A
VALUATION
OF
ANNUITIES
AND
LEASES certain,
FOR
A SINGLE LIFE.
WITH
TABLES,
SHOWING

At one View the Value of them, in Years, and the eighth Part of a Year, for every Year of Life, from the Age of 1 to 70, and from thence to the End of Life, for every Period of five Years:
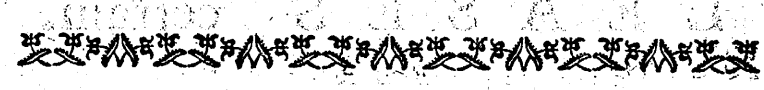
At the several Rates of 3, 4, 5, and 6 *per Cent.* Interest.

By *WEYMAN LEE*, Esq;
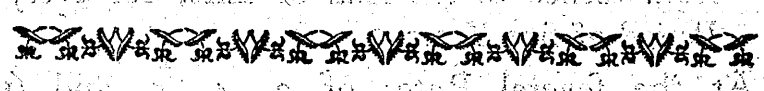Of the *Inner Temple.*

SECOND EDITION.
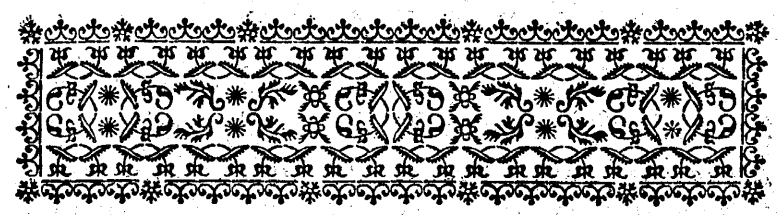
TO THE

READER.

MY Bookseller having call'd upon me for another Edition of the following Treatise, I have taken a Review of it, and find no Reason to alter my Sentiments in any Kind; but, on the contrary, I adhere to my fundamental Position, That the even Chance of the Duration of a Life, computed once only for the whole Life, is the sole true Measure of the Value of an Annuity depending on that Life. There have, indeed, of late appear'd in Print Attempts by several Persons to fix the Value of these Life-Annuities, most or all of which follow the same Track, and persue the Rule laid down by Dr. *Halley* for that Purpose: But I don't perceive that they have done any Thing but discover'd fresh Instances, to show, the Badness of that Rule, and that they falsify their own Schemes.

A 2   I shall

I fhall here take Notice, and that in very few Lines, of one Writer only, *viz.* Mr. *Stonehoufe*, who has wrote laft on this Subject. This Author, P. 17, gives us a Table, exhibiting *the Expectations of Life, or the Number of Years which a Perfon of any Age has an equal Chance to enjoy*; or, as I would call them, the Chances of Vitality on a Life. On a Life of ten Years he ftates the Chance there at 36-48, and of twenty Years at 28-55: The Difference between them is 7-93, or almoft eight Years; that is, in that Period of younger Life the Chance of Vitality has decreafd near eight Years. On a Life of forty Years Age he ftates the Chance there at 18-74, and of fifty Years Age at 15-28. The Difference between them is 3-46, or three Years and near an Half; that is, in that Period of elder Life the Chance of Vitality has decreafd only three Years and near an Half. Now this Want of Proportion in the Decreafe of the Expectations of Life, or Chances of Vitality, in thefe two Periods, is fo grofs and obvious, that it needs no Proof or Comment; and, if it did, this Author's two Tables immediately preceding, which are framed to fhow the Probability of Life from the Bills of Mortality of the City of

of *London*, do manifeft it moft evidently. From the Number given in one of thefe Tables, and on a Computation upon the other, it appears, that the Number of Perfons dying in the Period of younger Life is 31 out of 509 Perfons exifting at the Beginning of the Period; and the Number of Perfons dying in the Period of elder Life is 96 out of 307 Perfons exifting at the Beginning of the Period of elder Life. This Defect then of the Want of a proportional Decreafe is apparent: And, if any one will give themfelves the Trouble of making a Computation in the Manner I have done, they will find, as I did, that on thefe very Tables the real Decreafe of the Expectations of Life, or of the Chances of Vitality, for the City of *London*, for the Period of younger Life, is about one fixteenth Part of the Chances of Vitality for the whole Life, which is about two Years and one Eighth; and for the Period of elder Life, near one third Part of the like Chances; that is, near fix Years and three Quarters. Thefe, I fay, are the true Decreafes of the Expectations or Chances of Vitality of Life, for thefe two Periods of Life, if the Number of Perfons dying be the Rule of determining them: And this

Author

Author himfelf intended and thought he made ufe of it, but was miftaken in the Manner of applying it: And in Truth, if this be not the Rule, there neither is, nor can be, one. And there I reft that Matter.

Let us now look a little into his Tables for the Value of Annuities on a Life. In P. 42, &c. on a Life of fix Years Age, Intereft 3, the Value is ftated at 20-38; and the Term to which 20-38 is equal, or cor-refponds, is 32 ½; but the Expectation, or Chance of Vitality of fuch a Life, as before noticed here, on his own Tables, is 38 ½: So that there is a Difference of full fix Years between the Chance of the Duration of the Life, and the Chance of the Duration of the Annuity on that Life. It is urged, I know, that during thefe 38 ½ many Chances of Mortality do arife on that Life; and I admit it: But, in computing the Expectation or Chance of Vitality of the Life, thofe Chances of Mortality are already once deducted and difcounted; and, if we deduct and difcount them again in computing the Value of the Annuity, 'tis certain and plain that they are twice deducted and difcounted, merely to depretiate the Value of the Annuity. I ob-ferve farther, as on a Life of fix Years Age the

the Annuity is 20-38, and the Term to which that Value correfponds, Intereft 3, is 32 ½; fo an Annuity on the fame Life, Intereft 5, is 15-23, and the Term to which that Value correfponds is 29 ½: whereby there is a Variance of three Years between the Chance of the Duration of the Life, or of the Annuity, or both, according to the Dif-ference in the Intereft made ufe of in com-puting fuch Value. From hence I argue thus: The Life or Exiftence of a Life-Annuitant is a Matter of Fact: The Receipt of the Annuity by fuch Annuitant is likewife fo. Thefe two Matters of Fact do and ne-ceffarily muft concur; that is, they do and muft continue for, and determin at, one and the fame Time. Now, to fay that there is a Chance that one of thefe Facts fhall fubfift for a Time after all Chance of the other is extinguifhd and gone, and yet both thefe Events do and of Neceffity muft coincide, is contrary to common Senfe and Reafon, and borders upon a Contradiction in Terms. To this I muft add, that 'tis equally abfurd and contradictious to fay that the Chance of the Duration of the Annuity, or of the Life, which of Neceffity muft be the fame, fhould depend on, and be governd by, the Intereft

which

which is made ufe of to adjuft the Value of fuch Annuity: And yet this is apparently the Cafe on thefe Tables, and unavoidably will be fo on all that have been or can be form'd in Dr. *Halley's* Method, and in moft of his Problems and Solutions on Annuities for one, two, or three Lives, either joint or on Survivorfhip. But, if Perfons will give themfelves Leave to compute the Value of thefe Sort of Annuities upon fuch Principles, which have no juft Foundation in Fact or Reafon, and thofe who are concern'd in Point of Intereft will continue to make ufe of Tables fo form'd, till by dear-bought Experience they find out the Falfity of them; they muft, and for me they fhall, follow their own Imaginations: For I fhall give you no farther Trouble about the Matter; but fubfcribe myfelf

Your obedient

humble Servant,

Inner Temple,
March 8, 1754.

Weyman Lee.

A
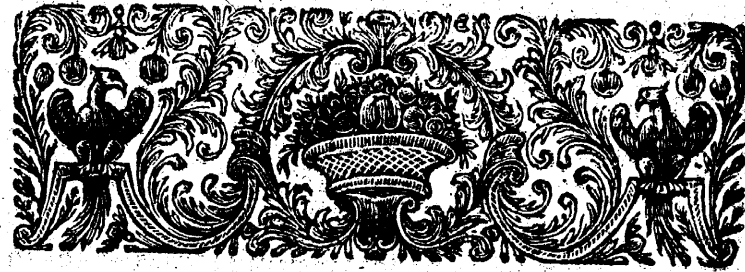
VALUATION

OF

ANNUITIES

FOR

LIVES, &c.

SOME Time ago, I publifh'd an Effay on the Method of afcertaining the Value of Annuities, and of Leafes reduced to Annuities certain, for one or more Lives; and therein I laid down this Pofition as the Groundwork, *viz.* that the even Chance of the Duration of a Life, computed once only for the whole Life, was the fole true Meafure of the Value of an Annuity

B   depending

depending on that Life. But fince fome Perfons have made Obfervations on this my Method, and others have form'd Valuations of thefe Eftates, and have defended and ftill adhere to the Method prefcribed by Dr. *Halley*; I think it will not be amifs to make fome Remarks upon them, partly to anfwer and remove the Objections which have been offer'd, but chiefly, becaufe it gives me an Opportunity to maintain and further confirm the fundamental Pofition.

Amongft my Adverfaries Mr. *H. B.* appears to be moft potent, and indeed the only one who deferves any ferious Confideration; and the Queftion between him and me is, Whether Dr. *Halley*'s Rule or mine be the righter, for forming an Eftimate of the Value of Annuities on a fingle given Life. The Dr's Rule is to find the Value of the Annuity for each Year of the given Life to an 100; which is done, and can be no otherwife done, but by computing the Chances of Mortality arifing in each Year, and deducting the Value thereof out of the Value of an Annuity abfolute for that Year, and then to put together all the Values fo collected, and the Sum total is the Value of the Annuity for the given Life. Mine is, to find to what Term of Years any Life is equal, or an even Chance may probably continue; and the Value of an Annuity for the given Life will be the fame as the Value of an Annuity for that Term of Years.

Now if my Rule be a right one, and from the flender Objections made to it, and the weak

Defence of the other by this Advocate, and, if this other could be defended, by this Hand it would have been defended, I am fully fatisfied that it is a right one; this my Rule, and the Tables form'd upon it, will have a manifeft Advantage over any other Rule, and any Tables that have been or can be planned from thence. By the other Rule, if the Value of an Annuity for a given Life, is to be computed, the Computer, on every Variation of the Rate of Intereft, whether real or on Suppofition only, muft, of Neceffity, calculate the Value of the Annuity for every individual Year, to which the Life of the Perfon nominated may poffibly continue, for each Year feparately and diftinctly, tho' it fhould happen to be 80, 90, or an 100 Years; and believe me, thofe who have tried it, have found this not only a tedious, but very intricate Affair: Nay, this elaborate and perplex Calculation does not attend only on every Change of the Rate of Intereft, but occurs again in fome Meafure on every Age of Life, tho' it differs one Year only, and even tho' the Rate of Intereft is, or is fuppofed to be, the fame. On the other Side, and upon my Rule, let the Rate of Intereft vary as often or as much as you will, either in Practice or Imagination, yet the Chance of the Duration of a Life of one and the fame Age, in one and the fame Country, will not vary, but perpetually be the fame, and equal to the fame Term of Years, and fuch Term of Years appears in my Tables on the firft View.

