $\varepsilon^C{}_t=1$); $B^h{}_{t+1}(x)$ denotes the nominal bond denominated in the country C's currency held by country h's agent x in period $t+1$; i_t denotes the nominal yield on the bond in terms of the country C's currency; $W^h{}_t(x)\ell^{sh}{}_t(x)$ is nominal labor income, where $W^h{}_t(x)$ denotes the nominal wage rate of labor supplied by household x in period t; $\int_0^{m_t}\Pi_t^A(z)dz$, $\int_{m_t}^{n_t}\Pi_t^B(z)dz$, and $\int_{n_t}^{1}\Pi_t^C(z)dz$ represent the total nominal profit flows of firms located at countries A, B, and C, respectively; $P^h{}_tC^h{}_t(x)$ represents nominal consumption expenditure; and $\tau^h{}_t$ denotes real lump-sum taxes. All variables in (2) are measured in per capita terms. In the government sector, we assume that government spending is purely dissipative and financed by lump-sum taxes and by seigniorage revenues derived from printing the national currency. Hence, the government budget constraint in country h is $G^h{}_t=s^h\tau^h{}_t+[(M^h{}_{t+1}-M^h{}_t)/P^h{}_t]$, where $G^h{}_t$ denotes the government spending in country h, $M^h{}_t$ is aggregate money supply, and s^h denotes the population share of country h in the world population. Countries B and C have an analogous government budget constraint.

In what follows, we mainly focus on the description of country A, because other countries are described analogously. In country A, firm $z\in[0,m_t]$ domestically hires a continuum of differentiated labor inputs and produces a unique product in a single location according to the CES production function:

$$y_{At}(z) = \left((1/3)^{-1/\phi}\int_0^{1/3}\ell_{At}(z,x)^{(\phi-1)/\phi}dx\right)^{\phi/(\phi-1)} \tag{3}$$

where $y_{At}(z)$ denotes the production of firm z in period t; $\ell_{At}(z,x)$ is firm z's input of labor from household x in period t; and $\phi>1$ is the elasticity of input substitution. Given the firm's cost minimization problem, firm z's optimal demand function for labor x is expressed as follows:

$$\ell_{At}(z,x) = (1/3)^{-1}(W_t^A(z)/W_t^A)^{-\phi}y_{At}(z) \tag{4}$$

where $W_t^A\equiv\left((1/3)^{-1}\int_0^{1/3}W_t^A(x)^{(1-\phi)}dx\right)^{1/(1-\phi)}$ is a price index for labor input. Similarly, the other countries' firms have an optimal demand function for labor x that is analogous to equation (4).

2. 1. 1. Definition of consumption basket

The consumption basket of household x living in country h at period t is:

$$C_t^h(x) = \left[\int_0^{m_t} c_{At}^h(z,x)^{(\theta-1)/\theta}dz + \int_{m_t}^{n_t} c_{Bt}^h(z,x)^{(\theta-1)/\theta}dz + \int_{n_t}^{1} c_{Ct}^h(z,x)^{(\theta-1)/\theta}dz\right]^{\theta/(\theta-1)} \quad (5)$$

where $\theta > 1$ is the elasticity of substitution among varieties produced within each country; and $c^h{}_{jt}(z, x)$ denotes consumption by household x located in country h of the good produced by firm z located in country j. From (5), the consumption-based price indexes is defined as:

$$P^h{}_t = \left[\int_0^{m_t} (P_{At}^h(z))^{1-\theta}dz + \int_{m_t}^{n_t} (P_{Bt}^h(z))^{1-\theta}dz + \int_{n_t}^{1} (P_{Ct}^h(z))^{1-\theta}dz\right]^{1/(1-\theta)}$$

where $P^h{}_{jt}(z)$ is the price in country h of the good produced by firm z in country j, $j = A, B, C$.

2. 1. 2. Household decisions

Households maximize the consumption index $C^h{}_t(x)$ subject to a given level of expenditure by optimally allocating differentiated goods produced in the three countries $c^h{}_{jt}(z, x)$, $j = A, B, C$. From this problem, we obtain the following private demand functions:

$$c^h{}_{jt}(z,x) = (P^h{}_{jt}(z)/P^h{}_t)^{-\theta} C^h{}_t(x) \quad (6)$$

In accordance with the NOEM literature, we assume that the consumption index of the government is the same as that of the household sector, which is given by (5). Therefore, the demand functions of the government for good j in the home and foreign countries are the same as those of the household sector. Summing the private and public demand functions and equating the resulting equation to the product of firm z located in country j yields the following market-clearing condition for any product z produced in country j:

$$\begin{aligned} y_{jt}(z) = {} & (P^A{}_{jt}(z)/P^A{}_t)^{-\theta}(C^A{}_t + G^A{}_t) + (P^B{}_{jt}(z)/P^B{}_t)^{-\theta}(C^B{}_t + G^B{}_t) \\ & + (P^C{}_{jt}(z)/P^C{}_t)^{-\theta}(C^C{}_t + G^C{}_t), \end{aligned} \quad (7)$$

where $C^A{}_t = \int_0^{1/3} C_t^A(x)dx$, $C^B{}_t = \int_{1/3}^{2/3} C_t^B(x)dx$, $C^C{}_t = \int_{2/3}^{1} C_t^C(x)dx$, $G^A{}_t = \int_0^{1/3} G_t^A(x)dx$, $G^B{}_t = \int_{1/3}^{2/3} G_t^B(x)dx$, and $G^C{}_t = \int_{2/3}^{1} G_t^C(x)dx$. From the law of one price and the purchasing power parity arising from symmetric preferences, (7) is rewritten as:

$$y_{jt}(z) = (P^j{}_{jt}(z)/P^j{}_t)^{-\theta}(C_t^w + G_t^w) \quad (8)$$

where $C^w{}_t \equiv C^A{}_t + C^B{}_t + C^C{}_t$, $G^w{}_t \equiv G^A{}_t + G^B{}_t + G^C{}_t$. In the second stage, households maximize

(1) subject to (2). The first-order conditions for this problem with respect to $B^h{}_{t+1}(x)$ and $M^h{}_t(x)$ can be written as:

$$1/C^h{}_t(x) = \beta(1+i_{t+1})E_t[(P^h{}_t/\varepsilon^h{}_t)/(P^h{}_{t+1}/\varepsilon^h{}_{t+1})C^h{}_{t+1}(x)], \tag{9}$$

$$M^h{}_t(x)/P^h{}_t = \chi C^h{}_t(x)[(1+i_{t+1})E_t\varepsilon^h{}_{t+1}/((1+i_{t+1})E_t\varepsilon^h{}_{t+1}-\varepsilon^h{}_t)], \tag{10}$$

where i_{t+1} is the nominal interest rate for country C's currency loans between periods t and $t+1$ and defined as usual by $1+i_{t+1}=(1+r_{t+1})E_t[(P^C{}_{t+1}/P^C{}_t)]$ and r_{t+1} denotes the real interest rate. Equation (9) is the Euler equation for consumption, and (10) is for the money-demand schedule.

Following the work of Corsetti and Pesenti (2001), we introduce nominal rigidities into the model in the form of one-period wage contracts. The nominal wages in period t are predetermined at the end of period $t-1$. In monopolistic labor markets, each household provides a single variety of labor input to a continuum of domestic firms. Hence, in country A, the equilibrium labor-market conditions can be expressed as $\ell_t^{sA}(x)=\int_0^{m_t}\ell_{At}(z,x)dz$, $x\in[0, 1/3]$, where the left-hand side represents the amount of labor supplied by household x, and the right-hand side represents firms' total demand for labor x. By taking W_t^A, P_t^A, and m_t as a given, then by substituting $\ell_t^{sA}(x)=\int_0^{m_t}\ell_{At}(z,x)dz$ and equation (4) into the budget constraint given by (2), and finally by maximizing the lifetime utility given by (1) with respect to the nominal wage $W_t^A(x)$, we obtain the following first-order condition for the optimal nominal wage, $W_t^A(x)$:

$$\phi(W_t^A(x)/P_t^A)^{-1}E_{t-1}[\kappa\ell_t^{sA}(x)^2] = (\phi-1)E_{t-1}[(\ell_t^{sA}(x)/C^A{}_t)] \tag{11}$$

The labor suppliers of countries B and C have analogous optimal wage conditions.

