Wage Taxation, Cross-Border Relocation of Workers and the Exchange Rate

Wataru JOHDO

Abstract

This paper presents a new open economy macroeconomics model that incorporates cross-border relocation of workers to analyze the effects of wage taxation in the home country. This taxation proves to produce not only worker relocation toward the foreign county, but also appreciation in the exchange rate through the increase in the relative consumption of the home country. We show that the more flexibility is the cross-border relocation of worker, the greater is the effect of the wage taxation on the relative consumption of the home country.

1 Introduction

Most new open economy macroeconomics (NOEM) models have assumed that workers are immobile across countries since the publication of Obstfeld and Rogoff's (1995) paper (e.g., Céspedes et al., 2000; Hau, 2000; Corsetti and Pesenti, 2001; Kollmann, 2001a, 2001b; Benigno, 2002; Chu, 2005; and Johdo, 2013a, 2013b, 2015, 2019a, 2019b, 2019c, 2019d, 2020a, 2020b). On the other hand, recently, the international worker moves aimed at higher wages has been expanded rapidly between emerging countries (e. g., China, India etc) and developed countries (e.g., the United States, Japan etc). However, in the new open economy macroeconomics literature, no one has considered how interaction between international worker movement and the exchange rate affects macroeconomics variables.

Despite the importance of examining the wage taxation, there are few theoretical works that analyze the effect of an increase in the wage tax within the open macroeconomic framework with cross-border relocation of workers. The study of Johdo (2010) is one of the few works that looks at the impact of a rise in wage tax on the cross-border relocation of workers and terms of trade, but that study did not examine the effects on consumptions and welfare. Obstfeld and Rogoff (1996) investigated the effect of an increase in labor income tax using the NOEM model. Their analysis includes many suggestions, but they examine the policy impact in the model of fixed location of labor, so it is difficult to understand the linkage between the cross-border relocation of labor and other key macroeconomic variables including the terms of trade (or the exchange rate).

The purpose of this paper is to consider how interaction between cross-border worker movements and the exchange rate movements affects the macroeconomic effects of wage taxation in the home country, and how this taxation affects another country' output and consumption through the international worker movement. In order to address these issues, in this paper, we propose a NOEM model that incorporates the cross-border movement of workers and nominal wage rigidities. In particular, in this model, the driving force in the cross-border worker relocation is the workers' real wage differential between the two trading countries. This implies that the cross-border movement of workers is affected by the nominal exchange rate. Accordingly, from this model, we can show the interaction between worker movement and the nominal exchange rate and illustrates how these factors affect consumption in both countries.

We conclude that an increase in the home country's wage tax rate induces relocation of workers of the home country towards the foreign country, and they leads to appreciation in the home currency through the increase in the relative home consumption. We also find that the more mobility is the cross-border relocation of workers, the greater is the effect of the wage tax shock on the relative consumption of the home country to the foreign country.

The remainder of this paper is structured as follows. In Section 2, we outline the features of the dynamic optimizing model. In Section 3, we present the symmetric equilibrium under flexible nominal wages. In Section 4, we present a log-linearized version of this model, and explain how exchange rate changes affect the international relocation of workers. In Section 5, we examine how an unanticipated increase in the wage tax rate affects the distribution of workers between two countries, the exchange rate, and cross-country differences in consumption. In Section 6, we examine the world welfare effects of the wage taxation. The final section summarizes the findings and concludes the paper.

2 The model

The size of the world population is normalized to unity. Workers in the interval $[0, n_t]$ locate in the home country, and the remaining $(n_t, 1]$ workers locate in the foreign country, where n_t is endogenous. On the production side, monopolistically competitive firms exist continuously in the world in the [0, 1] range, each of which produces a single differentiated product that is freely tradable. In addition, we assume that, firms in the interval $[0, s]$ locate in the home country, and the remaining $(s, 1]$ firms locate in the foreign country, where s is exogenous. Home and foreign households (or workers) have perfect foresight and share the same utility function. Households in each country derive utility from consuming a group of differentiated goods (defined later), gain from money holdings through liquidity services, and incur the cost of expending labor effort. The intertemporal objective of household $i \in (0, n_t)$ in the home country at time 0 is to maximize the following lifetime utility: [1]

$$U^i{}_0 = E_0 \sum_{t=0}^{\infty} \beta^t (\log C^i{}_t + \chi \log (M^i{}_t / P_t) - (\kappa/2)(\ell^{si}{}_t)^2), \tag{1}$$

where E_0 represents the mathematical expectation conditional on the information set made available to household i in period 0, β is a constant subjective discount factor $(0<\beta<1)$, $\ell^{si}{}_t$ is the amount of labor supplied by household i in period t, and the consumption index $C^i{}_t$ is defined as follows:

$$C^i{}_t = \left(\int_0^1 C^i_t(j)^{(\theta-1)/\theta} dj \right)^{\theta/(\theta-1)}, \theta > 1.$$

where θ is the elasticity of substitution between any two differentiated goods and $C^i{}_t(j)$ is the consumption of good j in period t for household i.[2] In addition, the second term in (1) is real money balances $(M^i{}_t/P_t)$, where $M^i{}_t$ denotes nominal money balances held at the beginning of period $t+1$, and P_t is the home country consumption price index (CPI), which is defined as $P_t = \left(\int_0^1 P_t(j)^{1-\theta} dj \right)^{1/(1-\theta)}$, where $P_t(j)$ is the home-currency price of good j in period t. Analogously, the foreign country CPI is $P_t^* = \left(\int_0^1 P_t^*(j)^{1-\theta} dj \right)^{1/(1-\theta)}$, where $P_t^*(j)$ is the foreign-currency price of good j in period t. Under the law of one price, we can rewrite the price indexes as

$$P_t = \left(\int_0^s P_t(j)^{1-\theta} dj + \int_s^1 (\varepsilon_t P_t^*(j))^{1-\theta} dj \right)^{1/(1-\theta)},$$

$$P_t^* = \left(\int_0^s (P_t(j)/\varepsilon_t)^{1-\theta} dj + \int_s^1 P_t^*(j)^{1-\theta} dj \right)^{1/(1-\theta)}.$$