Thefe Tables go farther, and fhow what is the Value of fuch Term, that is, what is the Value

of the Annuity for the given Life at the Rates of 3, 4, 5, and 6 *per Cent.* Intereſt. And if we could ſuppoſe that the Intereſt of Money ſhould fall from 3 to 2½, or to 2, which I do not ſuſpect will ever be the Caſe, at leaſt not ſo ſoon as ſome People vulgarly imagin ; yet the Term, to which the given Life is equal, will be perpetually the ſame, and will appear in theſe Tables ; and, ſuch Term being once known, the Value of ſuch Term will be eaſily found out by the common Tables, which ſhall then be in Uſe, for the Value of Terms for Years, at the Rates of 2 or 2 ½ ; for I make no Doubt, but that, if and when thoſe ſhall become the common Rates of Intereſt, ſuch Tables will be conſtructed, for they are very eaſily conſtructed, and will be in every body's Hand.

Mr. *H. B.* begins his Attack on the Eſſay, with ſome Obſervations on the Table for Annuities on a ſingle Life ; but ſince I have thought fit, for the Reaſons which I ſhall hereafter mention to alter it, there is no Need to take any Notice of them. From my Table he proceeds to confider the Rule which I juſt now mentiond, and ſays, he will give a few Reaſons why he thinks the very Foundation I go upon is wrong, and the Rule I firſt lay down not a right one. I am very much obliged to the Gentleman, that he will give but a few Reaſons againſt the Rule ; and more ſo, that he has reduced thoſe few to one ; but he is Maſter of his own Reaſon. The Rule I have already ſet forth ; and the Inſtance I gave of it, which 'tis neceſſary to repeat here, becauſe of frequent References

ferences to it hereafter, was this : The Number of Perſons of the Age of 10, of which we will ſuppoſe *A* to be, by a Calculation made on the *Breſlaw* Table, is 661 ; the Half of this is 330 ½ ; and this Number we find between 51 and 52, ſo that *A* has an even Chance to live 41 Years and ſomething over. A Term for 41 Years Intereſt computed at *6l. per Cent.* is in Value 15—12 Centeſimals ; therefore an Annuity for the Life of *A*, who has an even Chance to live 41 Years and beyond, muſt be worth 15—12 Centeſimals and above. This Author admits the Premiſſes to be true ; *viz.* that *A* has an even Chance to live 41 Years ; that a Term for 41 Years is in Value 15—12 ; but denies the Concluſion, *viz.* that an Annuity for his Life is worth 15—12. His Reaſon for denying this Concluſion runs thus : *That, tho' the Chances of A's dying before 51 or after 51 are equal in Number, yet they are not equal in Value.* He then eſtabliſhes the Truth of this Poſition by a Demonſtration, in two ſeveral Inſtances, and from thence, without further Proof or Allegation, concludes : *Conſequently, an Annuity for A's Life, who has an even Chance to live 41 Years and beyond, is not worth as much as an Annuity for 41 Years abſolute, that is, is not worth 15—12,* Q. E. D.

When he ſays, that theſe Chances are of unequal Value, I ſuppoſe he means, that the Annuities, attendant on thoſe Chances for each Year, are of unequal Value ; and he cannot well mean any Thing elſe, becauſe in any other Senſe the Poſition is not true ; he might have ſaved himſelf the
Trouble

Trouble of a Demonſtration, that theſe Chances are of unequal Value, ſince the Annuities attending them are Part in Poſſeſſion, and Part in Reverſion ; and, to be ſure, there could not be Occaſion for a Demonſtration, at leaſt, not in two Inſtances, to have ſhown that the Annuities for a Year of a Life muſt be of more or leſs Value, as they are more or leſs removed from the firſt Year's Annuity ; for Inſtance, that the preſent Value of an Annuity for the ſeventh Year of a Man's Life is greater than the Value of the like Annuity for his twentieth Year. Admitting then, that theſe Chances, or the Annuities attending them, are of unequal Value, to which, after ſuch an irrefragable Proof as two Demonſtrations, I ſhall certainly ſubmit ; Does it follow from thence, that the Rule and the Values produced by it are wrong ? This is the Concluſion which he draws ; but, as I apprehend, long before he is come at it, for theſe Premiſſes and theſe Demonſtrations do no more prove it, than they do, that the three Angles of an equilateral Triangle are equal to two right ones. He admits that the Rule does not direct to put in more Chances in Number, than the Annuitant has a Right to ; and he does not ſo much as aſſert, that it takes in any Chances of greater Value than he has a Right to ; and this ought not only to have been aſſerted, but proved too, otherwiſe the Conſequence he draws is a ſtrange one ; for the Argument, as it now ſtands, is no better than this : An Annuitant, by this Rule, has no more Chances in Number than he has a Right to ; but,

but, in Fact, there are ſome Chances of greater Value than others, without proving or even aſſerting, that any ſuch Chances of greater Value are inſerted in the Computation. Now ſince the Matter in Diſpute is not, whether ſuch Annuitant, by Virtue of the Rule, has more Chances in Number, but whether he has ſome better in Value, than he has a Right to : This is evidently to beg the Queſtion, and not to prove it, notwithſtanding the terrible Sting at the Tail of it ; for the Demonſtrations plainly go to one Propoſition, *viz.* to prove this ths or that is the Value of ſuch Chances ; and the Argument concludes to another, *viz.* to ſuch and ſuch Chances the Annuitant has a Right, and no other.

However, to help out a lame Argument, for ſuch I muſt call it, as here formd, I will admit, that the Rule does direct to put in, and that the Value produced thereby does include, all thoſe Chances which ariſe on the Life of *A*, before he arrives at the Age of 41 ; and that the Chances, ſo inſerted, are of greater Value, than thoſe which ariſe after he is paſt 41 Years. But not one Word of Proof is here ſo much as offerd, that theſe greater Values are not the real Values, which an Annuitant on the Life of *A* has the Chance to receive ; and in the Eſſay it is aſſerted, and ſo ſtrongly and fully proved, that theſe greater Values, and no other, are the very Values which ſuch an Annuitant has a Chance to receive, that I cannot well add any thing to confirm it.

But

But this Author, inftead of attempting to invalidate thefe Arguments, or to prove that the Annuitant on the Life of *A*, who has an even Chance to live 41 Years, has not an even Chance to enjoy his Annuity for as many Years as he has an even Chance to live, that is, 41 Years, or to enjoy thofe Chances which arife in thofe firft 41 Years of his Life; which he muft do, before he deftroys the Foundation of my Rule, and yet I may be confident he never will be able to do; inftead of this, he very valiantly demonftrates fomething, which, to be fure no Man did ever make any Doubt of, that the Chances, or the Values attending them, where fome are in Poffeffion, and fome are in Reverfion, are not all of equal Value.

Since this Author is fo full of, and fo learned in, the Doctrine of Chances, as he calls it, I will take the Liberty of offering to his Confideration an Argument or two, in Behalf of my Rule, according to his Way of Thinking. The Value of the Chances in this Cafe, depends on, or is one and the fame Thing, as, the Value of the Annuities dependant on thofe Chances. All the poffible Chances, which an Annuitant has on the Life of *A*, are agreed to be 661, and of thofe he has an even Chance to enjoy a Moiety, that is, 330 ½. The Annuities which attend thofe Chances for the whole Life, are Annuities for a Term of 90 Years, it being fuppofed poffible that he may live for fo many Years; the total Value of them for 90 Years, as I compute, is 16—58

16-58 Centefimals, and the Value of thofe in Poffeffion for 41 Years is 15-12, and of thofe in Reverfion 1-46. Since then the Annuitant on the Life of *A*, has an even Chance to 330 ½ Chances on his Life, and the Annuities will attend thofe Chances; and fince thefe 330 ½ Chances will, and from the Nature of the Life of Man of Neceffity muft, arife in the firft Part of thofe 90 Years, if they arife at all, and 'tis fuppofed in the Cafe to be an even Chance that fo many Chances will arife in his Life: He muft then of Neceffity have the Annuities attending thofe very Chances, that is, he muft have the Annuities for the firft 41 Years of Life. This Author, or whoever will undertake to anfwer this Argument, muft fhow that 'tis poffible for an Annuitant on the Life of *A* to have the Benefit of all thefe 330 ½ Chances, to which Number of Chances he has without Difpute a Right or an even Chance to enjoy, unlefs he does enjoy thofe which arife in the firft Part of Life, and in immediate Poffeffion, that is, thofe of the greateft Value; and this I am very fure he cannot fhow.

Again, Thefe Pofitions are not contefted: That an Annuity for a Term of 41 Years is in Value 15-12; and that an Annuitant for the Life of *A* has an even Chance that *A* lives for 41 Years. 'Tis a Maxim made ufe of by this Author, and is certainly a true one, that in eftimating the Value of Annuities for a Life all the poffible Chances of Life muft be computed, From hence I argue thus: To eftimate the Value

C   of

of an Annuity for the Life of *A*, we muſt compute all the poſſible Chances on the Life of *A*; the Chance that *A* lives for 41 Years is an even Chance, and conſequently is one of the poſſible Chances on the Life of *A*; therefore to eſtimate the Value of an Annuity for his Life we muſt compute this even Chance. The even Chance on this Life is to a Term for 41 Years. A Term for 41 is in Value 15-12. Therefore the Value of an Annuity for this Life is 15-12.

This Author, having by this and by no other Argument demoliſhd my Rule, even to the very Foundations, as he expreſſes himſelf, proceeds to conſider Dr. *Halley*'s Rule; and, having given it us, pronounces that 'tis moſt certainly right, my Objections notwithſtanding; and, to be even with him, I ſay, notwithſtanding his Defence, that 'tis certainly wrong. One of my Exceptions to his Rule, and which this Writer's Obſervations on it make neceſſary to repeat, is this: That the Value produced by his Rule is not the Value of an Annuity for the Life of *A*, but will be the Value of 90 ſeveral Annuities or Grants made to 90 ſeveral Perſons, ſuppoſe to N. 1, to N. 2, and ſo on to N. 90, formd in this Manner, *viz.* to N. 1 for a Year, if he ſhall live from 10 to 11 Years of Age; to N. 2 for a Year, if he ſhall live from 11 to 12 Years Age; and ſo on ſucceſſively to N. 90 for a Year, if he ſhall live from 99 to an 100 Years Age. I then affirm, that the Value of an Annuity for the Life of *A* abſolutely, or for 90 Years, if *A* ſo long lives,

is

is of a different and much greater Value than the Intereſt of N. 1, N. 2. and ſo on to 90, all put together. To this he ſays, that it woud take up too much Time to anſwer all the Arguments I have brought to ſupport this Aſſertion. This ſurely is a very ſmall Reaſon for not attempting to anſwer any of them, or the moſt material of them; and ſom People will be apt to ſurmiſe that he wanted ſomething elſe rather than Time to anſwer them. However, to be ſure it will be neceſſary for me to ſpend my Time in repeating and defending them; ſo that, ſince he has left them quiet and unanſwerd, I ſhall e'en leave them, in *ſtatu quo*, undefended.