2.2. Firm's decision

Since the country A-located firm z domestically hires labor, given $W^A{}_t$, $P^A{}_{At}$, and $P^A{}_t$, $C^w{}_t$, $G^w{}_t$, m_t, (3), subject to (8), the country A-located firm z faces the following profit-maximization problem:

$$\max_{P^A_{At}(z)}\Pi_{At}(z) = P^A{}_{At}(z)y_{At}(z)-\int_0^{1/3}W_t^A(z)\ell_{At}(z,x)dx = (P^A{}_{At}(z)-W^A{}_t)y_{At}(z)$$

$$\text{subject to } y_{At}(z) = (P^A{}_{At}(z)/P^A{}_t)^{-\theta}(C_t^w+G_t^w)$$

Given the above, the price mark-up is chosen according to:

$$P^A{}_{At}(z) = (\theta/(\theta-1)\,W^A{}_t \tag{12}$$

Since $W^A{}_t$ is a given, (12) yields $P^A{}_{At}(z) = P^A{}_{At}$, $z \in [0, m_t]$. Similarly, other firms located in different country have the price mark-up that is analogous to equation (12). By denoting the maximized real profit flows of country j-located firms as $\Pi_{jt}(z)/P^j{}_t$ and by substituting (8) and (12) into $\Pi_{jt}(z)$, we obtain

$$\Pi_{jt}(z)/P^j{}_t = (1/\theta)(P^j{}_{jt}(z)/P^j{}_t)^{1-\theta}(C_t^w + G_t^w) \tag{13}$$

2. 3.　Relocation behavior

The driving force of relocation to other countries is the differences in current real profits between two bounded countries. In particular, in this model, the international relocation of firms is addressed in a framework where some firms can relocate to another country when there is a difference in real profit flows between two adjacent countries. At each point in time, this adjustment mechanism for relocation between countries A and B is formulated as follows:

$$m_t - m_{t-1} = \gamma[\Pi_{At}(z)/P_t^A - \Pi_{Bt}(z)/P_t^B] = \gamma[\Pi_{At}(z)/P_t^A - (\varepsilon^A{}_t/\varepsilon^B{}_t)\Pi_{Bt}(z)/P_t^A] \tag{14}$$

Analogously, the adjustment mechanism for relocation between countries B and C is formulated as follows:

$$n_t - n_{t-1} = \gamma[\Pi_{Bt}(z)/P_t^B - \Pi_{Ct}(z)/P_t^C] = \gamma[\Pi_{Bt}(z)/P_t^B - \varepsilon^B{}_t\Pi_{Ct}(z)/P_t^B] \tag{15}$$

where $\gamma(0 \leq \gamma < \infty)$ is a constant positive parameter that determine the degree of firm mobility between two bounded countries: a larger value of γ implies higher firm mobility between countries.[9] The advantage of the above specific spatial structure of relocation of firms is as follows. First, from this framework, even if both the short-run equilibrium with nominal rigidities and the long-run equilibrium with flexible prices are taken into consideration, we can show analytically the general equilibrium effects of fiscal shocks. Secondly, if we do not assume such special relocation structure, we have to determine the number of firms by using free entry conditions where firms can adjust their production location by comparing their net profits among each location. However, this implies that in our sticky price model the international distribution of firms always adjusts more quickly than the adjustment of nominal prices at least in the short-run equilibrium, because the number of firms is determined instantaneously by the free entry conditions. We believe that it may not be supported in the real world that the international distribution of firms is determined more quickly than the adjustment of nominal prices. Therefore, to avoid this issue, our model needs to adopt the above specific spatial structure. Thirdly, there is a large empirical literature that shows the essential role of exchange rate depreciation (appreciation) in explaining inward (outward) FDI (see, e.g., Cushman, 1988, Caves, 1989, Froot and Stein, 1991, Campa, 1993, Swenson, 1993, Klein and Rosengren, 1994, Dewenter, 1995, Grosse and Trevino, 1996, Kogut and Chang, 1996, Blonigen, 1997, Bayoumi and Lipworth, 1998, Goldberg and Klein, 1998, Gopinath et al, 1998, Bénassy-quéré et al, 2001, Chakrabarti and Scholnick, 2002, Pain and Van Welsum, 2003, Kiyota and Urata, 2004, Bolling et al, 2007, Ang, 2008, Phillips and Ahmadi-Esfahani, 2008, Xing and Zhao, 2008, Osinubi and Amaghionyeodiwe, 2009, Udomkerdmongkol et al, 2009, and Takagi and Shi, 2011). Under our special spatial structure, we can show the results that the domestic currency depreciation (appreciation) induces global relocation of firms toward (away from) the domestic country, which is consistent with the above empirical evidences. Fourthly, from this special spatial structure, we can parameterize the degree of firm mobility ($=\gamma$) so that it is easy to see if and how the derived results depend on the degree of firm mobility. For example, as can be seen in section 4, equations (22) and (23) show that nominal exchange rate changes have greater effects on the relocation of firms as the larger is γ. Finally, as our model builds on this specific spatial structure, we can show cascading effects such that a given country's government spending can influence the international distribution of production, not only bilaterally, but also between third parties. In the standard monopolistic competition literature, the equilibrium is defined as the situation where the free-entry condition is imposed. Instead of the spatial structure as equations (17) and (18), if we assume the free-entry condition of firms, a rise in government spending changes m_t and n_t simultaneously. These simultaneous changes in m_t and n_t mean that firms in country A (C) can directly relocate to "distant" country C (A). This result implies that, under the free-entry condition of firms, the present model cannot generate the cascading effects through the international relocation of firms. On the other hand, in our model, the gradual firm relocation mechanism determines which firms are located across two "adjacent" countries first, and then the relocation of firms between third parties is determined. As can be seen in Section 5, from the assumption of the special relocation structure, we can show that a rise in government spending in country A first leads to the increase in the relative profit of firms located in country A (that is, $\Pi^{T}{}_{At}(z)/P_t^{AT} > \Pi^{T}{}_{Bt}(z)/P_t^{BT}$), and then leads to the relocation of some firms away from countries B to country A from equation (17). This firm relocation in turn increases the relative profits of firms located in country B because of the depreciation of country B's currency (that is, $\Pi^{T}{}_{Bt}(z)/P_t^{BT} > \Pi^{T}{}_{Ct}(z)/P_t^{CT}$), which causes firms located in country C to relocate to country B.

For the reasons given above, this paper contributes to the new open economy macroeconomics literature by providing an analytically tractable framework for the analysis of the macroeconomic consequences of government spending shocks under international relocation of firms among three countries. In addition, the main advantage of our specification on the spatial structure is that it is possible to study the implications of government spending under international firm mobility yet maintaining compatibility with the evidence found in the empirical literature on the relationship between exchange rate appreciation and outward FDI.

2. 4.　Market conditions

The equilibrium condition for the integrated international bond market is given by:

$$\int_0^{1/3} B_t^A(x)dx + \int_{1/3}^{2/3} B_t^B(x)dx + \int_{2/3}^{1} B_t^C(x)dx = 0 \tag{16}$$

Money markets are always assumed to be clear in all countries. Hence, the equilibrium conditions are given by $M^A{}_t = \int_0^{1/3} M_t^A(x)dx$, $M^B{}_t = \int_{1/3}^{2/3} M_t^B(x)dx$, and $M^C{}_t = \int_{2/3}^{1} M_t^C(x)dx$.

3. Steady state values

In this section, we derive the solution for a symmetric steady state in which all variables are constant, the initial net foreign assets are zero ($B^h{}_0=0$) and $G^h{}_0=0$, $h=A, B, C$. In the symmetric steady state, we drop the index value "x" from all variables in order to simplify notation. Then, we denote the steady-state values by using the subscript ss. In the symmetric steady state, given the Euler equation for consumption (equation (9)), the constant real interest rate is given by:

$$r_{ss} = (1-\beta)/\beta \equiv \delta \tag{17}$$

where δ is the rate of time preference. Because symmetry, which implies $C^h{}_{ss} = C^w{}_{ss}$, holds, the steady-state international allocations of firms are:

$$m_{ss} = 1/3 \tag{18}$$

$$n_{ss} = 2/3 \tag{19}$$

The steady state output levels are:

$$y_{jss}=\ell^{sh}{}_{ss}=C^{h}{}_{ss}=C^{w}{}_{ss}=((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2},\quad j,h=A,B,C \tag{20}$$

Substituting $C^{w}{}_{ss}$ from equation (20) into equation (13) yields the steady-state levels of real profit flows of country j-located firms, which have equal values:

$$\Pi_{jss}/P^{j}{}_{ss}=(1/\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2},\quad j=A,B,C \tag{21}$$

4. A log-linearized analysis

Following the work of Obstfeld and Rogoff (1995), for any variable X, we use $\widehat{X}$ to denote "short-run" percentage deviations from the initial steady-state value, and we use $\overline{X}$ to denote "long-run" percentage deviations from the initial steady-state value (see Appendix for the derivation of short-run and long-run fiscal policy effects).