Because there are no trade costs between the two countries, the law of one price holds for any variety j; i.e., $P_t(j) = \varepsilon_t P_t^*(j)$, where ε_t is the nominal exchange rate, defined as the home currency price per unit of foreign currency. Given the law of one price, a comparison of the above price indexes implies that purchasing power parity (PPP) is represented by $P_t = \varepsilon_t P_t^*$. We assume that there is an international risk-free real bond market and that real bonds are denominated in units of the composite consumption good. At each point in time, households receive returns on risk-free real bonds, earn wage income by supplying labor, and receive profits from firms. Thus, a typical domestic household faces the following budget constraint:

$$P_tB^i_{t+1}+M^i_t = P_t(1+r_t)B^i_t+M^i_{t-1}+(1-\tau^i_t)W^i_t\ell^{si}_t$$
$$+(\alpha/s)\left(\int_0^s \Pi_t(j)dj+\int_s^1 \varepsilon_t\Pi^*_t(j)dj\right)-P_tC^i_t+P_tT^i_t. \qquad (2)$$

where B^i_{t+1} denotes real bonds held by home agent i in period $t+1$, r_t denotes the real interest rate on bonds that applies between periods $t-1$, and t, $W^i_t\ell^{si}_t$ is nominal labor income, where W^i_t denotes the nominal wage rate of household i in period t, α denotes the extent to which firms are domestically owned; thus, α/s (resp. $\alpha^*/(1-s)$) denotes the share of firms' total profit flows that are repatriated to each home (resp. foreign) agent, where $0<\alpha<1$, $0<\alpha^*<1$, and $\alpha+\alpha^*=1$, $\int_0^s \Pi_t(j)dj$ (resp. $\int_s^1 \varepsilon_t\Pi^*_t(j)dj$) represents the total nominal profit flows of firms located at home (resp. abroad). In addition, $P_tC^i_t$ represents nominal consumption expenditure and T^i_t denotes real lump-sum transfers from the government in period t. Note that all variables in (2) are measured in terms of per unit of labor endowments. In the government sector, we assume that government spending is zero and that all seignorage revenues derived from printing the national currency are rebated to the public in the form of lump-sum transfers. Hence, the government budget constraint in the home country is $T_t=\tau_t/P_t+[(M_t-M_{t-1})/P_t]$, where $T_t=\int_0^{n_t} T^i_t di$, $\tau_t=\int_0^{n_t} \tau^i_t W^i_t\ell^{si}_t di$ and M_t is the aggregate money supply.

In the home country, firm $j\in[0, s]$ hires a continuum of differentiated labor inputs domestically and produces a unique product in a single location according to the CES production function, $y_t(j)=\left(n_t^{-1/\phi}\int_0^{n_t} \ell^{di}_t(j)^{(\phi-1)/\phi}di\right)^{\phi/(\phi-1)}$, where $y_t(j)$ denotes the production of home-located firm j in period t, $\ell^{di}_t(j)$ is the firm j's input of labor from household i in period t, and $\phi>1$ is the elasticity of input substitution. Given the home firm's cost minimization problem, firm j's optimal labor demand for household i's labor input is as follows:

$$\ell^{di}_t(j) = n_t^{-1}(W^i_t/W_t)^{-\phi}y_t(j) \qquad (3)$$

where $W_t\equiv\left(n_t^{-1}\int_0^{n_t} W_t^{i(1-\phi)}di\right)^{1/(1-\phi)}$ is a price index for labor input (i.e., an aggregate wage index), which represents the minimum cost of producing a unit of output $y_t(j)$ in period t.

We now consider the dynamic optimization problem of households. In the first stage, households in the home (resp. foreign) country maximize the consumption index $C^i{}_t$ (resp. $C^i{}_t{}^*$) subject to a given level of expenditure $P_tC^i{}_t=\int_0^1 P_t(j)C^i{}_t(j)dj$ (resp. $P_t^*C^i{}_t{}^*=\int_0^1 P_t^*(j)C^i{}_t{}^*(j)dj$) by optimally allocating differentiated goods. This static problem yields the following demand functions for good j in the home and foreign countries, respectively:

$$C^i{}_t(j)=(P_t(j)/P_t)^{-\theta}C^i{}_t,\quad C^i{}_t{}^*(j)=(P_t^*(j)/P_t^*)^{-\theta}C^i{}_t{}^*. \tag{4}$$

Aggregating the demands in (4) across all households worldwide and equating the resulting equation to the output of good j produced in the home country, $y_t(j)$, yields the following market clearing condition for any product j in period t:

$$y_t(j)=n_tC^i{}_t(j)+(1-n_t)C^i{}_t{}^*(j)=(P_t(j)/P_t)^{-\theta}C_t^w, \tag{5}$$

where $P_t(j)/P_t=P_t^*(j)/P_t^*$ from the law of one price, and $C_t^w\equiv(n_tC^i{}_t(j)+(1-n_t)C^i{}_t{}^*(j))$ is aggregate per capita world consumption. Similarly, for product j of the foreign located firms, we obtain $y_t^*(j)=n_tC^i{}_t(j)+(1-n_t)C^i{}_t{}^*(j)=(P_t^*(j)/P_t^*)^{-\theta}C_t^w$. Given a symmetric equilibrium in which $P_t(j)=P_t(h)$ and $P_t^*(j)=P_t^*(f)$, $\forall j$, the real prices can be rewritten as

$$P_t(h)/P_t=(P_t(h)/\varepsilon_t)/P_t^*=[s+(1-s)(\varepsilon_tP_t^*(f)/P_t(h))^{1-\theta}]^{-1/(1-\theta)}, \tag{6}$$

$$\varepsilon_tP_t^*(f)/P_t=P_t^*(f)/P_t^*=[s(\varepsilon_tP_t^*(f)/P_t(h))^{\theta-1}+(1-s)]^{-1/(1-\theta)}, \tag{7}$$

where $P_t(h)/P_t$ and $\varepsilon_tP_t^*(f)/P_t$ are the real prices of the home and foreign goods, which equal $(P_t(h)/\varepsilon_t)/P_t^*$ and $P_t^*(f)/P_t^*$, respectively. In the second stage, households maximize (1) subject to (2). The first-order conditions for this problem with respect to $B^i{}_{t+1}$ and $M^i{}_t$ can be written as

$$1/C^i{}_t=\beta E_t[(1+r_{t+1})/C^i{}_{t+1}], \tag{8}$$

$$M^i{}_t/P_t=\chi C^i{}_t((1+i_{t+1})/i_{t+1}), \tag{9}$$

where i_{t+1} is the nominal interest rate for home-currency loans between periods t and $t+1$, defined as usual by $1+i_{t+1}=(1+r_{t+1})E_t[(P_{t+1}/P_t)]$. Equation (8) is the Euler equation for consumption and (9) is the one for money demand. The terminal condition is $\lim_{T=\infty}[(1/\Pi_{\nu=t}^{t+T}(1+r_\nu)][B_{t+T+1}+(M_{t+T}/P_{t+T})]=0$.

In the monopolistic goods markets, each firm has some monopoly power over pricing.