He does indeed afterwards make an Attempt to prove, that an abſolute Annuity for this Life, which I contend to be in Value 15-12, is of no greater Value than theſe ſeveral Annuities or Grants, which I ſuppoſe to be made to 90 Perſons for 90 Years, when put together, that is, not more than 13-44. He ſays then, that 90 ſeveral Annuities granted to *A*, in this manner; the firſt Annuity for one Year, if he ſhall live from 10 to 11 Years Age; the ſecond for one Year, if he ſhall live from 11 to 12 Years Age; and ſo on ſucceſſively to a 90th Annuity for one Year, if the Nominee ſhall live to be an 100 Years old; will be of equal Value with one Annuity granted to him for his Life abſolutely: For the Chances of receiving the Annuity will be the ſame in both Caſes, becauſe the Chances of receiving it depend on the Chances of his Vitality. Here he alters

the

the State of the Cafe intirely from what I gave ;
for whereas I fuppofe thefe feveral Annuities or
Grants for 90 Years, determinable at the End of
each Year, to be made to 90 feveral Perfons on
their feveral Lives, he fuppofes thefe feveral de-
terminable Annuities to be granted to one and the
fame Perfon, and on one and the fame Life ;
which Alteration manifeftly varies the State of the
Chances. The Difference between the two Cafes
is evidently this ; that in the Manner I ftated it
there are as many Chances of Mortality as there
are Lives, *viz.* 90 ; but, in the Cafe as he ftates
it, there is but one Chance of Mortality, for
there is but one Life in the whole, *viz.* the Life
of *A.*

He afferts farther, that thefe feveral Annuities,
as he calls them, *will be of equal Value, whether
they are granted by one Perfon to one Perfon, or by
90 feveral Perfons to one Perfon ; and all equal to
an abfolute Annuity for a Life.* That they are fo
in the firft Cafe I admit, becaufe the Grants
amount to an abfolute Annuity for a Life ; but
that they are fo in the latter Cafe, as well on the Part
of the feveral Grantors as of the Annuitant, is a
Point at leaft doubtful ; and I fhall go farther and
affirm, that they are not in all Inftances equal,
and that the Reafon he gives to prove their Equa-
lity, *viz. That the Chances for paying and receiving
it muft in both Cafes be the fame,* is a Fact not in
all Cafes true, and therefore does not make good
his Affertion. In the firft Place, if feveral Grants
were made to one and the fame Perfon, *viz. A.*

by

by 90 diftinct Perfons in the manner here fup-
pofed, it might be a Queftion, whether fuch
Grants were properly an Annuity, or a Grant only
of fo many feveral Sums in Grofs made by fo many
diftinct Perfons. With Regard to *A*, the feveral
Sums fo granted, when put together, woud
amount to fomething which woud be equivalent to
an Annuity for his Life ; but *quoad* each feparate
Grant, they feem to be no other than a Grant
of fo many feveral Sums in Grofs. If we con-
ftrue them as an Annuity for the Life of *A*, a
Difpute woud then arife between *A* and the fe-
veral Grantors, whether the greater Sum, *viz.*
15-12, or the lefs Sum, *viz.* 13-44, was to be
paid to the Grantors in proportionable Parts. *A*
on his Side may allege, that the Grantors, on their
Ingagements for the refpective Sums to be paid
in futuro, are to receive no more than 13-44 in
the whole, and all put together, becaufe *quoad*
them the prefent Value of the Chances to pay
them, feverally and diftinctly confiderd, do
amount to no more. The feveral Grantors on the
other Side may allege, that *A* on the Pur-
chafe of fuch Grants is to pay for them in the
whole, and all put together, 15-12, becaufe *quoad*
him the prefent Value of all the Sums put toge-
ther which he has a Chance to receive, by the
Combination of their feveral Grants, amounts to
that greater Sum : And, believe me, fuch their
Allegation is founded on very juft and good Rea-
fons. If thefe Grants, fuppofed to be made by
90 feveral Perfons, are only fo many Sums in

Grofs,

Grofs, they are not pertinent to the prefent Pur-
pofe; and, if we conftrue them to amount to an
Annuity for Life, 'tis difputable at leaft which
of the two Sums is to be paid, the greater or the
lefs; and it may be Time enough to difcufs that
Controverfy when any fuch Grants fhall be made,
which is not very likely ever to come to pafs; this
imaginary Cafe therefore will avail nothing to de-
termin the Point in Difpute, but it muft be left to
be decided by other Arguments and upon other
Topics.

I affirm then, in the fecond Place, that this
Author's Reafon, *viz.* that the Chances of pay-
ing or receiving in both thefe Cafes are the fame,
do not prove that the Annuities in each are of
equal Value. In this Matter, the Chances of not
paying, or not receiving, are to be taken into the
Confideration as well as thofe of paying and re-
ceiving. This being premifed, I fay, that in the
Cafe of a fingle Grantor of an Annuity for the
Life of *A*, he has only one Chance during the
whole 90 Years that he fhall be exempted from
paying, but the feveral Grantors have, each in
their refpective Years, a Chance for fuch Exemp-
tion; confequently the Value of the fingle Chance
of not paying in the firft Cafe is not the fame
as the Value of the accumulated Chances in the
latter; therefore the Value of the Annuities at-
tending is not equal. Each of thefe Grantors of
the Sums in Grofs or annual Payments, on their
Grants, muft be allowd and have a Deduction
for fo many Chances as each of them has of not
paying

paying the Sum at the End of each Year, that is,
for fo many Chances of Mortality as arife on the
Life of the Annuitant in each Year refpectively.
And why is this done? Becaufe the Chance of
Mortality *quoad* each of them arifes in each Year,
and is confined to each Year refpectively. But
in the Cafe of a fingle Grantor, a Grantor of an
Annuity for a Life abfolutely, or a Combination
of Grantors who fhall make fo many Grants as to
be equal to an Annuity for a Life, this Chance
of Mortality, and confequently of not paying,
does not arife in that Manner, but does, and
from the Nature of the Life of Man of Necef-
fity muft arife at once, and once only; and confe-
quently on the Purchafe an Allowance and De-
duction can be made for one Chance only,
fince it can arife but once. Now that thefe
Chances of Mortality, for which an Allowance
and Deduction is to be made, in the Cafe of the
feveral Grantors, when all put together, are of
greater Value than the fingle Chance of Morta-
lity in the Cafe of the fingle Grantor is evident
from hence; becaufe the Chances in the firft
Cafe arife, fom in the firft Year, fom in the fe-
cond, and fo on fucceffively for 90 Years; where-
as the one Chance of Mortality in the latter Cafe
does not arife till the even Chance of the Morta-
lity of *A*, that is, not till the End of 41 Years;
and Chances in immediate Poffeffion are of much
greater Value than one or the like Number of
Chances in Reverfion after 41 Years, as this
Author has very learnedly demonftrated.

I have

I have ſuppoſed here, that a ſingle Grantor of an Annuity for a Life has but one Chance of the Mortality of his Annuitant throughout the whole 90 Years, and moſt certainly he has not. That every Man ſhall die once, and no Man more than once, muſt be admitted; but 'tis abſurd to ſay that a Perſon has more than one Chance to ſuffer that which 'tis impoſſible he ſhoud ſuffer more than once. However, if it be inſiſted on, in the Caſe put of 90 ſeveral Grants to *A*, that the Chance of receiving the Annuity is the ſame as in the Caſe of an Annuity abſolutely for his Life, and it ſeems to be the better Opinion, then *A* has in both Caſes the ſame Chance of Mortality againſt him, and that is one only: Then it will be inſiſted on by the other Side, that *A* having by ſuch a Combination of Grants put himſelf into ſuch a Condition, that he has only a ſingle Chance of Mortality againſt him, and of not receiving his Annuity, that then for theſe accumulated Grants he muſt pay the ſame Sum as he ſhoud do for an abſolute Annuity for his Life, that is, the greater Sum.

Hitherto we have been ſpeaking of and conſidering what woud be the Caſe of *A*, whom we have ſuppoſed to be 10 Years old only, on a ſingle Grant of an Annuity for his Life abſolutely, and on 90 ſeveral Grants to him by 90 Perſons, determinable by his Death within any one Year of 90 Years: And, if the Point in Diſpute remains there doubtful, the Inſtance which follows will put the Matter beyond all Doubt. We will ſuppoſe

ſuppoſe then that *B* is 80 Years old, that there are 20 Years of poſſible Life remaining to him, that he has from one Perſon an abſolute Annuity for his Life, and has likewiſe 20 ſeveral Grants made to him by 20 ſeveral Perſons determinable in the ſame Manner as in the former Caſe. The even Chance of the Duration of the Life of *B*, Computation being made of ſuch Chance by the *Breſlaw* Table, is 4 Years, or hardly ſo much; and an Annuity for 4 Years, Intereſt ſtated at 6 *l. per Cent.* is 3-46; an Annuity for the Life of *B* abſolutely is therefore in Value 3-46. An Annuity or Grant for the Life of *B*, when made by 20 ſeveral Perſons for 20 Years, determinable in the Manner aforeſaid, is equal to a Term for four Years and an Half, and is in Value 3-84, all which Facts are admitted by this Writer: From whence 'tis evident, as evident as Numbers or Figures can make it, that the Annuities in both Caſes are not equal. 'Tis true likewiſe, and as demonſtrable, in the Caſe of *B*, that the Chances of receiving or paying are not the ſame in both Inſtances, unleſs it be the ſame Thing to have a Chance of receiving 3-46, as 3-84; or *B* have a Chance of receiving the Annuity for his Life, after he is dead and gone.

He adds farther, that the Chances of Vitality cannot be increaſed or decreaſed by the Manner of granting the Annuity: But, I think, he ſhoud rather have ſaid, that they ought not, or ſhoud not be varied by the Manner of granting, for 'tis

'tis visible, that in Fact they are varied in the Case of *B*; for the Chance of Vitality on the single Annuity for his Life is only four Years, and very barely that; but on the Annuity by the accumulated Grants the Chance is four Years and an Half; and 'tis a Contradiction to common Sense to say he has any Chances of Vitality left on the accumulated Annuity, when on the single absolute Annuity they are all determind and gone.

I woud here recommend it to this Writer to reconcile these Matters, but that I am afraid he will be sick of his reconciling Function, when he finds, as he will do in an Instance herein after mentioned, how little Success he meets with in such Undertakings. I shall choose rather to observe that this Argument, as weak as it is, is the only one which this Author has offerd to maintain the Justice of Dr. *Halley*'s Rule, for making a Valuation of these Annuities; and proceed to consider what may seem to be alleged by him, in Answer to my main Exception to the Dr's. Rule.