By log-linearizing equations (14) and (15) around the symmetric steady state and by setting $\widehat{W}^{j}=\widehat{P}^{j}_{j}(z)=0$, $j=A$, B, C, we obtain the following log-linearized expression for the international distribution of firms:

$$\widehat{m}=3\gamma((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{3/2}(1/\kappa)^{1/2}(\widehat{\varepsilon}^{A}-\widehat{\varepsilon}^{B}) \tag{22}$$

$$\widehat{n}=(3/2)\gamma((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{3/2}(1/\kappa)^{1/2}\widehat{\varepsilon}^{B} \tag{23}$$

Equation (22) shows that under given ε^{B} and $\Pi_{Ct}(z)/P^{C}_{t}$, exchange rate depreciation of country A's currency ($\widehat{\varepsilon}^{A}-\widehat{\varepsilon}^{B}>0$) induces the relocation of firms located in country B towards the country A. Intuitively, with fixed nominal wages, which cause nominal product prices to be sticky because of the mark-up pricing by monopolistic product suppliers, depreciation in country A's currency increases relative production of country A's goods through the 'expenditure-switching effect'; i. e., $\widehat{y}^{A}-\widehat{y}^{B}=\theta(\widehat{\varepsilon}^{A}-\widehat{\varepsilon}^{B})$. This phenomenon increases the relative profits of country A-located firms, and consequently, firms located in country B relocate to the country A. Equation (22) also shows that nominal exchange rate changes have greater effects the greater is the flexibility of relocation (the larger is γ). By contrast, when relocation costs are high ($\gamma=0$), nominal exchange rate changes have a negligible effect on the relocation of firms. The intuition behind the impact of ε^{B} in equation (23) on the international relocation of firms between countries B and C can be explained analogously.

5. Government spending shocks

We then consider the effects of an unanticipated permanent rise in government spending in each country.

5.1. The case of $\widehat{G}^A=\overline{G}^A>0$, $\widehat{G}^B=\overline{G}^B=\widehat{G}^C=\overline{G}^C=0$

In this subsection, we focus on the impacts of a permanent government spending shock in country A. In this case, the closed-form solutions for the six key variables are as follows:

$$\hat{\varepsilon}^A-\hat{\varepsilon}^B = -\tilde{\delta}\left(\frac{\alpha_1}{(\alpha_2)^2-(\alpha_1)^2}\right)\widehat{G}^A > 0 \tag{24}$$

$$\hat{\varepsilon}^B = \tilde{\delta}\left(\frac{\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right)\widehat{G}^A > 0 \tag{25}$$

$$\widehat{m} = -3\gamma\theta_1\tilde{\delta}\left(\frac{\alpha_1}{(\alpha_2)^2-(\alpha_1)^2}\right)\widehat{G}^A > 0 \tag{26}$$

$$\widehat{n} = (3\gamma/2)\theta_1\tilde{\delta}\left(\frac{\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right)\widehat{G}^A > 0 \tag{27}$$

$$\widehat{C}^A-\widehat{C}^B = \tilde{\delta}\left(\frac{\alpha_1}{(\alpha_2)^2-(\alpha_1)^2}\right)\widehat{G}^A < 0 \tag{28}$$

$$\widehat{C}^A-\widehat{C}^C = \tilde{\delta}\left(\frac{\alpha_1-\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right)\widehat{G}^A < 0 \tag{29}$$

where

$$\alpha_1 = \tilde{\delta}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1+6\gamma\tilde{\theta}\theta_1+\tilde{\theta}(\theta-1) > 0 \tag{30}$$

$$\alpha_2 = -\left\{\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]+3\gamma\tilde{\theta}\theta_1\right\} < 0 \tag{31}$$

$$\theta_1 = ((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{3/2}(1/\kappa)^{1/2} > 0 \tag{32}$$

$$\tilde{\delta} \equiv (1+\delta)/\delta, \quad \tilde{\theta} \equiv (\theta-1)/\theta, \quad \tilde{\phi} \equiv (\phi-1)/\phi$$

Equations (24) and (25) indicate that an unanticipated government spending shock in country A leads to exchange rate depreciation in $\varepsilon^A-\varepsilon^B$ and ε^B, respectively. Equation (26) shows that an unanticipated government spending shock in country A causes country B firms to relocate to country A. Equation (27) shows that an unanticipated government spending shock in country A causes country C firms to relocate to country B. Equations (28) and (29) show that the relative consumption levels of country A decrease when there is an unanticipated government spending shock in country A.

The above results can be explained intuitively as follows. First, a rise in government spending in country A leads to crowding-out of country A consumption, because country A's government-spending rise does not increase the country A's output needed to sufficiently offset the rise in taxes. Hereafter, we call this phenomenon the 'crowding-out effect'. Consequently, with a given ε^B, the reduction in country A's consumption then leads to a depreciation of its currency ($\hat{\varepsilon}^A - \hat{\varepsilon}^B > 0$, see equation (24)). This scenario can be attributed to the demand for real money balances, which increases with consumption, and country A's currency must depreciate and decrease the supply of real money balances in country A to restore money market equilibrium. In turn, exchange rate depreciation causes a consumption switching because the world consumption demand shifts toward the country A's goods given the fall in the relative price of country A's goods. Under a given $\Pi_{Ct}(z)/P_t^C$, this phenomenon causes country B's firms to move to country A because of the increase in relative profits of firms located in country A ($\hat{m} > 0$, see equation (26)). The relocation increases labor demand in country A and decreases labor demand in country B accordingly, which raises the labor income of country A and decreases the labor income of country B. Hereafter, we call this phenomenon the 'AB relocation effect'. This phenomenon increases the consumption in country A while decreases the consumption in country B. Thus, $\hat{C}^A - \hat{C}^B$ is determined by the two conflicting mechanisms of the crowding-out effect and the AB relocation effect. However, from equation (28), a rise in government spending unambiguously leads to a decrease (rise) in the relative consumption of country A (B), $\hat{C}^A - \hat{C}^B < 0$.

In addition, from the decrease in the consumption of country B through the AB relocation effect, country B's currency must depreciate and decrease the supply of real money balances in country B to restore money market equilibrium ($\hat{\varepsilon}^B > 0$, see equation (25)). With a given $\Pi_{At}(z)/P_t^A$, this in turn causes country C's firms to relocate to country B because of the increase in the relative profits of firms located in country B ($\hat{n} > 0$, see equation (27)). This relocation then increases labor demand in country B and decreases labor demand in country C, which in turn raises labor income in country B and decreases labor income in country C. Hereafter, we call this phenomenon the 'BC relocation effect'. This phenomenon increases the consumption in country B while decreases the consumption in country C. Therefore, $\hat{C}^A - \hat{C}^C$ is determined by the three conflicting mechanisms of the crowding-out effect, the AB relocation effect, and the BC relocation effect. However, on the basis of equation (29), a rise in government spending unambiguously leads to a decrease (rise) in the relative consumption of country A (C), $\hat{C}^A - \hat{C}^C < 0$.

In sum, a permanent government spending shock in country A is detrimental to country A in terms of the relative consumption level. In other words, a permanent government spending shock in country A always benefits not only country B but also country C in terms of relative consumption.

Incidentally, we can see the impacts that the absence of relocation of firms ($\gamma=0$) has on the exchange rates and relative consumption levels. Substituting $\gamma=0$ into equations (24) to (32), we obtain:

$$\hat{\varepsilon}^A-\hat{\varepsilon}^B=-\tilde{\delta}(\alpha_1)^{-1}\hat{G}^A>0,\quad \hat{\varepsilon}^B=0,\quad \hat{m}=0,\quad \hat{n}=0,\quad \hat{n}-\hat{m}=0,$$
$$\hat{C}^A-\hat{C}^B=-\tilde{\delta}(\alpha_1)^{-1}\hat{G}^A<0,\quad \hat{C}^A-\hat{C}^C=-\tilde{\delta}(\alpha_1)^{-1}\hat{G}^A<0,$$

where

$$\alpha_1=\tilde{\delta}\left\{2\left(\frac{\theta-1}{1+\theta}\right)+\frac{1}{\theta}\right\}+\theta-1+\frac{1}{\theta}>0,\quad \alpha_2=0.$$

The above equations are the same as those in the *Redux* model of Obstfeld and Rogoff (1995, 1996).