Because home-located firm j hires labor domestically, given W_t, P_t and $C^w{}_t$, and subject to (3) and (5), home-located firm j faces the following profit-maximization problem: $\max_{P_t(j)} \Pi_t(j) = P_t(j) y_t(j) - \int_0^{n_t} W_t^i \ell_t^{di}(j) di = (P_t(j) - W_t) y_t(j)$, where $\Pi_t(j)$ denotes the nominal profit of home-located firm j. By substituting $y_t(j)$ from equation (5) into the firm's profit $\Pi_t(j)$ and then differentiating the resulting equation with respect to $P_t(j)$, we obtain the following price mark-up:

$$P_t(j) = (\theta/(\theta-1)) W_t. \tag{10}$$

Because W_t is given, from (10), all home-located firms charge the same price. In what follows, we define these identical prices as $P_t(j) = P_t(h)$, $j \in [0, s]$. These relationships imply that each home-located firm supplies the same quantity of goods, and hence each firm requires the same quantity of labor; i.e., $\ell^{id}{}_t(j) = \ell^{id}{}_t(h)$, $j \in [0, s]$, where the firm index j is omitted because of symmetry. The price mark-ups of foreign-located firms are identical because $P_t^*(j) = P_t^*(f)$, $j \in (s, 1]$. Substituting (5) and (10) into the real profit flows of the home- and foreign-located firms, $\Pi_t(h)/P_t$ and $\Pi_t(f)^*/P_t^*$, respectively, yields

$$\Pi_t(h)/P_t = (1/\theta)(P_t(h)/P_t)^{1-\theta} C^w{}_t, \quad \Pi_t(f)^*/P_t^* = (1/\theta)(P_t^*(f)/P_t^*)^{1-\theta} C^w{}_t. \tag{11}$$

The key feature of our model is that it allows workers to change their locations. Here, we assume that the driving force for cross-border relocation of workers to other countries is differences in current real wages between home- and foreign-located workers. This adjustment mechanism for relocation at time t is formulated as follows:

$$n_t - n_{t-1} = \gamma[(1-\tau^i{}_t) W^i{}_t/P_t - W^{i*}{}_t/P_t^*] = \gamma[(1-\tau^i{}_t) W^i{}_t/P_t - \varepsilon_t W^{i*}{}_t/P_t]. \tag{12}$$

where the third term can be rewritten by using PPP, and $\gamma (0 \leq \gamma < \infty)$ is a constant positive parameter that determines the degree of worker mobility between the two countries: a larger value of γ implies higher worker mobility between two countries. Intuitively, the parameter γ reflects the costs incurred by mobile workers in their new locations.

Following Corsetti and Pesenti (2001), we introduce nominal rigidities into the model in the form of one-period wage contracts under which nominal wages in period t are predetermined at time $t-1$ through negotiations between monopolistic labor suppliers and firms. In the monopolistic labor market, each household provides a single variety of labor input to a continuum of domestic firms that have market power over their labor inputs. Hence, the equilibrium labor-market conditions for the home and foreign countries imply that $\ell^{si}{}_t = \int_0^s \ell_t^{di}(j) dj$, $i \in [0, n_t]$ and $\ell^{si*}{}_t = \int_s^1 \ell_t^{di*}(j) dj$, $i \in (n_t, 1]$, respectively, where the left-hand sides represent the amounts of labor supplied by household i and the right-hand sides represent firms' total demand for household labor i. Taking W_t, P_t, $y_t(j)$, and n_t as given, by substituting $\ell^{si}{}_t=\int_0^s \ell_t^{di}(j)dj$ and equation (3) into the budget constraint, given by (2), and maximizing the lifetime utility, given by (1), with respect to the nominal wage $W^i{}_t$, we obtain the following first-order condition:

$$\phi(W^i{}_t/P_t)^{-1}E_{t-1}[\kappa\ell^{si}{}_t{}^2] = (1-\tau^i{}_t)(\phi-1)E_{t-1}[(\ell^{si}{}_t/C^i{}_t)]. \quad (13)$$

The right-hand side of (13) represents the marginal consumption utility of additional labor income resulting from an increase in labor effort. This term is positive because $\phi>1$. The left-hand side represents the marginal disutility of an associated increase in labor effort. Hence, each monopolistically competitive household uses (13) to set its wage rate.

Finally, the equilibrium condition for the integrated international bond market is given by $\int_0^{n_t} B_{t+1}^i di+\int_{n_t}^1 B_{t+1}^{i*} di=0$. The money markets are assumed always to clear in both countries, so that the equilibrium conditions are given by $M_t=\int_0^{n_t} M_t^i di$ and $M^*{}_t=\int_{n_t}^1 M_t^{i*} di$.

3　A symmetric steady state

In this section, we derive the solution for a symmetric steady state in which all exogenous variables are constant, initial net foreign assets are zero ($B_0=0$), $\tau_0=0$ and $s=\alpha=1/2$. The superscript i and the index j are omitted because households and firms make the same equilibrium choices within and between countries. Henceforth, we denote the steady-state values by using the subscript ss. In the symmetric steady state, in which all variables are constant in both countries, given the Euler equation for consumption (equation (8)), the constant real interest rate is given by $r_{ss}=(1-\beta)/\beta\equiv\delta$, where δ is the rate of time preference and r_{ss} is the steady-state real interest rate. In the symmetric steady state, $W^h{}_{ss}/P_{ss}=W^{f*}{}_{ss}/P^*{}_{ss}$ must be satisfied. From $W^h{}_{ss}/P_{ss}=(W^h{}_{ss}/W_{ss})(W_{ss}/P_{ss}(h))(P_{ss}(h)/P_{ss})$, $W^{f*}{}_{ss}/P^*{}_{ss}=(W^{f*}{}_{ss}/W^*{}_{ss})(W^*{}_{ss}/P^*{}_{ss}(f))(P^*{}_{ss}(f)/P^*{}_{ss})$, $P_{ss}=\varepsilon_{ss}P_{ss}{}^*$ and equation (10), we obtain $P_{ss}(h)=\varepsilon_{ss}P_{ss}{}^*(f)$. Substituting this into equations (6) and (7) yields steady-state real prices of

$$P_{ss}(h)/P_{ss} = (P_{ss}(h)/\varepsilon_{ss})/P_{ss}{}^* = \varepsilon_{ss}P_{ss}{}^*(f)/P_{ss} = P_{ss}{}^*(f)/P_{ss}{}^* = 1. \quad (14)$$

From (10) and (14), steady-state real wages are

$$W_{ss}/P_{ss} = W_{ss}{}^*/P_{ss}{}^* = (\theta-1)/\theta. \quad (15)$$

Because symmetry, which implies $n_{ss}=1-n_{ss}$, holds, the steady-state allocation of workers is

$$n_{ss} = 1/2. \tag{16}$$

In addition, from (13), we obtain

$$\ell^{s}{}_{ss} = \ell^{s*}{}_{ss} = C_{ss} = C^{*}{}_{ss} = C^{w}{}_{ss} = ((\phi-1)/\phi)^{1/2}((\theta-1)/\theta)^{1/2}(1/\kappa)^{1/2}. \tag{17}$$

4 The relationship between worker relocation and the exchange rate

Here, we solve a log-linear approximation of the system around the initial, zero-shock steady state with $B_{ss,0}=0$, which is the same as that derived in the previous section. We assume that the economy starts from the zero-shock steady state at period 0, and then in period 1, nominal wage rigidities arise. This implies that nominal wages cannot adjust instantaneously to an unanticipated wage tax policy in period 1. In addition, the condition for the optimal nominal wage (equation (13)) does not hold, and therefore, households are willing to satisfy any level of labor demand at predetermined wage rates because their (real) wage rates dominate their marginal labor costs. We refer to the period of the shock as the 'short-run'. Thereafter, in periods 2 and beyond, nominal wages adjust perfectly to their new steady-state values to be consistent with the optimal wage conditions, given by (13). In what follows, the time from period 2 onwards is referred to as the 'long run'.