The Exception I mean, and which is the plainest and strongest against the Dr's. Rule, may be reduced to this short Syllogism: Any Rule for computing the Value of Annuities for a Life, which produces such a Value as that one and the same Life shall be equal to a different Term, as the Rate of Interest varies, is a false Rule: This Rule does necessarily produce such a Value: Therefore is a false one. The first Proposition is so clear and obvious a Truth, that this Author does

does not offer to contradict it; nay, he goes farther, and by his own Position just mentiond, *viz.* That the Chances of Vitality cannot be increased or decreased by the Manner of granting an Annuity for a Life, he does effectually confirm it; for if, under the Direction of the Rule, such Sums are necessarily produced as the Value of an Annuity on a Life, as that the Term to which such Life is equal, or the Chance of the Duration of such Life, shall vary as the Rate of Interest varies; most certainly this is to make the Manner of granting the Annuities, and the Rate of Interest at which they are granted, to govern and to increase or decrease the Chance of Vitality on that Life. My second Premise I demonstrate by showing that Mr. *Richards*'s Tables, which were constructed by that Rule, do in Fact exhibit such a Value of these Annuities. This appears by an Inspection into his Tables, where the Value of an Annuity for a Life of 12 Years old, when Interest is computed at 4 *l. per Cent.* is 17-20 Centesimals, which is equal to a Term of 29-3-00; and when computed at 8 *l. per Cent.* is 10 67 Centesimals, which is equal only to a Term of 25—. To show that these Values do not proceed from any Error or Defect in the Operation made by Mr. *Richards*, but are a necessary Result from the Rule, and consequently the Rule a false one, I advance these two Propositions: That if a certain Value and Term corresponding are given, and out of that Value a proportionate Part be deducted; first, that the Term corresponding

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ing to the Value remaining will not bear the same Proportion to the Value remaining, as the original Term does to the original Value; secondly, that the Term correfponding to the Value remaining, after a Deduction at one Rate of Intereft, will greatly differ from the Term correfponding to the Value remaining, after a Deduction made at another Rate of Intereft: To which I add, that at 4 *l. per Cent.* the Difference of the Term is the leaft, and at 8 *l. per Cent.* the greater of the two. To prove thefe Pofitions I have fubjoind a Calculation which demonftrates them. Thefe Pofitions and the Calculation this Author has inferted at length in his Obfervations, *p.* the 26*th,* without taking Notice what introduced them, or how they are applied; however, he admits the Truth of them, and that from thence it neceffarily follows, if the proportionate Part deducted out of a given Value be an 100th Part only (my Calculation being made on an half Part of the Value) yet that the Term correfponding to the Value remaining, after the Deduction, muft be different at different Rates of Intereft. But he cannot agree that this is a Demonftration that the Rule is wrong. I did not lay down thefe Pofitions immediately and directly to prove that the Rule is wrong, but to prove that the Values given in the Tables were a neceffary Refult from the Rule; and they do demonftratively prove it, and he admits they do: And then indeed, if the Premiffes are true, and of one of them he takes no Notice or offers to oppofe, and

the

the other he cannot contradict without contradicting himfelf; the Confequence, that the Rule is a wrong one, will follow very clofe at the Heels of them, for the Conclufion is a neceffary and certain one, as neceffary and certain as any in *Euclid*: For if the Rule does neceffarily produce Values, which are certainly falfe, the Rule which produces them muft neceffarily be falfe likewife.

This Author however, notwithftanding this ftrong Proof, I may fay Demonftration, of the Falfity of the Rule, feeming to offer fomething againft it, I fhall confider it. He fays then, *that the Rule does not bid you deduct from the Value a Part proportionate to a Part deducted from a Term, but bids you deduct from the Value a Part proportionate to the Number of Chances of Mortality within that Year, which are Things widely different.* 'Tis in no fort material what are not the Directions of the Rule, nor whether they are Things widely different from the real Directions; however, I will admit this Allegation; but this Writer, I fuppofe, becaufe he coud fay nothing to the real Queftion, had a Mind to raife an Imagination, and palm it upon the Reader, that the Queftion was, whether this or that was the Direction of the Rule, and how far the one might differ from the other. Be that as it will, in the *Effay* 'tis faid, and here repeated by this Writer, that one of the Directions of the Rule is this, *viz.* from the Value of an abfolute Annuity for a Year to deduct a Part of fuch Value proportionate to the Number of Chances of Mortality arifing in that

Year.

Year. Now this is fo far from a Reafon for not agreeing with me, that it fhows, whether he fees it or not, or will acknowledge it, that he does or muft agree with me, and proves for me that my Syllogiftical Reafoning demonftrates the Falfity of the Rule. The Matter in Controverfy is, whether this Rule for finding the Value of Annuities on a Life be a good one; and, having denied it, amongft other Exceptions I alleged this as one, that it directs fome Deductions to be made in fuch a Manner as produces fuch a Value of thefe Annuities as make fomething to happen on the Life of Man, which is abfolutely inconfiftent with any Thing that ever can happen in the Life of Man. To this the Anfwer is no more or better than this: That the Rule is a good one, for that it directs thefe Deductions to be made in the Manner I allege, that is, in a Manner which produces thofe abfurd Values. If the Matter were here frefh, and no Objection had ever been made to the Rule, this woud have been a poor Argument to prove its Goodnefs; for 'tis in Effect begging the Queftion, and faying that the Rule and the Directions are fuch and fuch, for it does no more than repeat one of the Directions of the Rule, and tell us that it differs widely from another that is no Direction of the Rule, without offering one Word in Juftification of it. But now, when he is attempting to anfwer a Treatife, in which are many Objections to the Rule, and to that very Direction in particular, and in which 'tis alleged, that fuch and fuch abfurd Values in certain

certain Tables were produced by following that Direction, and demonftrated that thofe Values fo produced were a neceffary Refult from fo following it: When he tells us that the Rule bids us make the Deductions in the Manner alleged in the Exception; this manifeftly confirms the Exception, for it admits the Verity and Juftice of the Allegations; and, if thofe are true, I'm very confident that the Conclufion is right, that the Rule is not a good one.

Another Objection to the Rule mentiond in the *Effay*, and which this Writer's Obfervations upon it make neceffary here to repeat is this: That according to fuch Rule the Value of an Annuity for a Life of ten Years Age is 13-44 Centefimals, which is equal to a Term of 28-1-00; and the Value of an Annuity for a Life of 80 Years Age is 3-84 Centefimals, which is equal to a Term of 4-2-00; but from the *Breflaw* Table, from whence this Calculation of thefe Values was made, it appears that the even Chance of the Duration of the Life of 10 Years Age is 41 Years, and the even Chance of the Duration of the Life of 80 Years is not full 4 Years. From thence it was argued: That an Annuity for the Life of a Perfon aged 10 Years fhoud be in Value 13-44, that is, fhoud be equal to a Term of 28-1-00 only, when his Life on the even Chance of Duration is equal to 41 Years and above; and at the fame Time that an Annuity for the Life of a Perfon aged 80 fhoud be in Value 3-84, which is equal to a Term of 4-2-00,

4-2-00, when his Life on the even Chance of Duration is equal only to a Term of 4 Years: This is there said to be so palpable a Contradiction to common Sense, that nothing can maintain the Rule by which those Values were produced. The Jet of the Argument lies in this, and on this the Contradiction is grounded, *viz.* That the same Rule shoud in the Operation have such different Effects on the younger Life and the elder Life as to deprefs the Chance of Vitality of one below the even Chance, and exalt the other above it; which, in Effect and in other Words, is as much as to say, that in the one Case the Annuity will not have a Chance to continue so long as the Life will have a Chance to continue, and in the other Case will have a Chance to continue longer than the Life does: And, notwithstanding the learned Labors of this Author to reconcile this Matter, I think, and I believe most others will think, that it remains still in the same State of Inconsistency.

Let us now examin his Way of Reasoning here. He admits the Values of the Annuities, and the Chances of the Duration of the Lives, respectively, to be such as I have stated them; but a Repetition of those Computations, merely and by itself, sure enough does not prove either that the Values produced are the true Values of the Annuities, or that the Rule by which they were computed is a right one. This is neither better nor worse, than first to suppose and take for granted, that these Values thus produced are the true

true ones, and then to infer that the Rule by which they were computed is a right one; or *vice versâ*, to suppose and take for granted that the Rule by which they were computed is a good one, and from thence to infer that the Values are true ones; but this will not have much Weight with those who deny both that the Rule is a good one, and that these Values are the true ones.

However, to justify the Value given on the younger Life he urges, *p.* the 33*d*, that, when all the possible Chances are considerd, they will reduce it to an Equality in Value with a Term of 28-1-00. I observe here, that he speaks only of the possible Chances of Vitality, and not of the Value of those possible Chances; and I say, that all the possible Chances of Vitality have been considerd, and upon a Computation of them, it appears that 'tis an even Chance, that the Life will have Continuance for 41 Years, and this he will and does admit; therefore 'tis incumbent on him to prove that 'tis not an even Chance or Probability that the Annuity, which is to continue for the Life, will have a Duration for the same Term, as 'tis an even Chance or Probability that the Life will have a Duration, that is, for 41 Years: And until he does this, which I despair of ever seeing done, for indeed 'tis impossible to be done, the Difficulty will remain.

To justify the Value given on the elder Life, which seems to be the harder Piece of Work, he gives us, *p.* the 33*d*, a Computation of som of the possible Chances on such a Life, and refers us to *p.* the 18*th* for a like Computation on som of

the like poſſible Chances on the younger Life; and then concludes: *That it will be no Contradiction to common Senſe to ſay, that the Values of theſe two particular Annuities, produced by the Dr's Rule are the true Values of the Annuities on theſe Lives reſpectively: Becauſe in eſtimating the Value of an Annuity on a given Life all the poſſible Chances muſt be computed; and the Chances in the Caſe of the elder Life have a different Operation from what they have in the Caſe of the younger Life.* If he means here no more than that all the Chances of poſſible Life muſt be computed, as his Words import, I ſhall readily agree with him, becauſe we cannot compute the Value of an Annuity for a Life without eſtimating firſt the Chance of the Duration of that Life, and we cannot rightly eſtimate that Chance without computing all the poſſible Chances of that Life. When that is done according to the Rule laid down in the *Eſſay* for eſtimating the Value of Life Annuities, a Moiety of all the poſſible Chances gives us the even Chance of the Duration of the Life, the even Chance of the Duration of the Life gives us the Term to which the Life is equal, and that gives us the Value of the Annuity. But if he means here that the VALUE of all the poſſible Chances on the Life are to be computed, and the poſſible Chances of a Life and the Value of thoſe poſſible Chances are quite different Things, and will produce Annuities of very unequal Value; and this latter he muſt mean, if he intends to ſupport the Dr's Rule, for the Direction of that Rule is to take the VALUE of all the poſſible Chances

Chances of Life; then he begs the Queſtion, according to Cuſtom, inſtead of proving it. The Objection to the Rule was this; that it produces ſuch Values of Annuities, as that in the one Caſe Caſe they neceſſarily ſuppoſe the Chance of the Duration of the Annuity to fall ſhort of the even Chance of the Duration of the Life, and in the other to exceed it; and theſe Values of Annuities are in the *Eſſay* aſſerted, and moſt People I believe will agree, to be abſurd ones. 'Tis there demonſtrated, that theſe Values of Annuities are produced by the Rule, and particularly by its directing, or becauſe it directs, that the VALUE of all the poſſible Chances on Life be computed; and he concludes here that the Rule is a good one, without more ſaying, than that 'tis a good one, becauſe it directs the Computation of the Value of ſuch Chances to be made in a particular Manner, which Manner neceſſarily produces ſuch a Value of theſe Annuities as are manifeſtly abſurd ones.