5. 2.　The case of $\hat{G}^B=\overline{G}^B>0$, $\hat{G}^A=\overline{G}^A=\hat{G}^C=\overline{G}^C=0$

In this subsection, we focus on the impacts of a permanent government spending shock in country B. In this case, the closed-form solutions for the six key variables are as follows:

$$\hat{\varepsilon}^A-\hat{\varepsilon}^B=\tilde{\delta}\left(\frac{\alpha_1+\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right)\hat{G}^B<0 \tag{33}$$

$$\hat{\varepsilon}^B=-\tilde{\delta}\left(\frac{\alpha_1+\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right)\hat{G}^B>0 \tag{34}$$

$$\hat{m}=3\gamma\theta_1\tilde{\delta}\left(\frac{\alpha_1+\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right)\hat{G}^B<0 \tag{35}$$

$$\hat{n}=-(3\gamma/2)\theta_1\tilde{\delta}\left(\frac{\alpha_1+\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right)\hat{G}^B>0 \tag{36}$$

$$\hat{C}^A-\hat{C}^B=-\tilde{\delta}\left(\frac{\alpha_1+\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right)\hat{G}^B>0 \tag{37}$$

$$\hat{C}^B-\hat{C}^C=\tilde{\delta}\left(\frac{\alpha_1+\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right)\hat{G}^B<0 \tag{38}$$

The above results can be explained intuitively as follows. First, a rise in the government spending in country B leads to the crowding out of country B's consumption, because country B's government spending rise does not increase the country B's output needed to sufficiently offset the rise in taxes (the crowding-out effect). Consequently, under a given ε^A, the decrease in the consumption of country B then leads to exchange rate depreciation of country B's currency ($\hat{\varepsilon}^A - \hat{\varepsilon}^B < 0$, $\hat{\varepsilon}^B > 0$, see equations (33) and (34)). Furthermore, exchange rate depreciation causes a consumption switching because the world consumption demand shifts toward the country B's goods given the fall in the relative price of country B's goods. This phenomenon causes firms located in countries A and C to move to country B because of the increase in the relative profits of firms located in country B ($\hat{m} < 0$, $\hat{n} > 0$, see equations (35) and (36)). This relocation increases labor demand in country B and decreases labor demand in countries A and C, which in turn raises labor income in country B and decreases labor income in countries A and C. As a result, the relocation increases the consumption in country B, while it decreases the consumption in countries A and C. Thus, the government spending effect on $\hat{C}^A - \hat{C}^B$ is determined by the three conflicting mechanisms of the crowding-out effect, the AB relocation effect, and the BC relocation effect. However, from equation (37), a rise in government spending unambiguously leads to a rise (decrease) in the relative consumption of country A (B), $\hat{C}^A - \hat{C}^B > 0$.

Similarly, the impact of an increase in government spending in country B on $\hat{C}^B - \hat{C}^C$ is ambiguous. This is because the impact of an increase in government spending is also determined by three conflicting mechanisms: the crowding-out effect, the AB relocation effect, and the BC relocation effect. However, on the basis of (38), a rise in government spending unambiguously leads to a decrease (rise) in the relative consumption of country B (C), $\hat{C}^B - \hat{C}^C < 0$.

In sum, a permanent government spending rise in country B is detrimental to country B in terms of the relative consumption level. In other words, a permanent government spending rise in country B always benefits country A but also country C in terms of relative consumption level.

5. 3. The case of $\hat{G}^C = \overline{G}^C > 0$, $\hat{G}^A = \overline{G}^A = \hat{G}^B = \overline{G}^B = 0$

In this subsection, we focus on the impacts of a permanent government spending shock in country C. In this case, the closed-form solutions for the six key variables are as follows:

$$\hat{\varepsilon}^A - \hat{\varepsilon}^B = -\tilde{\delta}\left(\frac{\alpha_2}{(\alpha_2)^2 - (\alpha_1)^2}\right)\hat{G}^C < 0 \tag{39}$$

$$\hat{\varepsilon}^B = \tilde{\delta}\left(\frac{\alpha_1}{(\alpha_2)^2 - (\alpha_1)^2}\right)\hat{G}^C < 0 \tag{40}$$

$$\widehat{m} = -3\gamma\theta_1\tilde{\delta}\left(\frac{\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right)\widehat{G}^C < 0 \tag{41}$$

$$\widehat{n} = (3\gamma/2)\theta_1\tilde{\delta}\left(\frac{\alpha_1}{(\alpha_2)^2-(\alpha_1)^2}\right)\widehat{G}^C < 0 \tag{42}$$

$$\widehat{C}^B-\widehat{C}^C = -\tilde{\delta}\left(\frac{\alpha_1}{(\alpha_2)^2-(\alpha_1)^2}\right)\widehat{G}^C > 0 \tag{43}$$

$$\widehat{C}^A-\widehat{C}^C = \tilde{\delta}\left(\frac{\alpha_2-\alpha_1}{(\alpha_2)^2-(\alpha_1)^2}\right)\widehat{G}^C > 0 \tag{44}$$

The above results can be explained intuitively as follows. First, a rise in government spending in country C leads to the crowding out of country C's consumption, because country C's government spending rise does not increase the country C's output needed to sufficiently offset the rise in taxes (the crowding-out effect). The decrease in the consumption of country C through the crowding-out effect then leads to exchange rate depreciation of its currency ($\widehat{\varepsilon}^A=\widehat{\varepsilon}^B<0$, see equation (40)). However, at this stage, country A's currency relative to B's remains unchanged, because $\widehat{\varepsilon}^A-\widehat{\varepsilon}^B=0$. Furthermore, exchange rate depreciation causes a consumption switching because the world consumption demand shifts toward the country C's goods given the fall in the relative price of country C's goods. In turn, with a given $\Pi_{At}(z)/P_t^A$, this phenomenon causes country B's firms to relocate to country C because of the increase in the relative profits of firms located in country C ($\widehat{n}<0$, see equation (42)). This relocation increases labor demand in country C and decreases labor demand in country B, which increases labor income in country C and decreases labor income in country B accordingly (the BC relocation effect). As a result, the relocation increases the consumption in country C and decreases that of country B. Thus, $\widehat{C}^B-\widehat{C}^C$ is determined by the two conflicting mechanisms of the crowding-out effect and the BC relocation effect. However, on the basis of equation (43), such a government spending rise unambiguously leads to a rise (decrease) in the relative consumption of country B (C), $\widehat{C}^B-\widehat{C}^C>0$.

Furthermore, as discussed in the definition of the BC relocation effect, the rise in country C's government spending also decreases country B's consumption through firm relocation from country B to country C. From this result, country B's currency must depreciate to restore equilibrium in the market for real balances. This depreciation of country B's currency weakens the initial appreciation of its currency, and consequently the change in country A's currency relative to B's is negative ($\widehat{\varepsilon}^A-\widehat{\varepsilon}^B<0$, see equation (39)). Furthermore, this leads to reduction of the real prices of country B's goods relative to country A's goods, which causes the world demand to switch from the country A's goods to the country B's goods. These demand shifts increase the relative profits of firms located in country B, which cause firms located in country A to move to country B ($\widehat{m}<0$, see equation (41)). This relocation increases labor demand in country B and decreases labor demand in country A, which increases labor income in country B and decreases labor income in country A accordingly (the AB relocation effect). As a result, the relocation decreases the consumption in country A. Thus, $\widehat{C}^A-\widehat{C}^C$ is determined by the three conflicting mechanisms of the crowding-out effect, the AB relocation effect, and the BC relocation effect. However, on the basis of equation (44), a rise in government spending unambiguously leads to a rise (decrease) in the relative consumption of country A (C), $\widehat{C}^A-\widehat{C}^C>0$. This is because the decrease in $\widehat{C}^A-\widehat{C}^C$ through the AB and BC relocation effects is dominated by the country C's consumption reduction through the crowding-out effect.

In sum, a permanent government spending shock in country C is detrimental to country C in terms of the relative consumption revel. In other words, a permanent government spending shock in country C always benefits not only country A but also country B in terms of relative consumption.

6. Conclusion

In this paper we considered the question of how allowing for international relocation of firms among three countries affects the impacts of government spending shocks on relative consumption and exchange rate. From this analysis, we showed explicitly the macroeconomic effects of government spending shocks that lead to firm relocation among three countries, and a rise in government spending of one of the three countries always depreciates its currency and decreases its relative consumption levels, while it can be beneficial for the neighboring countries.