For any variable X, we use $\widehat{X}$ to denote 'short-run' percentage deviations from the initial steady-state value; i.e., $\widehat{X}=dX_1/X_{ss,0}$, where $X_{ss,0}$ is the zero-shock initial steady-state value and subscript 1 denotes the period in which the shock takes place. These short-run percentage deviations are consistent with the length of nominal wage contracts. Thus, nominal wages and goods prices can be determined as $\widehat{W}=\widehat{W}^{*}=\widehat{P}(h)=\widehat{P}^{*}(f)=0$ in the short-run log-linearized equations. In addition, we use $\overline{X}$ to denote 'long-run' percentage deviations from the initial steady-state value; i. e., $\overline{X}=dX_2/X_{ss,0}=dX_{ss}/X_{ss,0}$, which is consistent with flexible nominal wages. Note that $X_2=X_{ss}$ because the new steady state is reached at period 2.

By log linearizing equation (12) around the symmetric steady state and setting $\widehat{P}(h)=\widehat{P}^{*}(f)=0$, we obtain the following log-linearized expression for the international distribution of workers:

$$\widehat{n} = -2\gamma((\theta-1)/\theta)[d\tau+\widehat{\varepsilon}]. \tag{18}$$

Equation (18) shows that an exchange rate depreciation induces global relocation of workers towards the foreign country for a given level of the wage tax. Equation (18) shows that nominal exchange rate changes have greater effects when the flexibility of relocation is greater (when γ is larger). By contrast, when relocation costs are high ($\gamma=0$), nominal exchange rate changes have a negligible effect on the relocation of workers. In addition, from equation (18), for a given level of the exchange rate, a wage tax increase by the home country ($d\tau>0$) leads workers to relocate to the foreign country, i.e., $\hat{n}<0$. This is because an increase in τ leads to a decrease in the after-tax relative real wage of workers located in the home country, which then leads to the relocation of some workers away from the home to the foreign country.

Analogously, in the long run, we obtain the following log-linearized expression for the international distribution of workers:

$$\bar{n} = -2\gamma((\theta-1)/\theta)[d\tau+\bar{\varepsilon}-(\overline{W}-\overline{W}^*)]. \tag{19}$$

From this equation, the long-run change in the spatial distribution of workers is affected positively by the difference in nominal wages between home- and foreign-located workers and affected negatively by the exchange rate. In addition, a wage tax increase by the home country ($d\tau>0$) leads workers to relocate into the foreign country in the long run.

Thus, from equations (18) and (19), we find that the exchange rate is one of the important determinants of the cross-border relocation of workers. In addition, from (18) and (19), increasing the wage tax rate has both short-term and longer-term cross-border relocation of workers to the country with lowered wage tax rate.

5 Analysis of wage taxation

We now consider the macroeconomic effects of an unanticipated increase in the home wage tax. As assumed in the previous section, the economy starts from the zero-shock steady state in period 0, and nominal wage rigidities and the relocation of workers characterize the economy for period 1. The short-run closed-form solution for the international distribution of workers is as follows:

$$\hat{n} = -2\gamma((\theta-1)/\theta)(1-\tilde{\theta})d\tau<0, \quad (0<\tilde{\theta}\leq 1), \tag{20}$$

where

$$\tilde{\theta} \equiv \left\{4\gamma\tilde{\gamma}((\theta-1)/\theta)^2\left[\delta+\frac{2}{\theta+1+4\gamma((\theta-1)/\theta)}\right]\Big/[1+\delta\tilde{\gamma}+2\delta\tilde{\gamma}((\theta/2)+2\gamma)((\theta-1)/\theta)^2]\right\} > 0$$

and $\tilde{\gamma} \equiv \left\{1/\theta+2((\theta-1)/\theta)\left[\frac{\theta+4\gamma((\theta-1)/\theta)}{\theta+1+4\gamma((\theta-1)/\theta)}\right]\right\}^{-1} > 0.$

The result in (20) shows that an increase in the home country's wage tax leads to the relocation of some workers away from the home country to the foreign country. Analogously, the long-run closed-form solution for the international distribution of workers is as follows:

$$\bar{n} = -2\gamma((\theta-1)/\theta)\left[\frac{1}{\theta+1+4\gamma((\theta-1)/\theta)}\right](\theta+1-\tilde{\theta})d\tau < 0. \tag{21}$$

The result in (21) also shows that an increase in the home country's wage tax leads to the relocation of some workers away from the home country to the foreign country. Next, we analyze the influence of the wage tax policy shock on the nominal exchange rate and short-run relative consumption. The closed-form solutions for these variables are as follows:

$$\hat{C}-\hat{C}^* = \tilde{\theta}d\tau. \tag{22}$$

$$\hat{\varepsilon} = -\tilde{\theta}d\tau. \tag{23}$$

Equation (22) shows that the domestic wage tax policy shock leads to an increase in the relative home consumption. Intuitively, an increase in the home wage tax results in less differentiated workers located in the home country because of relocation of some workers towards the foreign country. This then leads to an increase in the labor demand for home located workers per capita. This is because the distribution of firms is assumed to be fixed so that the total labor demand for home located workers remains unchanged. Therefore, the relocation raises relative home labor incomes per capita, which raises relative home consumption per capita. Because of this mechanism, the home currency must appreciate to restore equilibrium in the market for real balances (see equation (23)). This is because the real money demand for liquidity services is increasing in consumption per capita.

Equation (22) also shows that the tax policy shocks have greater effects the greater is the flexibility of relocation (the larger is γ). This is because the greater is the flexibility of relocation, the greater must be the shift in workers towards the foreign country for a given rise in the home wage tax rate.