I have ſaid that the poſſible Chances of a Life and the VALUE of thoſe Chances are different Things, and ſo moſt certainly they are; and tho' this Author does not ſee it, or may not be ready to acknowledge it, that there is in Truth the Matter in Controverſy, whether the Computation is to be made by the poſſible Chances of a Life, or by the Value of thoſe poſſible Chances. I ſay 'tis to be made in the firſt Manner, and that ſuch Computation will give us the Duration or Chance of the Duration of the Life; that the Duration of the Life is the Meaſure of the Duration of the Annuity depend-

ing on that Life, and the Duration of the Annuity the Meafure of the Value of it. On the other Side, if a Computation be made of the VALUE of fuch poffible Chances, that is done, and can be done no otherwife, than by computing the Value of the feveral Annuities attending on fuch poffible Chances in every Year of Life, and through the whole Life, and then affuming that thofe feveral Annuities put together conftitute the Value of the Annuity for the the Life. Now this does directly and obvioufly make the Chance of the Duration of the Life to depend on the Value of the Annuity; for, in every Computation which has been made of the Value of Annuities on Lives conftructed in this Manner, we have feen that the Fact is fuch, that in all Inftances where the Intereft is varied, and confequently the Value of the Annuity varied, there the Chance of the Duration of one and the fame Life is varied likewife, and that the one increafes or decreafes when and in Proportion as the other does; and yet 'tis moft undoubtedly true, and this Author afferts it, over and over again, that the Value of the Annuity is not to increafe or decreafe, or in any Shape to have an Influence on the Chance of the Duration of the Life, and yet in Practice he generally contradicts it.

He adds farther, that the poffible Chances in the Cafe of the elder Life have a quite different Operation from what they have in the other Cafe of a younger Life, and from thence woud infer
that

that the Values of the Annuity on thefe two Lives are or may each be right; but this furely is a ftrange Inference, for he gives that as a Reafon for the Rectitude of the Rule and for the Truth of the Values, which I have given, and moft certainly is a good one, for Falfity of both. When I confiderd the Value of the Annuity on thefe two Lives as produced by one and the fame common Rule, I faid that it coud not be a good one for eftimating the Values, and gave this Reafon for it, *viz.* becaufe in the Operation upon it the Effects in the Cafe of the younger Life, and in the elder were fo different, as to deprefs the Chance of Vitality on one Life below the even Chance, and to exalt the other above it. The Anfwer he gives to this is no more than a bare Repetition of the Grounds of my Objection to the Rule and the Values in thefe Inftances produced by it, with fom fmall Difference in the Expreffion, *viz.* the Rule is a good one, and the Values of thefe Annuities are right; becaufe the Value of the poffible Chances, which the Rule directs to be computed, have a different Operation in the younger and in the elder Life. This, I fay, is and will remain an unanfwerable Objection to the Rule, until it can be fhown, and I may be confident it can never be fhown, that the Value of thefe poffible Chances, or rather the Value of the Annuities attending thofe Chances, can in fom Inftances abridge the Chance of Vitality fo as to make it lefs, and in other Inftances inlarge it fo as to make it greater than the even Chance: Or, as this Author expreffes

preffes himfelf, that the Manner of granting thefe Annuities, whether on a younger or on an elder Life, can increafe or decreafe the Chance of Vitality. That the Operations on the Value of the poffible Chances on the younger and the elder Life, in their Way of computing them, have different Produces is a Thing vifible, and is the Matter of the Objection or the Grounds of it; and to anfwer this the Author fhoud have fhown fuch a Difference between the Nature of the Life of the young Perfon, and the old one, as fhoud reconcile this Difference in the Produces; but that being impoffible to be done, as moft certainly it is, for the Chance of Vitality of one to 41 Years is the very fame Thing, and collected in the fame Manner, as the Chance of Vitality of the other to 4 Years, he contents himfelf with repeating the Subftance of the Reafon and Grounds of the Objection, and there leaves the Matter.

However, to be more particular, he attempts to fhow that 13-44 may be the Value of an Annuity for a Life of ten Years Age, and 3-84 the Value of one for a Life of 80, tho' neither of them be confiftent with the even Chance of Vitality of the refpective Lives, which he muft and does admit. For this Purpofe he makes a Computation of the Value of the Chance which *A*, for Inftance, fuppofed to be 10 Years old, has whether he fhall die in the firft Year, or live to be 90 Years old, and the Sum arifing is 1-59. He makes the like Computation of the Value of the like Chance on the Life of *B*, for Inftance, fuppofed

pofed to be 80 Years old, and the Sum arifing there is 4-5. If we inquire how the Computations of the Values of thefe particular poffible Chances on thefe two Lives made by this Author, and his Reafoning upon them, are to be applied; I can make nothing other or farther of them than this.

The Dr's Rule for eftimating the Value of Annuities a Life may be a right one, altho, in computing the Value of the poffible Chances of thefe two Lives, the Produces are fo different as not to coincide with the even Chance of Vitality, which is the Point he undertakes to prove; for here is a Rule, he fays, of his own, for computing fom of the particular poffible Chances on thefe two Lives, and on the Computations made thereby it appears that one is in Value 1-59, and the other 4-5. Very true; but thefe two Values do in Fact coincide with the even Chance of Vitality, as we fhall fee prefently, and he himfelf fays they do, and takes fom Pains to prove it. The Argument then will ftand thus: This Obfervator's Rule for computing the partiular Chances on thefe two Lives is a good one, as I do admit it is, becaufe the Values produced by the Rule do coincide with, and are computed by the even Chance of Vitality; therefore Dr. *Halley*'s Rule for computing the Value of the Chances for the whole Life (or of an Annuity for the whole Life) of thefe two Perfons is a good one, which produces fuch Values as do not in either of them coincide with the even Chance of Vitality. I fhoud fpeak too
favourably

of thefe two Computations, and the Application of them to the prefent Purpofe, if I fhoud fay of them only that they prove nothing towards maintaining the Doctor's Rule; for fom People will be apt to furmife that they prove rather the contrary, that no Computations on the poffible Chances of Life, or the Values of them, are right but what do coincide, or at leaft are confiftent, with the even Chance of Vitality of Life.——

To fupport fuch Surmife of theirs, they make Ufe of the two Computations made by this Writer to fhow that the Sums 1-59 is the Value of one of the Chances, and 4-5 the Value of the other; but, fince they are both formed by the fame Pattern, I will mention one only——He fays, then; *That the Number of Perfons dying in the firft Year of the Life of A is 8, the Number of Perfons living of the Age of 90 is 8; confequently it is an even Chance whether A is one of the firft 8, or one of the laft 8; that is, the Chances of A's dying the firft Year, or living to 90, are equal. If he dies the firft Year, he lofes the Value of an Annuity for 41 Years, which is worth 15-12 Centefimals; whereas, if he lives to 90, he wins the Value of an Annuity for 39 Years, in Reverfion after a Term for 41 Years, which is worth no more than 1-59 Centefimals.* I obferve here that this Author, as great a Mafter as he is of Numbers and Figures, is guilty of a Miftake in computing the Value of the reverfionary Intereft of the 39 Years, and ftating it at 1-59, when 'tis only 1-38; for the whole

whole Term of 80 Years is, in the common Tables, fet at 16-50 Value; and, fince the Intereft of 41 Years in Poffeffion is 15-12, the Intereft of the 39 Years in Reverfion can be no more than 1-38. But to let that pafs, for in the prefent Argument 'tis not at all material which is the truer Value; and we will fuppofe 1-59 to be the right one; I admit then that the Number of Chances are equal; that the Chance here is an even one; and that 1-59, the Sum produced, is the right Value: But it happens very unluckily that this Argumentation, when applied to the prefent Purpofe, does directly and evidently prove my Pofition, and deftroy the other. Mine is, that the even Chance of the Duration of the Life, computing one Chance only in the whole Life, is the Meafure of the Value of the Annuity for that Life; and Dr. *Halley*'s and this Author's is, that the Value of the Chance for each Year of the poffible Life, computing as many Chances as there are Years in the poffible Life, is fuch Meafure. Now in the Cafe here ftated 'tis fuppofed, as the Foundation of the Truth of the Value produced, that the Chance is an even one; and in the Computation itfelf 'tis taken for granted, that the Chance of the Duration of the Life is the real Meafure of the Value of the Annuity for the Life. 'Tis here laid down that *A. by his Death in the firft Year will lofe an Annuity for* 41 *Years which is in Value* 15-12; but by his Death at *That* Time he can lofe nothing but an Annuity for his Life, for he can

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lofe nothing but what he has a Right or a Chance to have, therefore fuch Annuity muft be equal to a Term for 41 Years, and be in Value 15-12. And fince the even Chance of the Duration of the Life is a for a Term of 41 Years, and a Term for 41 Years is in Value 15-12, and the Lofs by his Death in the firft Year is fuch a Term and fuch a Value: It follows from thence, that the Chance of the Duration of the Life is here made the Meafure of the Value of the Annuity for the Life, according to my Pofition.

I cannot forbear obferving here, how naturally and eafily this Author falls into the Notion, that the Chance of the Duration of a Life is the Meafure of the Value of an Annuity on fuch Life; for his Reafoning is formed on Purpofe to prove that fuch Chance is not the Meafure, and that 15-12, the Sum produced by fuch Rule, is not the Value; yet even here he fuppofes that Sum to be the true Value of the Annuity, tho' at the fame Time he makes Ufe of it as the Medium to prove that 'tis not the true Value. If then 1-59 be the Value of this particular Chance on fuch Life, and for that Reafon, *viz.* becaufe 'tis an even Chance; I don't fee why I may not conclude that 15-12 is the Value of an Annuity for the younger Life, and 3-46 of an Annuity for the elder Life, beforementioned: For the even Chance is that the younger Perfon does not die before the End of 41 Years, and a Term for 41 Years is in Value 15-12; and the even Chance is that the elder Perfon dies in 4 Years, and a Term

Term for 4 Years is in Value 3-46. On the other Side, and from hence, I think I may infer that the Values of the Annuities on thefe Lives produced by Dr. *Halley*'s Rule muft be falfe, fince one falls greatly fhort of the Value of the even Chance of the Duration of the Life, and the other exceeds it. Therefore, upon the Whole, I recommend it to the Author of thefe Obfervations, and leave it with him, to anfwer his own Arguments, as well as mine, for my fundamental Pofition; that the fingle Chance of the Duration of a Life, computed once for all his Life, is a good Rule, and the only Rule, to compute the Value of an Annuity depending on that Life.