Appendix

Long-run equilibrium conditions

The long-run equilibrium conditions of the model are derived. By log-linearizing the model around the initial, zero-shock symmetric steady state with $B_{ss,0}=0$, we obtain the following equations to characterize the long-run equilibrium of the system:

$$\overline{P}^A = \overline{M}^A - \overline{C}^A, \quad \overline{P}^B = \overline{M}^B - \overline{C}^B, \quad \overline{P}^C = \overline{M}^C - \overline{C}^C \tag{A.1}$$

$$\overline{C}^A = \delta\overline{B}^A + ((\theta-1)/\theta)(\overline{W}^A - \overline{P}^A + \overline{\ell}^{As}) + (1/3\theta)[\overline{\Pi}^A + \overline{\Pi}^B + \overline{\Pi}^C + 2\overline{\varepsilon}^A - \overline{\varepsilon}^B] - (1/\theta)\overline{P}^A - \overline{G}^A \tag{A.2}$$

$$\overline{C}^B = \delta\overline{B}^B + ((\theta-1)/\theta)(\overline{W}^B - \overline{P}^B + \overline{\ell}^{Bs}) + (1/3\theta)[\overline{\Pi}^A + \overline{\Pi}^B + \overline{\Pi}^C - \overline{\varepsilon}^A + 2\overline{\varepsilon}^B] - (1/\theta)\overline{P}^B - \overline{G}^B \tag{A.3}$$

$$\overline{C}^C = \delta\overline{B}^C + ((\theta-1)/\theta)(\overline{W}^C - \overline{P}^C + \overline{\ell}^{Cs}) + (1/3\theta)[\overline{\Pi}^A + \overline{\Pi}^B + \overline{\Pi}^C - \overline{\varepsilon}^A - \overline{\varepsilon}^B] - (1/\theta)\overline{P}^C - \overline{G}^C \tag{A.4}$$

$$\overline{y}^A = \theta(\overline{P}^A - \overline{P}^A_A) + \overline{C}^W + \overline{G}^W, \quad \overline{y}^B = \theta(\overline{P}^B - \overline{P}^B_B) + \overline{C}^W + \overline{G}^W, \quad \overline{y}^C = \theta(\overline{P}^C - \overline{P}^C_C) + \overline{C}^W + \overline{G}^W \tag{A.5}$$

$$\overline{C}^W + \overline{G}^W \equiv (1/3)(\overline{C}^A + \overline{G}^A) + (1/3)(\overline{C}^B + \overline{G}^B) + (1/3)(\overline{C}^C + \overline{G}^C) = (1/3)\overline{y}^A + (1/3)\overline{y}^B + (1/3)\overline{y}^C \equiv \overline{y}^W \tag{A.6}$$

$$\overline{m} = (3\gamma/\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}[\overline{\Pi}^A - \overline{\Pi}^B - \overline{\varepsilon}^A + \overline{\varepsilon}^B] \tag{A.7}$$

$$\overline{n} = (3\gamma/2\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}[\overline{\Pi}^B - \overline{\Pi}^C - \overline{\varepsilon}^B] \tag{A.8}$$

$$\overline{\Pi}^A = (1-\theta)\overline{P}^A_A + \theta\overline{P}^A + \overline{C}^W + \overline{G}^W \tag{A.9}$$

$$\overline{\Pi}^B = (1-\theta)\overline{P}^B_B + \theta\overline{P}^B + \overline{C}^W + \overline{G}^W \tag{A.10}$$

$$\overline{\Pi}^C = (1-\theta)\overline{P}^C_C + \theta\overline{P}^C + \overline{C}^W + \overline{G}^W \tag{A.11}$$

$$\overline{y}^A = \overline{\ell}^{Ad}, \quad \overline{y}^B = \overline{\ell}^{Bd}, \quad \overline{y}^C = \overline{\ell}^{Cd} \tag{A.12}$$

$$\overline{\ell}^{As} = \overline{m} + \overline{\ell}^{Ad}, \quad \overline{\ell}^{Bs} = 2\overline{n} - \overline{m} + \overline{\ell}^{Bd}, \quad \overline{\ell}^{Cs} = -2\overline{n} + \overline{\ell}^{cd} \tag{A.13}$$

$$\overline{P}^A_A = \overline{W}^A, \quad \overline{P}^B_B = \overline{W}^B, \quad \overline{P}^C_C = \overline{W}^C \tag{A.14}$$

$$\overline{P}^A - \overline{P}^B = \overline{\varepsilon}^A - \overline{\varepsilon}^B, \quad \overline{P}^B - \overline{P}^C = \overline{\varepsilon}^B, \quad \overline{P}^A - \overline{P}^C = \overline{\varepsilon}^A \tag{A.15}$$

$$\overline{\ell}^{As} = \overline{W}^A - \overline{P}^A - \overline{C}^A, \quad \overline{\ell}^{Bs} = \overline{W}^B - \overline{P}^B - \overline{C}^B, \quad \overline{\ell}^{Cs} = \overline{W}^C - \overline{P}^C - \overline{C}^C, \tag{A.16}$$

where $\overline{B} \equiv dB_{t+1}/C^W_{ss,0}$, which $C^W_{ss,0}$ is the initial value of world consumption.[10] The equations in (A. 1) correspond to the money-demand equations. Equations (A. 2, 3, 4) represent the long-run change in incomes (returns on real bonds, real labor incomes, and real profit incomes), which are equal to the long-run changes in consumption in each country. The equations in (A. 5) represent the world demand schedules for home and foreign products. Equation (A. 6) is the world goods-market equilibrium condition. Equations (A. 7) and (A. 8) are the dynamic relocation equations. The equations in (A. 9, 10, 11) are the nominal profit equations for firms. The equations in (A. 12) represent the production technology, and those in (A. 13) represent the long-run labor-market clearing conditions for both countries. The equations in (A. 14) represent the optimal pricing equations for firms in each country. Equation (A. 15) is the purchasing power parity equation. The equations in (A. 16) represent the first-order conditions for optimal wage setting.

Subtracting (A. 3) from (A. 2) yields the long-run response of relative per capita consumption levels,

$$\overline{C}^A-\overline{C}^B=(\delta/P^C)(\overline{B}^A-\overline{B}^B)+((\theta-1)/\theta)(\overline{\ell}^A-\overline{\ell}^B)+((\theta-1)/\theta)(\overline{W}^A-\overline{W}^B-\overline{P}^A+\overline{P}^B)$$
$$-(\overline{G}^A-\overline{G}^B) \tag{A. 17}$$

Subtracting (A. 4) from (A. 3) yields the long-run response of relative per capita consumption levels,

$$\overline{C}^B-\overline{C}^C=(\delta/P^C)(\overline{B}^B-\overline{B}^C)+((\theta-1)/\theta)(\overline{\ell}^B-\overline{\ell}^C)+((\theta-1)/\theta)(\overline{W}^B-\overline{W}^C-\overline{P}^B+\overline{P}^C)$$
$$-(\overline{G}^B-\overline{G}^C) \tag{A. 18}$$

Substituting (A. 9), (A. 10), (A. 11), (A. 14), and (A. 15) into equations (A. 7) and (A. 8), respectively, yields

$$\overline{m}=3\gamma\theta_1[\overline{\varepsilon}^A-\overline{\varepsilon}^B-(\overline{W}^A-\overline{W}^B)], \tag{A. 19}$$
$$\overline{n}=(3\gamma/2)\theta_1[\overline{\varepsilon}^B-(\overline{W}^B-\overline{W}^C)] \tag{A. 20}$$

From equations (A. 5), (A. 12), (A. 13), (A. 14), and (A. 15), we obtain

$$\overline{\ell}^{As}-\overline{\ell}^{Bs}=2(\overline{m}-\overline{n})+\theta[\overline{\varepsilon}^A-\overline{\varepsilon}^B-(\overline{W}^A-\overline{W}^B)], \tag{A. 21}$$
$$\overline{\ell}^{Bs}-\overline{\ell}^{Cs}=4\overline{n}-\overline{m}+\theta[\overline{\varepsilon}^B-(\overline{W}^B-\overline{W}^C)] \tag{A. 22}$$

From equations (A. 15) and (A. 16), we obtain

$$\overline{\ell}^{As}-\overline{\ell}^{Bs}+\overline{C}^A-\overline{C}^B=\overline{W}^A-\overline{W}^B-(\overline{\varepsilon}^A-\overline{\varepsilon}^B), \tag{A. 23}$$
$$\overline{\ell}^{Bs}-\overline{\ell}^{Cs}+\overline{C}^B-\overline{C}^C=\overline{W}^B-\overline{W}^C-\overline{\varepsilon}^B \tag{A. 24}$$