6 World welfare effect

Following Obstfeld and Rogoff (1995), who ignore the welfare effect of real balances, we focus on the real component of an agent's utility, which comprises terms involving consumption and labor effort. By defining this real component as $U_0{}^R$, we can rewrite equation (1) as $U_0{}^R=\sum_{t=0}^{\infty}\beta^t(\log C_t-(\kappa/2)\ell_t^{s2})$. The impact of unanticipated wage tax policy shocks on domestic welfare is as follows:

$$d\hat{U}^R = (1/2)\{((1+\delta)/\delta)\tilde{\theta}-((\phi-1)/\phi)((\theta-1)/\theta)[4\gamma((\theta-1)/\theta) -[4\gamma((\theta-1)/\theta)+\theta]\tilde{\theta}]\}\left[1+(1/\delta)\left[\frac{1}{\theta+1+4\gamma((\theta-1)/\theta)}\right]\right]d\tau. \quad (24)$$

Analogously, the impact on foreign welfare is

$$d\hat{U}^{R*} = -(1/2)\{((1+\delta)/\delta)\tilde{\theta}-((\phi-1)/\phi)((\theta-1)/\theta)[4\gamma((\theta-1)/\theta) -[4\gamma((\theta-1)/\theta)+\theta]\tilde{\theta}]\}\left[1+(1/\delta)\left[\frac{1}{\theta+1+4\gamma((\theta-1)/\theta)}\right]\right]d\tau. \quad (25)$$

Here we can derive the world welfare effect of a rise in the home wage tax. For this purpose, we define world welfare as $U_t{}^W=n_tU_t{}^R+(1-n_t)U_t{}^{R*}$, where $n_{0,ss}=1/2$ denotes the initial steady-state distribution of workers and $U^R_{0,ss}=U^{R*}_{0,ss}$. From (24) and (25), the world welfare effect of an increase in the home wage tax rate is

$$d\hat{U}^W = dn(U^R_{0,ss}-U^{R*}_{0,ss})+(1/2)(d\hat{U}^R+d\hat{U}^{R*}) = 0. \quad (26)$$

This shows that changes in the world welfare in response to the home wage tax shocks offset each other between the two countries under the international worker mobility.

7 Conclusion

We have analyzed an increase in the wage tax rate of the home country on the exchange rate, consumption and world welfare using a new open economy macroeconomics model that incorporates international worker mobility and nominal wage rigidity. The main findings of our analysis are that i) an unanticipated increase in the home country's wage tax rate induces relocation of workers towards the foreign country, and the tax policy always leads to appreciation in the exchange rate through the increase in the relative home consumption, ii) the more flexibility is the labor relocation, the greater is the effect of the wage tax shock on both the exchange rate and the relative home consumption.

Appendix

Long-run equilibrium conditions

We derive the long-run equilibrium conditions of this model. By log-linearizing the model around the initial, zero-shock symmetric steady state with $B_{ss,0}=0$, we obtain the following equations that characterize the long-run equilibrium of the system:

$$\overline{P} = \overline{M} - \overline{C}, \quad \overline{P}^* = \overline{M}^* - \overline{C}^* \tag{A. 1}$$

$$\overline{C} = \delta\overline{B} + ((\theta-1)/\theta)(\overline{W}(h) - \overline{P} + \overline{\ell}^s) + (1/2\theta)[\overline{\Pi}(h) + \overline{\Pi}(f)^* + \overline{\varepsilon}] - (1/\theta)\overline{P} \tag{A. 2}$$

$$\overline{C}^* = -\delta\overline{B} + ((\theta-1)/\theta)(\overline{W}^*(f) - \overline{P}^* + \overline{\ell}^{s*}) + (1/2\theta)[\overline{\Pi}(h) - \overline{\varepsilon} + \overline{\Pi}(f)^*] - (1/\theta)\overline{P}^* \tag{A. 3}$$

$$\overline{y} = \theta(\overline{P} - \overline{P}(h)) + \overline{C}^W, \quad \overline{y}^* = \theta(\overline{P}^* - \overline{P}^*(f)) + \overline{C}^W \tag{A. 4}$$

$$\overline{C}^W \equiv (1/2)\overline{C} + (1/2)\overline{C}^* = (1/2)\overline{y} + (1/2)\overline{y}^* \equiv \overline{y}^W \tag{A. 5}$$

$$\overline{y} = \overline{n} + \overline{\ell}^d, \quad \overline{y}^* = -\overline{n} + \overline{\ell}^{d*} \tag{A. 6}$$

$$\overline{\ell}^s = \overline{\ell}^d, \quad \overline{\ell}^{s*} = \overline{\ell}^{d*} \tag{A. 7}$$

$$\overline{n} = -2\gamma((\theta-1)/\theta)(d\tau + (\overline{P} - \overline{P}^*) - (\overline{W} - \overline{W}^*)) \tag{A. 8}$$

$$\overline{\Pi}(h) = (1-\theta)\overline{P}(h) + \theta\overline{P} + \overline{C}^W, \quad \overline{\Pi}^*(f) = (1-\theta)\overline{P}^*(f) + \theta\overline{P}^* + \overline{C}^W \tag{A. 9}$$

$$\overline{P} = (1/2)\overline{P}(h) + (1/2)[\overline{\varepsilon} + \overline{P}^*(f)], \quad \overline{P}^* = (1/2)[\overline{P}(h) - \overline{\varepsilon}] + (1/2)\overline{P}^*(f) \tag{A. 10}$$

$$\overline{P}(h) = \overline{W} \quad \overline{P}^*(f) = \overline{W}^* \tag{A. 11}$$

$$\overline{\varepsilon} = \overline{P} - \overline{P}^* \tag{A. 12}$$

$$\overline{\ell}^s = \overline{W} - \overline{P} - \overline{C}, \quad \overline{\ell}^{s*} = \overline{W}^* - \overline{P}^* - \overline{C}^* \tag{A. 13}$$

where $\overline{B} \equiv dB_{t+1}/C^W_{ss,0}$, with ${C^W}_{ss,0}$ being the initial value of world consumption.[3)] The equations in (A. 1) correspond to the money-demand equations. Equations (A. 2) and (A. 3) represent the long-run change in incomes (returns on real bonds, real labor incomes, and real profit incomes), which equal the long-run changes in consumption in each country. The equations in (A. 4) represent the world demand schedules for home and foreign products. Equation (A. 5) is the world goods-market equilibrium condition. The equations in (A. 6) represent the production technology, and those in (A. 7) represent the long-run labor-market clearing conditions for both countries. Equation (A. 8) is the cross-border relocation of workers. The equations in (A. 10) are the price index equations. The equations in (A. 11) represent the optimal pricing equations for firms in each country. Equation (A. 12) is the purchasing power parity equation. The equations in (A. 13) represent the first-order conditions for optimal wage setting.