I muft, in the next Place, take Notice of the Remarks made on the Effay made by Mr. *Richards*, often therein mentioned, were it only to let him know that I have read them; but that I defpife the foul-mouthed and fcurrilous Language which he has fo plentifully beftowed on me and my Performance, and for his Arguments and Objections againft it, as he calls them, that I think they are already anfwered or obviated, as far as the Matter of them deferves and requires. All I fhall add is, that I would recommended it to him to look into his own firft Treatife on this Subject, if he thinks it worth his Trouble, which few perhaps befides himfelf will do, and compare what he fays in P. 27 and 28. with what he fays in P. 83, *&c.* In P. 27, having made a Suppofition that three Perfons are to enjoy an Eftate by equal Portions during their joint Lives, Re-

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mainder to the Survivors and Survivor, he makes a Quære what is the Value of each Man's Right; and, in the next Page, folves it thus—*That each Perfon has a Right to one Third of the Value of their joint Lives, and to Half the Value of the Expectation after the Deceafe of either of the other two, and alfo to the whole Value of the Expectation of both the others; which three Sums added together give the Value for each.*—In P. 83, &c. he fuppofes a Man to give his three Daughters an Annuity of 20*l.* per *Annum* each for their Lives; and to order, that, when any of them dies, the Annuity of the Sifter dying fhoud fall to the other two in equal Shares; and, at the Death of another, that the whole fhoud belong to the Survivor for her Life. He goes on then to compute the Value of this Gift to each of thefe Women; and gives us the Value of the Share of the eldeft Sifter thus: —Firft, 20*l.* per *Annum* during her own Life. Secondly, 10*l.* per *Annum* in Reverfion of the youngeft, to continue from thence during the joint Lives of the other two. Thirdly, 10*l.* per *Annum* in Reverfion of the Second, to continue from thence during the joint Lives of the Firft and Third. Fourthly, 40*l.* per *Annum* in Reverfion of the two youngeft Lives during her own Life. Now, unlefs this Writer does, and until he has reconciled his Solution in the firft Paffage, in which he allots to each Perfon three Sums only, with his Computation in the fecond Paffage, where he brings to Account four Sums: Which I fuppofe he will be in no great Hafte to attempt; and,

and, if he does, will never be able to effect: Som People will, perhaps, call in Queftion his Faculties, Algebraical, Mathematical, or Intellectual, and, for my own Part, I fhall never fay a Word more to fuch a ———.

To Mr. *Hodgfon*, who ftiles himfelf *F. R. S.* and Mafter of the Royal Mathematical School, who has lately publifhed fom Operations and Tables on the Value of Annuities for Lives, I have fomthing more to fay, becaufe he builds, in Part at leaft, on the fame Foundation on which Dr. *Halley* did, and has revived, feemingly, and unneceffarily I think at this Time of Day, the Controverfy on the Rule for making the Valuation of thefe Annuities. In his Preface, by Way of Apology for framing a new Table of Mortality for *London*, to ferve the Place of the old one of *Breflaw*; he tells us, that at *Breflaw* one Half of the People that are born there live 'till they are about 41 Years of Age, in *London* that one Half die before they are of the Age of 10 Years. Where and whence this Writer pickd up this Notion about the Age to which the People who are born at *Breflaw* do live, or who fhould tell him fo, I cannot devife; for Dr. *Halley*, in his Treatife on this Subject, tells us, in fo many Words, that it appears from the *Breflaw* Table, and in Fact it does fo appear, that one Half of thofe who are born there are dead in 17 Years: But, this not being material to the main Defign, let it pafs. In his Book, after giving us, for four or five Pages, a Calculation of the Chance of the Duration of

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Lives for *London* ; tho' by the Way, this long Detail, even were it a good one, is very little to the Purpose, unlefs fuch Chance be the Meafure of the Value of the Annuities depending on thofe Lives ; he fays, that from thence the Value of an Annuity for a fingle Life of a given Age, or the Value of any limited Time the Perfon has a Probability of living, at any Rate of Intereft, may be found. He proceeds, by Way of Inftance of a Perfon of 10 Years Age, and at 3 *l. per Cent.* Intereft, to compute the Value of an Annuity for his Life of 1 *l. per Annum* ; and having told us the prefent Value of 1 *l.* payable at the End of one Year, when 'tis an abfolute Certainty, and computed what muft be dedudted thereout on the Life Annuity for that Year, on Account of the Chance of Mortality arifing in that Year ; and having gone through the like Operation on the fecond and the third Year of the Life of fuch Perfon ; he concludes, that, *by repeating thefe Operations to the utmoft Extent of poffible Life, the Value of* 1 l. per Annum, *for a Life of* 10 *Years old, will be found to be* 20-16-46, *or* 20-1646 *Years Purchafe :* And then adds, that *the Tables there following were conftructed after this Manner.* I obferve here, that this Method of computing the Value of Annuities on a fingle Life, tho' it may vary in the Expreffion and Shape of it, yet in Subftance is the fame as Dr. *Halley* prefcribed for that Purpofe ; and therefore, that all the Exceptions taken to this Rule, as Dr. *Halley's,* will equally lie againft it now adopted and abetted

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by this Author. This being premifed, I fay, that this Writer has either himfelf been impofed on, or attempts to impofe upon his Reader, when he tells him that the Tables there following were conftructed in the Manner mentioned ; for, whatever was the Manner in which they were conftructed, 'tis certain it was not this ; and he himfelf fhows it demonftratively, and fo plainly and palpably, that one may read it as he runs. He fays, on computing the Value of an Annuity for a Life of 10 Years Age, the Value or Years Purchafe appears to be, when computed by his Method, 20-1646 ; which I believe to be right, becaufe I have made the Computation on the fame Life, at the like Intereft, and by his Rule, and I make the Value 20-1623 ; and this very fmall Difference may poffibly arife by his inferting an Account of the Fractions at the End of each Year, and my omitting them. But in the Table, which follows in the very next Page, he gives 19-5761 as the Value of an Annuity for a Life of 10 Years of Age, Intereft computed at the Rate of 3 *l. per Cent.* when, in the Paragraph juft preceding, he had computed and ftated it, and at that Rate of Intereft, at 20-1646. Now the Sum 20-1646 is equal to a Term of $31\frac{1}{8}$, and the Sum 19-5761 only to a Term of $29\frac{2}{8}$; fo that there is the Difference of a Year and an Half in the Term to which one and the fame Life is equal, as computed in the precedent Paragraph and as given in the fubfequent Table. At the Rate of 5 *l. per Cent.* there is the like Variation, but on

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the contrary Part; for the Value, on computing by the Rule given by this Writer, is only 15-0430, which is equal to a Term of 28½, but in the Tables is put down 15-3588, which is equal to a Term of 29⅖. At the Rate of 4*l. per Cent.* indeed, the Value arifing on a Computation by the Rule and that inferted in the Tables is one and the fame, *viz.* 17-2738, which is equal to a Term of 29⅖. From thefe Calculations and Facts, 'tis evident that the Value of this Annuity, at the Rate of 3*l.* and 5*l. per Cent.* was not collected and afcertained on a Computation by the Rule, for they are not agreeable to it; but, on the Contrary, it muft have been firft fuppofed that the Values at the Rates of 3*l.* and 5*l.* were equal to the fame Term to which the Value at 4*l.* is equal, and then a Value correfponding to that Term is inferted as the Value of the Annuity at the Rates of 3*l.* and 5*l.* The Truth is, and in his Table, P. 40. and which he calls, *The Value of Lives upon Annuities certain,* 'tis manifeft, that this Writer did mean and intend that the Value of Annuities for a Life of the Age of 10, and indeed of all other Ages of Life, at all Rates of Intereft, fhoud refpectively correfpond to one and the fame Term in an Annuity certain; for this Table exhibits them all, at every Rate of Intereft, as equal to one and the fame Value in an Annuity certain, which is the fame Thing as faying, that they are all equal to one and the fame Term in an Annuity certain : And that is the very Point for which I contend, and ought

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to be, but is not preferved, nor can poffibly be obtained by Dr. *Halley*'s Rule, and in the *Effay* have demonftrated it, tho' by this Author's new-fangled Regulator it may.

Now this Method of conftructing the Tables of thefe Values does not only contradict the Manner by which he fays, in the precedent Page, he conftructed thofe Tables, but 'tis likewife an abfolute and total Departure from the Rule laid down by Dr. *Halley,* obferved by others, and pretended to be affumed by this Author, except in the fingle Article of 4*l. per Cent.* and is conformable to the Rule laid down in the *Effay,* fo far forth as it makes the Term in an Annuity on one and the fame Life to be the fame as the Term in an Annuity certain, at all Rates of Intereft. He does indeed by this Means, in Part, obviate or evade the main Objection made to Dr. *Halley*'s Rule, but we fhall fee prefently that my Exception to this Method remains as ftrong, at the Rate of 4 *l. per Cent.* as it does againft the old Rule, where the Rate of 6 *l. per Cent.* is ufed. The Exception I mean is, that, by Virtue and in Perfuance of the old Rule, fuch a Sum is produced as the Value of a Life Annuity, as that one and fame Life fhall be equal to a different Term as the Rate of Intereft varies; and it is demonftrated that this muft be the Cafe, and admitted fo to be by Mr. *H. B.* whom I have beforementiond; and confequently that the Duration of a Life or the Chance of it depends on the Rate of Intereft, and muft, and in Fact is found to vary as oft as that

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varies. Since then the Term in the Value of an Annuity at every other Rate of Intereſt different from it is by this Method to be aſſized and brought to the Standard of the Term in the Value of an Annuity at the Rate of 4 *l.* then the Chance of Duration of Lives depends on that particular Rate of Intereſt as the common Standard. Now if the Queſtion were asked, on what Pretence or by what Authority this particular Rate of Intereſt is made the Regulator here, I think no tolerable Anſwer coud be given but this, that at preſent this is the common Price of Money. Be it ſo : But what if it ſhoud ſo happen that in 20 or in 50 Years Time, more or leſs, the Price ſhoud alter, and ſink to three, or riſe again to five ? Why, we muſt then change our Standard as the Caſe ſhall come out ; and whereas 29⅞ on a Life of ten Years Age is now the Term to which an Annuity on ſuch a Life is equal, then *in futuro,* if the Price ſhoud be three, an Annuity on ſuch a Life woud be equal to a Term of 31⅘, and, if it ſhoud riſe to five, woud be equal to a Term of 28½. If we look backwards, we find the Price of Money about 50 or 60 Years ago to have been ſix, and about an hundred Years ago eight ; and in the firſt Caſe the Term in the Annuity for the like Life woud be 27¼, and in the latter about 25, and then theſe Terms reſpectively muſt be lookd upon as the Term to which an Annuity on ſuch a Life, at the ſeveral Times, and at the different Rates of Intereſt then courant, muſt be reduced as to a common Standard. Now this Alteration of the common

common Standard changes the Chance of the Duration of a Life of one and the ſame Age, in the ſame Manner and as effectually as the Change of the Rate of Intereſt, at any one Time, does change the Chance of the Duration of one and the ſame Life at that Time : And I believe no Body will be ſo hardy as to aſſert, that the Chance of the Duration of a Life of one and the ſame Age is not the ſame now as it was 50 or 100 Years ago, or that it will not continue the ſame for 50 or an 100 Years to come ; ſo that the Difference between the Dr's Rule and this Regulator woud be this and no more than this, that by the Uſe of the former the Chance of the Duration of one and the ſame individual Life is varied as the Intereſt varies, and the Life is made ſhorter or longer, as that is higher or lower ; and by the Uſe of the latter, that ſuch Chance woud vary on a Life of one and the ſame Age, in the ſame Manner and Proportion ; from whence it follows, that this new Regulator is liable to the very ſame Objection as the old Rule.