From (A. 15), (A. 17) and (A. 18),

$$\overline{C}^A-\overline{C}^B=(\delta/P^C)(\overline{B}^A-\overline{B}^B)+((\theta-1)/\theta)(\overline{\ell}^A-\overline{\ell}^B)+((\theta-1)/\theta)(\overline{W}^A-\overline{W}^B-(\overline{\varepsilon}^A-\overline{\varepsilon}^B))$$
$$-(\overline{G}^A-\overline{G}^B), \tag{A. 25}$$
$$\overline{C}^B-\overline{C}^C=(\delta/P^C)(\overline{B}^B-\overline{B}^C)+((\theta-1)/\theta)(\overline{\ell}^B-\overline{\ell}^C)+((\theta-1)/\theta)(\overline{W}^B-\overline{W}^C-\overline{\varepsilon}^B)$$
$$-(\overline{G}^B-\overline{G}^C) \tag{A. 26}$$

Substituting (A. 23) into (A. 25) yields

$$\overline{C}^A-\overline{C}^B=(\delta/P^C)(\overline{B}^A-\overline{B}^B)+((\theta-1)/\theta)(\overline{\ell}^A-\overline{\ell}^B)+((\theta-1)/\theta)(\overline{\ell}^A-\overline{\ell}^B+\overline{C}^A-\overline{C}^B)$$
$$-(\overline{G}^A-\overline{G}^B) \tag{A. 27}$$

Substituting (A. 24) into (A. 26) yields

$$\overline{C}^B-\overline{C}^C=(\delta/P^C)(\overline{B}^B-\overline{B}^C)+((\theta-1)/\theta)(\overline{\ell}^B-\overline{\ell}^C)+((\theta-1)/\theta)(\overline{\ell}^B-\overline{\ell}^C+\overline{C}^B-\overline{C}^C)$$
$$-(\overline{G}^B-\overline{G}^C) \quad \text{(A. 28)}$$

Substituting (A. 23) into (A. 19) yields

$$\overline{m}=-3\gamma\theta_1[\overline{\ell}^A-\overline{\ell}^B+\overline{C}^A-\overline{C}^B] \quad \text{(A. 29)}$$

Substituting (A. 24) into (A. 20) yields

$$\overline{n}=-(3/2)\gamma\theta_1[\overline{\ell}^B-\overline{\ell}^C+\overline{C}^B-\overline{C}^C] \quad \text{(A. 30)}$$

Substituting (A. 23), (A. 29), and (A. 30) into (A. 21) yields

$$(1+6\gamma\theta_1+\theta)(\overline{\ell}^{As}-\overline{\ell}^{Bs})=-(6\gamma\theta_1+\theta)(\overline{C}^A-\overline{C}^B)+3\gamma\theta_1[\overline{\ell}^B-\overline{\ell}^C+\overline{C}^B-\overline{C}^C] \quad \text{(A. 31)}$$

Substituting (A. 24), (A. 29), and (A. 30) into (A. 22) yields

$$\overline{\ell}^B-\overline{\ell}^C=-\left(\frac{6\gamma\theta_1+\theta}{1+6\gamma\theta_1+\theta}\right)(\overline{C}^B-\overline{C}^C)+\left(\frac{3\gamma\theta_1}{1+6\gamma\theta_1+\theta}\right)(\overline{\ell}^A-\overline{\ell}^B+\overline{C}^A-\overline{C}^B) \quad \text{(A. 32)}$$

Substituting (A. 32) into (A. 31) yields

$$\overline{\ell}^A-\overline{\ell}^B=-\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\overline{C}^A-\overline{C}^B)$$
$$+\left[\frac{3\gamma\theta_1}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\overline{C}^B-\overline{C}^C) \quad \text{(A. 33)}$$

Substituting (A. 33) into (A. 27) yields

$$\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}(\overline{C}^A-\overline{C}^B)$$
$$=(\delta/P^C)(\overline{B}^A-\overline{B}^B)+\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\overline{C}^B-\overline{C}^C)-(\overline{G}^A-\overline{G}^B) \quad \text{(A. 34)}$$

Substituting (A. 32) and (A. 33) into (A. 28) yields

$$\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}(\overline{C}^B-\overline{C}^C)$$
$$=(\delta/P^C)(\overline{B}^B-\overline{B}^C)+\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\overline{C}^A-\overline{C}^B)-(\overline{G}^B-\overline{G}^C) \quad \text{(A. 35)}$$

Short-run equilibrium conditions and the effects of relative government spending shocks

The short-run equilibrium conditions of the model are derived. By log-linearizing the model around the initial, zero-shock symmetric steady state with $B_{ss,0}=0$, we obtain the following equations to characterize the short-run equilibrium of the system:

$$\overline{C}^A = \hat{C}^A + (\delta/(1+\delta))\overline{r} + \overline{\varepsilon}^A - \hat{\varepsilon}^A \tag{A. 36}$$

$$\overline{C}^B = \hat{C}^B + (\delta/(1+\delta))\overline{r} + \overline{\varepsilon}^B - \hat{\varepsilon}^B \tag{A. 37}$$

$$\overline{C}^C = \hat{C}^C + (\delta/(1+\delta))\overline{r} \tag{A. 38}$$

$$\hat{M}^A - \hat{P}^A = \hat{C}^A - \overline{r}/(1+\delta) - (\overline{P}^A - \hat{P}^A)/\delta - \overline{\varepsilon}^A/\delta + \hat{\varepsilon}^A/\delta \tag{A. 39}$$

$$\hat{M}^B - \hat{P}^B = \hat{C}^B - \overline{r}/(1+\delta) - (\overline{P}^B - \hat{P}^B)/\delta - \overline{\varepsilon}^B/\delta + \hat{\varepsilon}^B/\delta \tag{A. 40}$$

$$\hat{M}^C - \hat{P}^C = \hat{C}^C - \overline{r}/(1+\delta) - (\overline{P}^C - \hat{P}^C)/\delta \tag{A. 41}$$

$$\begin{aligned}\overline{B}^A/P^C = &-((\theta-1)/\theta)\hat{P}^A + ((\theta-1)/\theta)(\hat{m} + \hat{\ell}^{Ad}) \\ &+ (1/3\theta)[\hat{\Pi}^A + \hat{\Pi}^B + \hat{\Pi}^C + 2\hat{\varepsilon}^A - \hat{\varepsilon}^B - 3\hat{P}^A] - \hat{C}^A - \hat{G}^A\end{aligned} \tag{A. 42}$$

$$\begin{aligned}\overline{B}^B/P^C = &-((\theta-1)/\theta)\hat{P}^B + ((\theta-1)/\theta)(2\hat{n} - \hat{m} + \hat{\ell}^{Bd}) \\ &+ (1/3\theta)[\hat{\Pi}^A + \hat{\Pi}^B + \hat{\Pi}^C - \hat{\varepsilon}^A + 2\hat{\varepsilon}^B - 3\hat{P}^B] - \hat{C}^B - \hat{G}^B\end{aligned} \tag{A. 43}$$

$$\begin{aligned}\overline{B}^C/P^C = &-((\theta-1)/\theta)\hat{P}^C + ((\theta-1)/\theta)(-2\hat{n} + \hat{\ell}^{Cd}) \\ &+ (1/3\theta)[\hat{\Pi}^A + \hat{\Pi}^B + \hat{\Pi}^C - \hat{\varepsilon}^A - \hat{\varepsilon}^B - 3\hat{P}^C] - \hat{C}^C - \hat{G}^C\end{aligned} \tag{A. 44}$$

$$\hat{y}^A = \theta\hat{P}^A + \hat{C}^W + \hat{G}^W, \quad \hat{y}^B = \theta\hat{P}^B + \hat{C}^W + \hat{G}^W, \quad \hat{y}^C = \theta\hat{P}^C + \hat{C}^W + \hat{G}^W \tag{A. 45}$$

$$\hat{y}^A = \hat{\ell}^{Ad}, \quad \hat{y}^B = \hat{\ell}^{Bd}, \quad \hat{y}^C = \hat{\ell}^{Cd} \tag{A. 46}$$

$$\hat{\Pi}^A = \theta\hat{P}^A + \hat{C}^W + \hat{G}^W, \quad \hat{\Pi}^B = \theta\hat{P}^B + \hat{C}^W + \hat{G}^W, \quad \hat{\Pi}^C = \theta\hat{P}^C + \hat{C}^W + \hat{G}^W \tag{A. 47}$$