Subtracting (A. 3) from (A. 2) yields the long-run (from period $t+1$ onwards) response of relative per capita consumption levels,

$$\overline{C}-\overline{C}^{*} = 2\delta\overline{B}+((\theta-1)/\theta)(\overline{W}-\overline{W}^{*}-(\overline{P}-\overline{P}^{*})+\overline{\ell}^{s}-\overline{\ell}^{s*}) \tag{A.14}$$

From (A. 8) and (A. 12), we obtain

$$\overline{n} = -2\gamma((\theta-1)/\theta)(d\tau+\overline{\varepsilon}-(\overline{W}-\overline{W}^{*})) \tag{A.15}$$

From equations (A. 4), (A. 6), (A. 7) and (A. 11), we obtain

$$\overline{\ell}^{s}-\overline{\ell}^{s*} = 2\overline{n}+\theta[\overline{\varepsilon}-(\overline{W}-\overline{W}^{*})] \tag{A.16}$$

From equation (A. 8) and (A. 16), we obtain

$$\overline{\ell}^{s}-\overline{\ell}^{s*} = 4\gamma((\theta-1)/\theta)d\tau+(4\gamma((\theta-1)/\theta)+\theta)(\overline{\varepsilon}-(\overline{W}-\overline{W}^{*})) \tag{A.17}$$

From equations (A. 12) and (A. 13), we obtain

$$\overline{\ell}^{s}-\overline{\ell}^{s*}+\overline{C}-\overline{C}^{*} = \overline{W}-\overline{W}^{*}-\overline{\varepsilon} \tag{A.18}$$

Substituting (A. 18) into equation (A. 17) yields

$$\overline{\ell}^{s}-\overline{\ell}^{s*} = -\left[\frac{\theta+4\gamma((\theta-1)/\theta)}{\theta+1+4\gamma((\theta-1)/\theta)}\right](\overline{C}-\overline{C}^{*})+\left[\frac{4\gamma((\theta-1)/\theta)}{\theta+1+4\gamma((\theta-1)/\theta)}\right]d\tau \tag{A.19}$$

From (A. 12) and (A. 14), we obtain

$$\overline{C}-\overline{C}^{*} = 2\delta\overline{B}+((\theta-1)/\theta)(\overline{W}-\overline{W}^{*}-\overline{\varepsilon}+\overline{\ell}^{s}-\overline{\ell}^{s*}) \tag{A.20}$$

Then, substituting (A. 18) into equation (A. 20) yields

$$\overline{C}-\overline{C}^{*} = 2\delta\overline{B}+((\theta-1)/\theta)(2(\overline{\ell}^{s}-\overline{\ell}^{s*})+\overline{C}-\overline{C}^{*}) \tag{A.21}$$

Substituting (A. 18) into equation (A. 15) yields

$$\overline{n} = -2\gamma((\theta-1)/\theta)[d\tau-(\overline{\ell}^{s}-\overline{\ell}^{s*})-(\overline{C}-\overline{C}^{*})] \tag{A.22}$$

In addition, substituting (A. 19) into equation (A. 22) yields

$$\overline{n} = -2\gamma((\theta-1)/\theta)\left\{\left[\frac{\theta+1}{\theta+1+4\gamma((\theta-1)/\theta)}\right]d\tau-\left[\frac{1}{\theta+1+4\gamma((\theta-1)/\theta)}\right](\overline{C}-\overline{C}^{*})\right\} \tag{A.23}$$

Finally, substituting (A. 19) into (A. 21) yields the following long-run response of relative consumption levels:

$$\overline{C}-\overline{C}^{*}$$
$$=2\delta\left\{\frac{1}{\theta}+2((\theta-1)/\theta)\left[\frac{\theta+4\gamma((\theta-1)/\theta)}{\theta+1+4\gamma((\theta-1)/\theta)}\right]\right\}^{-1}\overline{B}$$
$$+((\theta-1)/\theta)\left\{\frac{8\gamma((\theta-1)/\theta)}{\theta+1+4\gamma((\theta-1)/\theta)}\right\}\left\{\frac{1}{\theta}+2((\theta-1)/\theta)\left[\frac{\theta+4\gamma((\theta-1)/\theta)}{\theta+1+4\gamma((\theta-1)/\theta)}\right]\right\}^{-1}d\tau \tag{A. 24}$$

Equation (A. 24) shows that a home country trade surplus permanently raises home consumption relative to foreign consumption. Equation (A. 24) can be rewritten as

$$\overline{C}-\overline{C}^{*}=2\delta\tilde{\gamma}\overline{B}+((\theta-1)/\theta)\left\{\frac{8\gamma((\theta-1)/\theta)}{\theta+1+4\gamma((\theta-1)/\theta)}\right\}\tilde{\gamma}d\tau \tag{A. 25}$$

where

$$\tilde{\gamma}\equiv\left\{1/\theta+2((\theta-1)/\theta)\left[\frac{\theta+4\gamma((\theta-1)/\theta)}{\theta+1+4\gamma((\theta-1)/\theta)}\right]\right\}^{-1}>0 \tag{A. 26}$$

Given (A. 1) and (A. 12), subtracting the foreign money-demand equation from its home counterpart yields

$$\bar{\varepsilon}=\overline{M}-\overline{M}^{*}-(\overline{C}-\overline{C}^{*}) \tag{A. 27}$$

Equation (A. 27) states that the long-run exchange rate change depends on the difference between the long-run change in the nominal money supply and the relative change in long-run consumption.

Short-run equilibrium conditions

We derive short-run equilibrium conditions of this model. By log-linearizing the model around the initial, zero-shock symmetric steady state with $B_{ss,0}=0$, we obtain the following equations that characterize the short-run equilibrium of the system:

$$\overline{C}=\widehat{C}+(\delta/(1+\delta))\bar{r},\quad \overline{C}^{*}=\widehat{C}^{*}+(\delta/(1+\delta))\bar{r} \tag{A. 28}$$

$$\widehat{M}-\widehat{P}=\widehat{C}-\bar{r}/(1+\delta)-(\overline{P}-\widehat{P})/\delta,\quad \widehat{M}^{*}-\widehat{P}^{*}=\widehat{C}^{*}-\bar{r}/(1+\delta)-(\overline{P}^{*}-\widehat{P}^{*})/\delta \tag{A. 29}$$

$$\overline{B}=-((\theta-1)/\theta)\widehat{P}+((\theta-1)/\theta)(\hat{n}+\hat{\ell}^{d})+(1/2\theta)[\widehat{\Pi}(h)+\widehat{\Pi}(f)^{*}+\hat{\varepsilon}-2\widehat{P}]-\widehat{C} \tag{A. 30}$$

$$-\overline{B}=-((\theta-1)/\theta)\widehat{P}^{*}+((\theta-1)/\theta)(-\hat{n}+\hat{\ell}^{d*})+(1/2\theta)[\widehat{\Pi}(h)-\hat{\varepsilon}+\widehat{\Pi}(f)^{*}-2\widehat{P}^{*}]-\widehat{C}^{*} \tag{A. 31}$$

$$\hat{y}=\theta\widehat{P}+\widehat{C}^{W},\quad \hat{y}^{*}=\theta\widehat{P}^{*}+\widehat{C}^{W} \tag{A. 32}$$

$$\hat{\ell}^{d}=-\hat{n}+\hat{y},\quad \hat{\ell}^{d*}=\hat{n}+\hat{y}^{*} \tag{A. 33}$$

$$\widehat{\Pi}(h)=\theta\widehat{P}+\widehat{C}^{W},\quad \widehat{\Pi}^{*}(f)=\theta\widehat{P}^{*}+\widehat{C}^{W} \tag{A. 34}$$