This Writer can himſelf give the beſt Account, what induced him in his Book to recommend and make an Operation by the Rule, but in conſtructing his Tables to give it up and deſert it, and to fall into this new-invented Method ; but 'tis ſuſpectable that he ſaw and felt the Weight of the Exception to the Rule, and not being able to anſwer it, as I may be confident he was not, he enterd into this Practice to elude or to cover it. But, whatever his Reaſon might be for uſing theſe Artifices,

tifices, he has faild of Succefs; as we fhall fee prefently he has done likewife in putting in Practice the fame little Arts, with Refpect to another material Exception to the Rule mentiond in the *Effay*. The Exception I mean here is; that in ufing the Rule prefcribed, in the younger Life, the Term, to which the Chance of the Duration of the Life is equal, is longer than the Term to which the Value of the Annuity for the Life is equal: And on the Reverfe, in older Life, the Term, to which the Value of the Annuity for the Life is equal, is longer than the Term to which the Chance of the Duration of the Life is equal: And this in the *Effay* was faid to be contradictory to common Senfe and Reafon; and notwithftanding the learned Endeavors of Mr. *H. B.* in his Obfervations before-mentiond, to reconcile thefe Inconfiftencies, I ftill adhere to my former Opinion, and am confirmd in it, on feeing the great Labor he has taken, and the little Succefs he has met with, in the Attempt. Let us examin now how this Matter ftands on computing the Value of thefe Annuities by this new Method. In the Cafe of the Value of an Annuity for a younger Life, fuppofe of ten Years Age, 'tis equal to a Term of $29\frac{7}{8}$, and the even Chance of the Duration of fuch a Life is 35 Years, Computation in both Inftances being made by this Writer's Tables, fo that the Difference amounts to a feventh Part of the whole Term, and fomething over : But in the Cafe of an Annuity for the older Life, fuppofe of 80 Years of Age, 'tis fet down as equal to a Term of

of 4-9103, which is four Years and eleven Months, wanting about three Days; and if we inquire, to what Term fuch a Life on the even Chance is equal, this Writer *p.* the *9th* fays between 4 and 5 Years; in *p.* the *7th* five Years; and, if we compute fuch Chance by his Bills of Vitality and Mortality for *London*, I find, and if any one elfe will give themfelves the Trouble of computing it, as I have done, he will find, that the even Chance is juft four Years and ten Months; fo that the Duration of the Annuity exceeds the Chance of the Duration of the Life, as well in Computations by this new Method, as by the old Rule; and therefore both of them abfurd and defencelefs.

After all, what do thefe little Subterfuges fignify? Even juft nothing at all; for Mr. *H. B.* a ftrenuous Advocate for Dr. *Halley*'s Rule, has admitted the Fact to be fuch as I have ftated it on an Operation made by fuch his Rule, and, on the like Operation by this Writer's Rule, the Fact muft come out nearly the fame, and confequently my Exception to the old Rule and this new Method will be equally ftrong, tho' perhaps it may not appear in a Light fo clear, even tho' the Chance of the Duration of this Life were fuppofed to be full five Years, and to exceed the Term in the Annuity, or that the Terms in both were equal. The Way of Reafoning here ftands thus : Since by Virtue of the Rule the Term in an Annuity for a Life is diminifhed and made lefs than the Term to which the Life on an even Chance is equal, 'tis neceffary that a certain and regular

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Proportion of fuch Decreafe fhoud be preferved through all the Stages of Life; but here, where in younger Life the Term in the Life is 35 Years, and the Term in the Annuity is 29 $\frac{7}{8}$, and in the elder the Term in the Life is 5 Years (fuppofing it were fo) and the Term in the Annuity four Years and eleven Months, or very near it; the Decreafe in one Cafe is one Year in feven, and in the other one Month only in 55 Months.—And if we fhoud fuppofe that in fom Inftance the Term in the Chance of the Life was the fame as the Term in the Annuity, which muft be the Cafe in fom one Inftance, tho' not here: And if it fhoud be faid that there the Value of the Annuity woud be the right one; I will admit it to be fo, and admit it for that Reafon, becaufe it has fortuitoufly happend in that Cafe, that the Term in the Life and the Term in the Value are the fame and coincide. But let us look a little into the State of an Annuity on a Life ftill older, fuppofe of 90 Years Age, and fee how Matters ftand there. The even Chance of fuch Life, *p.* the 9*th*, is faid to be a Term of three Years, and the Value of an Annuity for fuch Life given in his Tables is 1-8772; but this is not near equal to a Term even of two Years, for a Term of two Years is in Value 1-9135, fo that the Difference between them will exceed a whole Year; and confequently the Decreafe in this Cafe is one third at leaft of the Term, and this is a greater Decreafe than in a Life of ten Years Age, for there the Decreafe is only a feventh Part of the Term. We find then that to the

the old Difficulties on the Rule an Addition is here made of another; and, unlefs Mr. *Hodgfon* can difcover a better Solution of them than *H. B.* has done, thefe Irregularities in the Proportions of the Decreafes, believe me, do effectually deftroy the Juftice of the Regulator.

In the *Effay* very little Notice is taken of Mr. *Hayes*'s Method of Valuing Annuities on Lives, becaufe his Tables drawn for that Purpofe feemd to me fo abfurd and wide of the Mark, that I coud not not think that any one, of tolerable Underftanding in Things of this Nature, woud be guided by them: But fince his Treatife on this Subject has lately received a new Impreffion, in a very fuperb Drefs, I muft fay fomething to it now. I obferve he gives us the Age of 30 as the firft Age of Life, on which he begins his Valuation of Annuities; but for what Reafon he omits all the Stages of Life precedent to that of thirty I cannot imagin, unlefs he were of Opinion that the Age of thirty was the beft Age of Life: But for this Surmife there's no fort of Foundation in Nature or Experience; and on the contrary the *Breflaw* Table, the Bills of Mortality for *London,* and thofe of the Nominees in the Government Annuities on Survivorfhip, which is the beft Guide of all, do plainly fhow, that 'tis quite a vulgar Error. In *p.* the 1*ft* of his Book, as the Foundation of his fubfequent Calculations, he tells us, *that the Annuities, calculated in the following Pages, fhow, at one View, the prefent Value upon the Life of any Age from* 30 *to* 73, *according to the* Chance

Chance of an Annuitant's living to the Extremity of the common oldest Age of Life, and this is done upon the Suppofition of the various Degrees of Probability, which Lives of different Ages have to continue in Being. In *p.* the 22*d*, we have a Table, shewing how long an Annuitant muft live to receive the Value of an 100 *l.* funk, fuppofed to be given for his Annuity, according to the Chances of Life, at the feveral Rates of 4, 5, 6, 7 and 8 *per Cent.* Intereft: And, in *p.* the 74*th* to 127*th*, we have a Table, a very long Table, for the Value of an Annuity upon a Life from the Age of 30 to 73, at the like Rates of Intereft, and thefe I believe, do very well agree and coincide with each other. But then thefe two Tables, if right, do manifeftly demonftrate, that this Writer's firft and fundamental Pofition is abfolutely falfe; or the Pofition, if that be right, as undoubtedly it is, demonftrates that the Tables are all falfe, for thefe Tables coud not be framed, and 'tis vifible they were not framed, according to, or with any Regard had to the Principle there laid down, *viz.* the Chance or Probability of the Life of an Annuitant, for there can be no Chance or Probability of that happening in Life, which is impoffible ever to happen. I confefs it exceeds my Comprehenfion, by what Means or on what Grounds this Author coud be induced to fuppofe, even in the firft Inftant, and flatly in Contradiction to his own firft and fundamental Pofition, that the Chance or Probability of a Life of one and the fame Perfon coud by any Ways or Means whatever

ever be varied; and yet his Tables here are founded upon a Suppofition, that fuch Chance does vary as the Rate of Intereft varies, for his firft Table gives us on a Life of 30 Years Age the Chance or Probability of his living for 25 Years at 4 *per Cent.* Intereft, and at 8 *per Cent.* for 16 Years only; and the Inftance following fhows moft plainly that this is intirely falfe.

From his Table in P. 22, *&c.* which fhows how long an Annuitant muft live to receive the Value of the Money he gave for his Annuity, we find that if he buys at 4 *l. per Cent.* Intereft he muft live 25 Years, and if he buys at 8 *l. per Cent.* Intereft he muft live 16 Years. Now if one and the fame Perfon buys two Annuities for his own Life, and purchafes one at the Rate of 4 *l. per Cent.* Intereft, and the other at the Rate of 8 *l. per Cent.* then fuch Annuitant, in order to receive the Value of his Principal Money on one of the Annuities, muft live 25 Years, and, with Regard to the other, he need to live only 16 Years: But 'tis in Nature impoffible that his Life fhoud continue for 25 Years, if it ends in 16 Years; and almoft a Contradiction in Terms, to fay, that the fame Annuitant has a Chance or Probability that he fhall live 25 Years, and at the fame Time a Chance or Probability that he fhall live only 16 Years. Befides, thefe two Bargains can't poffibly be fair and juft ones on both Sides, for if the Annuitant has a Chance or Probability to live 25 Years, he has over-reachd the Vender who fold at 8 *l. per Cent.* becaufe with Regard to

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him the Annuitant will have been reimburfed the Principal, or received the Value, in 16 Years: and, on the other Side, if the Annuitant has a Chance to live only 16 Years, then with Regard to the Grantor at 4*l. per Cent.* the Annuitant himfelf will have been cheated, becaufe there will be nine Years wanting to make up his Principal, or to receive the Value of the Money given on the Purchafe. In fhort, if this Writer's fundamental Pofition, *viz.* that the Chance or Probability of the Continuance of a Life be the Meafure of the Value of an Annuity for the Life, be a right one, as I make no Doubt it is; or indeed if he intended to make Ufe of fuch Chance as his Rule for conftructing his Tables; inftead of his long Rows of Numbers and Figures, which are extended for 50 Pages together and upwards, he might even as well have fet out fo many broken Reeds, for he that makes Ufe of one as his Guide, in the Valuation of Life Annuities, will be led into grofs and very mifchievous Errors, as furely as he will be deceived in his walking who leans on fuch rotten Staffs for his Support. This Writer, I think, fomewhere in his Treatife, infinuates, that he grounded his Calculations and Rules on the Practices of fom of the great Companies in *London*, or, *vice verfâ*, that they grounded their Practice on his Rule; but, whichfoever of the two were the Cafe, I believe moft People at prefent will be ready to pronounce, that fom Events have fufficiently damned both the Rules and the Practice.