$$\hat{m} = (3\gamma/\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}[\hat{\Pi}^A - \hat{\Pi}^B - \hat{\varepsilon}^A + \hat{\varepsilon}^B] \tag{A. 48}$$

$$\hat{n} = (3\gamma/2\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}[\hat{\Pi}^B - \hat{\Pi}^C - \hat{\varepsilon}^B] \tag{A. 49}$$

$$\begin{aligned}\hat{C}^W + \hat{G}^W &\equiv (1/3)(\hat{C}^A + \hat{G}^A) + (1/3)(\hat{C}^B + \hat{G}^B) + (1/3)(\hat{C}^C + \hat{G}^C) \\ &= (1/3)\hat{y}^A + (1/3)\hat{y}^B + (1/3)\hat{y}^C \equiv \hat{y}^W\end{aligned} \tag{A. 50}$$

$$\begin{aligned}\hat{P}^A = (2/3)\hat{\varepsilon}^A - (1/3)\hat{\varepsilon}^B, \quad \hat{P}^B = -(1/3)\hat{\varepsilon}^A + (2/3)\hat{\varepsilon}^B, \\ \hat{P}^C = -(1/3)\hat{\varepsilon}^A - (1/3)\hat{\varepsilon}^B\end{aligned} \tag{A. 51}$$

$$\hat{\ell}^{As} = \hat{m} + \hat{\ell}^{Ad}, \quad \hat{\ell}^{Bs} = 2\hat{n} - \hat{m} + \hat{\ell}^{Bd}, \quad \hat{\ell}^{Cs} = -2\hat{n} + \hat{\ell}^{Cd} \tag{A. 52}$$

where we set nominal wages and prices of goods as $\hat{W}^h = \hat{P}^j_j(z) = 0$, h, $j = A, B, C$, for the above short-run log-linearized equations. The equations in (A. 36, 37, 38) are the Euler equations. The equations in (A. 39, 40, 41) describe equilibrium in the money markets in the short run. The equations in (A. 42, 43, 44) are linearized short-run current account equations. The equations in (A. 45) represent the world demand schedules for representative country j products ($j = A, B, C$). Equation (A. 46) is the production function. The equations in (A. 47) are the nominal profit equations for representative country j firms ($j = A, B, C$). Equations (A. 48) and (A. 49) are the dynamic relocation equations. Equation (A. 50) is the world goods-market equilibrium condition. Equation (A. 51) is the price index equation in the short run. The equations in (A. 52) represent the short-run labor-market clearing conditions for both countries.

By subtracting (A. 43) from (A. 42), we obtain

$$(\overline{B}^A-\overline{B}^B)/P^C = -((\theta-1)/\theta)(\widehat{P}^A-\widehat{P}^B)+2((\theta-1)/\theta)(\widehat{m}-\widehat{n})+((\theta-1)/\theta)(\widehat{\ell}^{Ad}-\widehat{\ell}^{Bd}) \\ -(\widehat{C}^A-\widehat{C}^B)-(\widehat{G}^A-\widehat{G}^B) \quad (A.53)$$

Substituting (A. 47) and (A. 51) into (A. 48) and (A. 49), respectively, yields

$$\widehat{m} = (3\gamma/\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}(\theta-1)(\hat{\varepsilon}^A-\hat{\varepsilon}^B), \quad (A.54)$$

$$\widehat{n} = (3\gamma/2\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}(\theta-1)\hat{\varepsilon}^B \quad (A.55)$$

From equations (A. 45), (A. 46), and (A. 51) we obtain the following relative labor demand

$$\widehat{\ell}^{Ad}-\widehat{\ell}^{Bd} = \theta(\widehat{P}^A-\widehat{P}^B) = \theta(\hat{\varepsilon}^A-\hat{\varepsilon}^B) \quad (A.56)$$

Subtracting (A. 55) from (A. 54) yields

$$\widehat{m}-\widehat{n} = 3\gamma\theta_1(\hat{\varepsilon}^A-\hat{\varepsilon}^B)-(3/2)\gamma\theta_1\hat{\varepsilon}^B \quad (A.57)$$

Substituting (A. 51), (A. 56), and (A. 57) into (A. 53) yields

$$(\overline{B}^A-\overline{B}^B)/P^C = 2\tilde{\theta}[3\gamma\theta_1(\hat{\varepsilon}^A-\hat{\varepsilon}^B)-(3/2)\gamma\theta_1\hat{\varepsilon}^B]+\tilde{\theta}(\theta-1)(\hat{\varepsilon}^A-\hat{\varepsilon}^B) \\ -(\widehat{C}^A-\widehat{C}^B)-(\widehat{G}^A-\widehat{G}^B) \quad (A.58)$$

From (A. 36), (A. 37), and (A. 38)

$$\overline{C}^A-\overline{C}^B = \widehat{C}^A-\widehat{C}^B, \quad (A.59)$$

$$\overline{C}^B-\overline{C}^C = \widehat{C}^B-\widehat{C}^C \quad (A.60)$$

Substituting (A. 59) and (A. 60) into (A. 34) yields

$$(1/P^C)(\overline{B}^A-\overline{B}^B) = \delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}(\widehat{C}^A-\widehat{C}^B) \\ -\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\widehat{C}^B-\widehat{C}^C)+\delta^{-1}(\widehat{G}^A-\widehat{G}^B) \quad (A.61)$$

Substituting (A. 61) into (A. 58) yields

$$\left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1\right\}(\widehat{C}^A-\widehat{C}^B) \\ -\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\widehat{C}^B-\widehat{C}^C)+\tilde{\delta}(\widehat{G}^A-\widehat{G}^B) \\ = \tilde{\theta}\theta_1[6\gamma(\hat{\varepsilon}^A-\hat{\varepsilon}^B)-3\gamma\hat{\varepsilon}^B]+\tilde{\theta}(\theta-1)(\hat{\varepsilon}^A-\hat{\varepsilon}^B) \quad (A.62)$$

From (A. 39), (A. 40), (A. 41), (A. 51), (A. 59) and (A. 60),

$$\widehat{C}^A - \widehat{C}^B = -(\hat{\varepsilon}^A - \hat{\varepsilon}^B), \tag{A. 63}$$

$$\widehat{C}^B - \widehat{C}^C = -\hat{\varepsilon}^B \tag{A. 64}$$

From (A. 62), (A. 63) and (A. 64), we obtain

$$\begin{aligned}
&\tilde{\delta}(\widehat{G}^A - \widehat{G}^B) \\
&= \left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1+6\gamma\theta_1\tilde{\theta}+\tilde{\theta}(\theta-1)\right\}(\hat{\varepsilon}^A-\hat{\varepsilon}^B) \\
&\quad -\left\{\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]+3\gamma\theta_1\tilde{\theta}\right\}\hat{\varepsilon}^B
\end{aligned} \tag{A. 65}$$

(A. 65) can be rewritten as

$$\tilde{\delta}(\widehat{G}^A - \widehat{G}^B) = \alpha_1(\hat{\varepsilon}^A - \hat{\varepsilon}^B) + \beta_1\hat{\varepsilon}^B \tag{A. 66}$$

where

$$\tilde{\delta} = \frac{1+\delta}{\delta}, \tag{A. 67}$$

$$\begin{aligned}
\alpha_1 = &\left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1+6\gamma\theta_1\tilde{\theta}+\tilde{\theta}(\theta-1)\right\} \\
&> 0,
\end{aligned} \tag{A. 68}$$

$$\beta_1 = -\left\{\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]+3\gamma\theta_1\tilde{\theta}\right\} < 0 \tag{A. 69}$$

Subtracting (A. 44) from (A. 43) and considering (A. 45), (A. 46), (A. 51), (A. 54) and (A. 55) yields

$$(\overline{B}^B - \overline{B}^C)/P^C = 6\gamma\theta_1\tilde{\theta}\hat{\varepsilon}^B - 3\gamma\theta_1\tilde{\theta}(\hat{\varepsilon}^A - \hat{\varepsilon}^B) + \tilde{\theta}(\theta-1)\hat{\varepsilon}^B - (\widehat{C}^B - \widehat{C}^C) - (\widehat{G}^B - \widehat{G}^C) \tag{A. 70}$$

Substituting (A. 59) and (A. 60) into (A. 35) yields

$$\begin{aligned}
&\frac{1}{P^C}(\overline{B}^B - \overline{B}^C) \\
&= \delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{6\gamma\theta_1+\theta}{1+6\gamma\theta_1+\theta}\right]-\tilde{\theta}-2\tilde{\theta}\left[\frac{3\gamma\theta_1}{1+6\gamma\theta_1+\theta}\right]\left[\frac{3\gamma\theta_1}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]\right\}(\widehat{C}^B - \widehat{C}^C) \\
&\quad -\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\widehat{C}^A - \widehat{C}^B) + \delta^{-1}(\widehat{G}^B - \widehat{G}^C)
\end{aligned} \tag{A. 71}$$