$$\hat{n}=-2\gamma((\theta-1)/\theta)(d\tau+\hat{\varepsilon}) \tag{A. 35}$$

$$\widehat{C}^W \equiv (1/2)\widehat{C}+(1/2)\widehat{C}^* = (1/2)\hat{y}+(1/2)\hat{y}^* \equiv \hat{y}^W \tag{A. 36}$$

$$\widehat{P} = (1/2)\hat{\varepsilon}, \quad \widehat{P}^* = -(1/2)\hat{\varepsilon} \tag{A. 37}$$

$$\hat{\ell}^s = \hat{\ell}^d, \quad \hat{\ell}^{s*} = \hat{\ell}^{d*} \tag{A. 38}$$

where we set nominal wages and prices of goods as $\widehat{W}=\widehat{W}^*=\widehat{P}(h)=\widehat{P}^*(f)=0$ in the above short-run log-linearized equations. The equations in (A. 28) are the Euler equations. Those in (A. 29) describe equilibrium in the money markets in the short run. The equations in (A. 30) and (A. 31) are linearized short-run current account equations. The equations in (A. 32) represent the world demand schedules for representative home and foreign products. Equation (A. 33) is the production function. The equations in (A. 34) are the nominal profit equations for representative home and foreign firms. Equation (A. 35) is the cross-border relocation of workers. Equation (A. 36) is the world goods-market equilibrium condition. Equation (A. 37) is the price index equation in the short run. The equations in (A. 38) represent the short-run labor-market clearing conditions for both countries. Subtracting (A. 31) from (A. 30) yields

$$2\overline{B} = -((\theta-1)/\theta)(\widehat{P}-\widehat{P}^*)+((\theta-1)/\theta)(\hat{\ell}^s-\hat{\ell}^{s*})+(1/\theta)[\hat{\varepsilon}-(\widehat{P}-\widehat{P}^*)]-(\widehat{C}-\widehat{C}^*) \tag{A. 39}$$

Given equations (A. 32), (A. 33), and (A. 37) and subtracting, relative labor demand is

$$\hat{\ell}^s-\hat{\ell}^{s*} = -2\hat{n}+\theta(\widehat{P}-\widehat{P}^*) \tag{A. 40}$$

From (A. 40), the relative labor demand change is proportional to the change in the relative product demand change, and hence it depends on consumption switching. From (A. 35), the short-run change in cross-border relocation of workers is as follows:

$$\hat{n} = -2\gamma((\theta-1)/\theta)(d\tau+\hat{\varepsilon}) \tag{A. 41}$$

Substituting (A. 37), (A. 40), and (A. 41) into (A. 39) yields

$$\overline{B} = 2\gamma((\theta-1)/\theta)^2 d\tau+(\theta/2+2\gamma)((\theta-1)/\theta)^2\hat{\varepsilon}-(1/2)(\widehat{C}-\widehat{C}^*) \tag{A. 42}$$

Given (A. 28), subtracting the foreign Euler equation from its home counterpart yields the following relative per capita consumptions dynamics:

$$\overline{C}-\overline{C}^* = \widehat{C}-\widehat{C}^* \tag{A. 43}$$

From (A. 29), subtracting the foreign money-demand equation from its home counterpart yields

$$\widehat{M}-\widehat{M}^{*}-\hat{\varepsilon}=\widehat{C}-\widehat{C}^{*}-(\bar{\varepsilon}-\hat{\varepsilon})/\delta \tag{A. 44}$$

In what follows, we assume that the nominal money supply is held constant in both countries, so that $\widehat{M}=\overline{M}=\widehat{M}^{*}=\overline{M}^{*}=0$.

The derivation of $\hat{\varepsilon}$ and $\widehat{C}-\widehat{C}^{*}$

We consider the macroeconomic effects of an unanticipated rise in the wage tax. Substituting (A. 43) and $\bar{\varepsilon}$ from equation (A. 44) into equation (A. 27) yields[4)]

$$\hat{\varepsilon}=-(\widehat{C}-\widehat{C}^{*}) \tag{A. 45}$$

A second schedule in $\hat{\varepsilon}$ and $\widehat{C}-\widehat{C}^{*}$ may be derived by using the short-run and long-run current account equations for both countries. Substituting equation (A. 43) into equation (A. 25) yields

$$\widehat{C}-\widehat{C}^{*}=2\delta\tilde{\gamma}\overline{B}+((\theta-1)/\theta)\left\{\frac{8\gamma((\theta-1)/\theta)}{\theta+1+4\gamma((\theta-1)/\theta)}\right\}\tilde{\gamma}d\tau \tag{A. 46}$$

From equations (A. 42) and (A. 46), we obtain the following relationship between the exchange rate change and the relative consumption change:

$$\begin{aligned}&2\delta\tilde{\gamma}((\theta/2)+2\gamma)((\theta-1)/\theta)^{2}\hat{\varepsilon}\\&=(1+\delta\tilde{\gamma})(\overline{C}-\overline{C}^{*})-4\gamma\tilde{\gamma}((\theta-1)/\theta)^{2}\left[\delta+\frac{2}{\theta+1+4\gamma((\theta-1)/\theta)}\right]d\tau\end{aligned} \tag{A. 47}$$

Now, we can combine these two curves to solve jointly for $\hat{\varepsilon}$ and $\widehat{C}-\widehat{C}^{*}$. From equations (A. 45) and (A. 47), the exchange rate change is

$$\hat{\varepsilon}=-\tilde{\theta}d\tau,\quad(0<\tilde{\theta}\leq1) \tag{A. 48}$$

where

$$\tilde{\theta}\equiv\left\{4\gamma\tilde{\gamma}((\theta-1)/\theta)^{2}\left[\delta+\frac{2}{\theta+1+4\gamma((\theta-1)/\theta)}\right]\Big/[1+\delta\tilde{\gamma}+2\delta\tilde{\gamma}((\theta/2)+2\gamma)((\theta-1)/\theta)^{2}]\right\}>0 \tag{A. 49}$$

and the relative consumption change is

$$\widehat{C}-\widehat{C}^{*}=\tilde{\theta}d\tau \tag{A. 50}$$

From (A. 35) and (A. 48), we obtain

$$\widehat{n} = -2\gamma((\theta-1)/\theta)(1-\tilde{\theta})d\tau < 0 \tag{A. 51}$$

From (A. 23) and (A. 50), we obtain

$$\overline{n} = -2\gamma((\theta-1)/\theta)\left[\frac{1}{\theta+1+4\gamma((\theta-1)/\theta)}\right](\theta+1-\tilde{\theta})d\tau < 0 \tag{A. 52}$$