Substituting (A. 71) into (A. 70) yields

$$\left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{6\gamma\theta_1+\theta}{1+6\gamma\theta_1+\theta}\right]-\tilde{\theta}-2\tilde{\theta}\left[\frac{3\gamma\theta_1}{1+6\gamma\theta_1+\theta}\right]\left[\frac{3\gamma\theta_1}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]\right\}+1\right\}(\widehat{C}^B-\widehat{C}^C)$$

$$-\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right](\widehat{C}^A-\widehat{C}^B)+\delta^{-1}(\widehat{G}^B-\widehat{G}^C)$$

$$=6\gamma\theta_1\tilde{\theta}\tilde{\varepsilon}^B-3\gamma\theta_1\tilde{\theta}(\tilde{\varepsilon}^A-\tilde{\varepsilon}^B)+\tilde{\theta}(\theta-1)\tilde{\varepsilon}^B \tag{A. 72}$$

From (A. 63), (A. 64) and (A. 72), we obtain

$$\delta^{-1}(\widehat{G}^B-\widehat{G}^C)=-\left\{\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]+3\gamma\theta_1\tilde{\theta}\right\}(\tilde{\varepsilon}^A-\tilde{\varepsilon}^B)$$

$$+\left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{6\gamma\theta_1+\theta}{1+6\gamma\theta_1+\theta}\right]-\tilde{\theta}\right.\right.$$

$$\left.\left.-2\tilde{\theta}\left[\frac{3\gamma\theta_1}{1+6\gamma\theta_1+\theta}\right]\left[\frac{3\gamma\theta_1}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]\right\}+1+6\gamma\theta_1\tilde{\theta}+\tilde{\theta}(\theta-1)\right\}\tilde{\varepsilon}^B \tag{A. 73}$$

(A. 73) can be rewritten as

$$\delta^{-1}(\widehat{G}^B-\widehat{G}^C)=\alpha_2(\tilde{\varepsilon}^A-\tilde{\varepsilon}^B)+\beta_2\tilde{\varepsilon}^B \tag{A. 74}$$

where

$$\alpha_2=\beta_1=-\left\{\delta^{-1}\left[\frac{6\gamma\theta_1\tilde{\theta}}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]+3\gamma\theta_1\tilde{\theta}\right\}, \tag{A. 75}$$

$$\beta_2=\alpha_1=\left\{\delta^{-1}\left\{1+2\tilde{\theta}\left[\frac{(6\gamma\theta_1+\theta)(1+6\gamma\theta_1+\theta)-9\gamma^2\theta_1^2}{(1+6\gamma\theta_1+\theta)^2-9\gamma^2\theta_1^2}\right]-\tilde{\theta}\right\}+1+6\gamma\theta_1\tilde{\theta}+\tilde{\theta}(\theta-1)\right\} \tag{A. 76}$$

Derivation of the impacts of permanent government spending shocks

From (A. 66) and (A. 74), we obtain

$$\tilde{\varepsilon}^A-\tilde{\varepsilon}^B=-\tilde{\delta}\left[\frac{\beta_2}{\alpha_2\beta_1-\alpha_1\beta_2}\right](\widehat{G}^A-\widehat{G}^B)+\tilde{\delta}\left[\frac{\beta_1}{\alpha_2\beta_1-\alpha_1\beta_2}\right](\widehat{G}^B-\widehat{G}^C), \tag{A. 77}$$

$$\tilde{\varepsilon}^B=\tilde{\delta}\left[\frac{\alpha_2}{\alpha_2\beta_1-\alpha_1\beta_2}\right](\widehat{G}^A-\widehat{G}^B)-\tilde{\delta}\left[\frac{\alpha_1}{\alpha_2\beta_1-\alpha_1\beta_2}\right](\widehat{G}^B-\widehat{G}^C) \tag{A. 78}$$

From $\alpha_1=\beta_2$ and $\alpha_2=\beta_1$, (A. 77) and (A. 78) can be rewritten as

$$\tilde{\varepsilon}^A-\tilde{\varepsilon}^B=-\tilde{\delta}\left[\frac{\alpha_1}{(\alpha_2)^2-(\alpha_1)^2}\right](\widehat{G}^A-\widehat{G}^B)+\tilde{\delta}\left[\frac{\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right](\widehat{G}^B-\widehat{G}^C), \tag{A. 79}$$

$$\hat{\varepsilon}^B = \tilde{\delta}\left[\frac{\alpha_2}{(\alpha_2)^2-(\alpha_1)^2}\right](\hat{G}^A-\hat{G}^B)-\tilde{\delta}\left[\frac{\alpha_1}{(\alpha_2)^2-(\alpha_1)^2}\right](\hat{G}^B-\hat{G}^C) \quad (A.80)$$

The relative consumption changes are

$$\hat{C}^A-\hat{C}^B = -(\hat{\varepsilon}^A-\hat{\varepsilon}^B) \quad (A.81)$$

$$\hat{C}^B-\hat{C}^C = -\hat{\varepsilon}^B \quad (A.82)$$

$$\hat{C}^A-\hat{C}^C = -(\hat{\varepsilon}^A-\hat{\varepsilon}^B)-\hat{\varepsilon}^B \quad (A.83)$$

In addition, from (A. 54) and (A. 55), the log-linearized expressions for the international distribution of firms are

$$\hat{m} = (3\gamma/\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}(\theta-1)(\hat{\varepsilon}^A-\hat{\varepsilon}^B), \quad (A.84)$$

$$\hat{n} = (3\gamma/2\theta)((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}(\theta-1)\hat{\varepsilon}^B \quad (A.85)$$

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Notes

1) For a survey, see Lane (2001), Lane and Ganelli (2003), Vanhoose (2004), and Coutinho (2005).

2) Other related NOEM models incorporating cross-border relocation of firms include Johdo (2015, 2019a, and 2019b).

3) However, one exception is Johdo (2019d), who attempts to present a new NOEM model with international relocation of firms in a three-country context and succeeds in showing explicitly the effects of one country's monetary expanding on consumption of the three countries and the exchange rate. Another exception is Johdo (2020), who examine the impacts of one country's deregulation policy on the distribution of firms among three countries, exchange rate, and relative consumption.

4) However, from the results of numerical simulations, Cavallari (2010) predicts that consumption spillovers between two countries are higher under endogenous entry compared with the model with no entry.

5) In Cavallari (2010), consumers' preferences are modelled as Cobb–Douglas utility function across the traded and nontraded goods. In addition, in her model, international trade is costly, incurring an iceberg-type transport cost per unit sold abroad.

6) Cavallari (2010) focuses only on the short-run equilibrium with nominal rigidities when she studies the implication of firms' entry for monetary policy shocks. This implies that, Cavallari (2010) overlooks the long-run equilibrium where flexible prices exist and money is neutral. In general, in the perfect-foresight model, the long-run impacts of policy shocks also have an important role in affecting the consumer's dynamic behavior, as in Obstfeld and Rogoff (1995).

Of course, in her model, because a unitary elasticity of substitution in consumption between tradable goods and nontradable goods is assumed, as in Corsetti and Pesenti (2001), the current account is balanced in any point in time provided initial nonmonetary wealth is zero. Consequently, in her model, the short-run equilibrium is determined independently of the long-run equilibrium. However, if nontradable goods are excluded from her model, or if a more general specification of consumption preferences is assumed, a current account surplus (or deficit) is created, and then the short-run equilibrium affects the long-run flexible price equilibrium.

7) Cavallari (2010) allows for pricing to market (PTM) behavior, in which foreign multinational firms active in the home market can price their goods in terms of the home currency. In the NOEM literature, for example, Betts and Devereux (2000) show that domestic monetary expansion is a 'beggar-thy-neighbor' policy if there is a high degree of PTM behavior in both countries.

8) However, empirical evidence shows that there are large margins of equity bias even among highly integrated economies. For detailed arguments regarding home equity bias, see Obstfeld and Rogoff (2001).

9) These adjustment equations could be justified by assuming that there is a cost to relocating; hence, only a fraction of firms will relocate.

10) In a symmetric steady state, initial net foreign assets are zero; i.e., $B_0=0$. Following Obstfeld and Rogoff (1995), we scale bond holdings by using the initial level of world consumption, $C^W_{ss,0}$.

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