Next, we derive absolute home and foreign consumption, and absolute home and foreign labor supply. From equations (A. 4), (A. 7), (A. 10), (A. 11), and (A. 13), we obtain $\overline{C}^W=0$. Hence, from (A. 28), we obtain $\widehat{C}^W=-[\delta/(1+\delta)]\overline{r}$. In addition, from (A. 29) and the assumption $\widehat{M}=\overline{M}=\widehat{M}^*=\overline{M}^*=0$, we obtain $\widehat{C}^W=[1/(1+\delta)]\overline{r}$. Hence, we obtain $\widehat{C}^W=0$. Therefore, from equation (A. 50) and $\widehat{C}^W=0$, the change in the short-run level of absolute home consumption is

$$\widehat{C} = (1/2)\tilde{\theta}d\tau \tag{A. 53}$$

Furthermore, from $\overline{C}^W=0$, the change in long-run absolute consumption is

$$\overline{C} = (1/2)\tilde{\theta}d\tau \tag{A. 54}$$

Analogously, the changes in short-run and long-run foreign absolute consumption are

$$\widehat{C}^* = -(1/2)\tilde{\theta}d\tau, \quad \overline{C}^* = -(1/2)\tilde{\theta}d\tau \tag{A. 55}$$

Next, we calculate the short-run levels of absolute labor supply, $\widehat{\ell}^s$ and $\widehat{\ell}^{s*}$. From equations (A. 40) and (A. 41), the relative labor supply change is

$$\widehat{\ell}^s-\widehat{\ell}^{s*} = -2\widehat{n}+\widehat{\ell}^d-\widehat{\ell}^{d*} = -2\widehat{n}+\widehat{y}-\widehat{y}^* = \{4\gamma((\theta-1)/\theta)-[4\gamma((\theta-1)/\theta)+\theta]\tilde{\theta}\}d\tau \tag{A. 56}$$

From equations (A. 33), (A. 36), (A. 38), and $\widehat{C}^W=0$, we obtain

$$\widehat{C}^W = \widehat{\ell}^{sW} = 0 \tag{A. 57}$$

From equations (A. 56), and (A. 57), we obtain

$$\widehat{\ell}^s = (1/2)\{4\gamma((\theta-1)/\theta)-[4\gamma((\theta-1)/\theta)+\theta]\tilde{\theta}\}d\tau \tag{A. 58}$$

Analogously, the short-run foreign absolute labor supply is

$$\widehat{\ell}^{s*} = -(1/2)\{4\gamma((\theta-1)/\theta)-[4\gamma((\theta-1)/\theta)+\theta]\tilde{\theta}\}d\tau \tag{A. 59}$$

Substituting equations (A. 43) and (A. 50) into equation (A. 19) yields

$$\bar{\ell}^{s}-\bar{\ell}^{s*}=\left[\frac{1}{\theta+1+4\gamma((\theta-1)/\theta)}\right][4\gamma((\theta-1)/\theta)-[\theta+4\gamma((\theta-1)/\theta)]\tilde{\theta}]d\tau \quad \text{(A. 60)}$$

From equation (A. 60) and $\bar{C}^{W}=\bar{\ell}^{sW}=0$, the changes in long-run absolute labor supply are

$$\bar{\ell}^{s}=\frac{1}{2}\left[\frac{1}{\theta+1+4\gamma((\theta-1)/\theta)}\right][4\gamma((\theta-1)/\theta)-[\theta+4\gamma((\theta-1)/\theta)]\tilde{\theta}]d\tau \quad \text{(A. 61)}$$

$$\bar{\ell}^{s*}=-\frac{1}{2}\left[\frac{1}{\theta+1+4\gamma((\theta-1)/\theta)}\right][4\gamma((\theta-1)/\theta)-[\theta+4\gamma((\theta-1)/\theta)]\tilde{\theta}]d\tau \quad \text{(A. 62)}$$

Following Obstfeld and Rogoff (1995), who ignore the welfare effect of real balances, we focus on the real component of an agent's utility, which comprises terms involving consumption and labor effort. By defining this real component as $U_0{}^{R}$, we can rewrite equation (1) as $U_0{}^{R}=\sum_{t=0}^{\infty}\beta^{t}(\log C_t-(\kappa/2)\ell_t^{s2})$. Given that the new steady state is reached after just one period, total differentiation of this equation yields

$$dU_0{}^{R}=\hat{C}-\kappa\ell_{0,ss}^{s2}\hat{\ell}^{s}+(1/\delta)(\bar{C}-\kappa\ell_{0,ss}^{s2}\bar{\ell}^{s}) \quad \text{(A. 63)}$$

where $\ell^{s}{}_{0,ss}$ denotes the initial steady-state level of labor supply. Substituting (17), (A. 53), (A. 54), (A. 58), and (A. 61) into (A. 63) yields

$$d\hat{U}^{R}=(1/2)\{((1+\delta)/\delta)\tilde{\theta}-((\phi-1)/\phi)((\theta-1)/\theta)[4\gamma((\theta-1)/\theta)$$
$$-[4\gamma((\theta-1)/\theta)+\theta]\tilde{\theta}]\}\left[1+(1/\delta)\left[\frac{1}{\theta+1+4\gamma((\theta-1)/\theta)}\right]\right]d\tau \quad \text{(A. 64)}$$

Analogously, from (17), (A. 55), (A. 59), and (A. 62), the impact on foreign welfare is

$$d\hat{U}^{R*}=-(1/2)\{((1+\delta)/\delta)\tilde{\theta}-((\phi-1)/\phi)((\theta-1)/\theta)[4\gamma((\theta-1)/\theta)$$
$$-[4\gamma((\theta-1)/\theta)+\theta]\tilde{\theta}]\}\left[1+(1/\delta)\left[\frac{1}{\theta+1+4\gamma((\theta-1)/\theta)}\right]\right]d\tau \quad \text{(A. 65)}$$

Here we can derive the world welfare effect of a rise in the home wage tax. For this purpose, we define world welfare as $U_t{}^{W}=n_tU_t{}^{R}+(1-n_t)U_t{}^{R*}$, where $n_{0,ss}=1/2$ denotes the initial steady-state distribution of workers and $U_{0,ss}{}^{R}=U_{0,ss}{}^{R*}$. From (A. 64) and (A. 65), the world welfare effect of an increase in the home wage tax rate is

$$d\hat{U}^{W}=dn(U_{0,ss}{}^{R}-U_{0,ss}{}^{R*})+(1/2)(d\hat{U}^{R}+d\hat{U}^{R*})=0 \quad \text{(A. 66)}$$

(A. 66) is equivalent to (26).

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Notes

1) In what follows, we mainly focus on the description of the home country because the foreign country is described analogously.

2) Throughout the paper, we also use the index $j \in [0, 1]$ to refer to the product of firm j.

3) In a symmetric steady state, initial net foreign assets are zero; i.e., $B_0 = 0$. Following Obstfeld and Rogoff (1995), we scale bond holdings by using the initial level of world consumption, $C^W_{ss,0}$.

4) Given (A. 43), comparing equations (A. 27) and (A. 45) therefore implies that $\hat{\varepsilon} = \bar{\varepsilon}$, which shows that the change in the exchange rate is permanent following a one-off unanticipated rise in the wage tax.

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