I fhall

I fhall not need to add any Thing farther on this Subject, unlefs fomewhat to inform the Reader on what Grounds I framed the Tables for the Value of an Annuity on a fingle Life, and in what Manner they were conftructed; fo that he may be able to judge for himfelf how far he may depend on them. I adhere then to the Pofition laid down in the *Effay*, that the Chance of the Duration of the Life is the only fure Meafure of the Value of an Annuity depending on that Life; for no material Objection to it has occurred to me which has not been obviated or fufficiently removed. The Chance of Vitality being then the Ground-work, in order to inveftigate this, I reviewed the Bills of Mortality for *London*; and having an Opportunity of feeing and confidering an Account of the Mortality of the Nominees in the Government Annuities on Survivorfhip, I had fpecial Regard to fuch Account; fince it containd a Bill of Mortality of Individuals without Variation, and therefore much the beft Guide to difcover and fix the Chance of Vitality.——

Now, fince there is one certain Stage of Life which muft have the greateft Chance of Vitality; for thofe Chances, from the Beginning of Life, do increafe to a certain Time, and from thence decreafe to the End of Life; my firft Inquiry was what might be that Year or Years of Life wherein the Chance of Vitality was greateft.——To fix this Point, I confulted firft the *London* Bills of Mortality, and thofe of the Nominees in the Government Annuities, and from thefe fixed the Age

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between fix and ten, including both, as the beſt Stage of Life, on the following Grounds. On a View of thoſe *London* Bills, it appears that the Number of Perſons dying between 10 and 20 Years Age, is 30 in a 1000; and in the Period between 5 and 10 Years age, tho' no more than Half the Length of the former Period, the Number is 38 in 1000; and in theſe two Periods the Proportion of Mortality is the loweſt: But then, tho' the Proportion of Mortaliry in theſe two Periods is not the ſame, and in the Period of younger Life is much the greater, I conſiderd that the Children of the Ages between five and ten were reſpectively and gradually removed farther from the Extremity of Life, and that the one might nearly counterbalance the other, I ſtated all thoſe five Years at one and the ſame Chance of 35 Years. I then examind how the Caſe ſtood in the Account of the Mortality of the Nominees in the Government Annuities, and found it to ſtand thus: The Perſons were nominated ſome time before *Midſummer* 1693; and at *Chriſtmas* 1693, the Account I have ſeen being drawn up from *Chriſtmas* to *Chriſtmas*, the Number of Perſons then ſurviving was 1009, and at *Chriſtmas* 1728 there were dead 505; ſo that in 35 Years one Half exactly were dead; and I ſuppoſe that all the Perſons nominated, and when nominated, at an Average, might be ſomething between the Age of ſix and ten. To ſatisfy myſelf farther in this Point, I made a Computation on the Bills of Mortality for *London*, and obſerved the

the ſame Method preſcribed by Dr. *Halley*, and made Uſe of by him and Mr. *Hodgſon*, and another on the ſame Bills in a Method formd by myſelf, and the Produce for the even Chance of Vitality for a Life of ten Years Age is, upon all the Operations, very nearly the ſame; and, in ſom of the Ages of Life younger than ten, this Chance runs rather higher.——

Having thus ſtated the Chances of Vitality for the beſt of Age of Life, *viz.* of ſix to ten, and ſuppoſed it to be equal to a Term of 35 Years; and ſince an Annuity for ſuch Lives will be equal to the ſame Term; moſt People, I believe, will think the Eſtimate ſufficiently high, even for that very beſt Age. From thence I proceeded to the Stages of Life precedent and ſubſequent to this, and propoſed to ſettle the Chances of theſe by inquiring into and fixing the Proportions of the Decreaſes of them from the ſaid Term of 35 Years, for every ſingle Year, or for every Period of ten Years, reſpectively. Here I was in Hopes of ſaving myſelf ſom Trouble, by uſing the Labors of ſom of my Predeceſſors who have wrote on this Subject, and particularly of Mr. *Hodgſon,* who in his Treatiſe, P. 6, has given us a Computation of theſe Chances, from every Period of ten Years of Life, from ten Years Age to the Extremity of Life. For this Purpoſe, I threw ſuch his Computation into a ſhort Sketch, exhibited at the End, in Scheme No. 1. but, on doing this, I diſcovered immediately that ſuch Scheme did not give us a right Partition of thoſe

thofe 35 Years or Chances for all the feveral Periods of ten Years there mentiond. The Defect in it appears almoft on the Face of the Scheme; but, in the Inftance which follows, 'tis fo grofs, that it cannot fail of being feen, when pointed out, by every Man of common Underftanding. That the Chance of Vitality on a Life for one Year does fink in that Proportion in which a certain Number of Perfons living at the Beginning of fuch Year does die before the End of it, is a plain and an indubitable Truth: And yet the Decreafe of thefe Chances given in the Scheme, in fom Periods of ten Years, where we throw fuch fingle Years into a Period of ten Years, runs counter to that Pofition, directly, and to a very wide Degree. For Inftance, in the Period from ten to twenty Years Age, the Chance of Vitality is funk eight Years, and confequently in Proportion for each fingle Year of thefe ten Years, and in the fucceeding Period, 'tis funk only five Years and an Half; and yet the Perfons dying in the firft Period are only 31 out of 490 living at the Beginning of it, and in the fecond the Perfons dying are 94 out of 459 living; from whence it appears that the Number of Perfons dying in the fecond Period is much more than double the Number of thofe dying in the firft, and yet the Chances of Vitality funk in fuch fecond Period are not near three Quarters of thofe funk in the firft: Therefore the Partition in this Scheme, not giving the true Decreafes of thefe Chances of Vitality for fom Periods and Years of

Life,

Life, cannot give us a juft Partition of thefe Chances for the whole Life. From thence it will follow, that the Tables for the Value of Annuities for a Life, conftructed by Mr. *Hodgfon*, do not give us the true Value of them, even tho' they were in all other Inftances faultlefs; for that the Value muft depend on fuch Chances, and he himfelf fuppofes as much, and yet thefe Chances themfelves, in fom Parts of Life, are not rightly adjufted, by this Method; and this is the Cafe in all the Stages of Life from the 10th to the 60th Age of Life, or near it; and, believe me, this is a very large, and, in all Refpects, a confiderable Part of Life.——

I cannot forbear adding a few more Words on this Subject, becaufe I think that, from this Writer's own Pofition, it may be demonftrated, that his Repartition of the Chances of Vitality, at leaft for a Life between 10 and 20, muft be wide of the Mark. In his making the Computation of thefe Chances, I fuppofe that he made Ufe of his own Tables of Vitality for *London*, by computing in what Time the Number of Perfons living of the Age of 10, of 20, and 30, were refpectively reduced to an Half, and then taking fuch Time as the Term to which Perfons of thofe refpective Ages were equal; for that, on fuch Computation, and on fuch Operation, they do come out to be the fame as we find them in his Treatife, *viz.* 35, 27, and 21½. Now as the Decreafes of the Chances between 10 and 20, and between 20 and 30, appeared to be unproportionate,

tionate, the firſt being eight Years by the Death of only 31 Perſons out of 490, and the ſecond being no more than five and an Half by the Death of 94 out of 459; I was ſo much ſurprized, as to miſtruſt this Method ſo far, as to examin it more accurately than I had done when I wrote my *Eſſay*, for there I gave into and made Uſe of it; but, on a cloſer Examination and frequent Trial of it, I am ſatisfied that the Rule will not hold in all Inſtances, tho' in ſom it does. I lookd then into the Poſition on which this Writer grounds and makes the Calculation of the Chances which immediately there follows; and, on a Calculation made by the Method preſcribed by that preliminary Poſition, it appears, that 'tis 14, and much above, to 1, that a Perſon of 10 Years old lives to the End of the ſucceeding Period of 10 Years; and that 'tis only 5, and a little above, to 1, that a Perſon of 20 Years lives to the End of the ſucceeding Period of ten Years: From whence it follows, that the Chances of Vitality for the Lives in the Period between 10 and 20 Years, is very far from being rightly apportiond; —for he who is 10 Years of Age, and has the Odds of 14 to 1 to live 10 Years, has evidently a greater Chance of living to be 20 Years old, than he that is 20 Years of Age, and has only the Odds of 5 to 1 to live 10 Years, has of living to be 30 Years old: And yet this Author's Computation of the Chances of Vitality of a Perſon of 10 Years, and a Perſon of 20 Years Age, makes theſe Chances of the Perſon of the younger Life

be greatly leſs than the ſame Chances on the older Life. This being the Caſe, on the Method made Uſe of by this Author for adjuſting the Chances of Vitality, I conſulted Dr. *Halley* on this Subject, but I find that he preſcribes and makes Uſe of the ſame Method as this Author does; but I muſt beg Leave, in this as well as in ſom other Matters of this Kind, to diſſent from them both, and to affirm that this Rule does not in all Inſtances, tho' it may do in ſom, give us the even Chances of Vitality, and that for the Reaſon and on the Computations juſt now made; and thoſe are built upon the Foundations laid down by Dr. *Halley* himſelf, where he gives the Rule to find the Odds, which a Perſon of a given Age has to live for a given Term. Being forced thus to reſort to a different Method, I formd that which follows, both to aſcertain the even Chance of Vitality for a Life, and for any Period of Years: And this differs very little from that given by the Dr. for calculating the Odds of Vitality for a certain Number of Years, in another Shape indeed, and with ſom Additions.

The Rule then is this: To take the Number of Perſons living at the Beginning of any Year or Term of Years, and the Number living at the end of that Year or Term Years; and if we ſubtract the one from the other, the *Remanet* is the Number of Perſons dying in that Year or Term of Years: Next, to take the Number of Years, which we may call Chances of Vitality, to which a Perſon of the Age named may live, if we compute to the Ex-

tremity of Life: and then to inveftigate what is the Proportion in fuch Years or Chances of Vitality, which the Number of Perfons living at the Beginning of fuch Year or Term of Years bears to the Number of thofe dead at the End of the Year or Term of Years. To perform this, we muft multiply the Number dead in the Year or Term of Years by the Number of Years or Chances of Vitality through the whole poffible Life, and divide the Produce by the Number living at the Beginning of the Year or Term of Years: And the Quotient will give us the Chances of Vitality funk in that Year or Term of Years: And, if we deduct the Number of Chances funk in that Time out of the Number of Chances fubfifting at the Beginning, the *Remanet* will be the Number of Chances due and belonging to the Life propofed, and the Term to which fuch Life will have an even Chance of Duration.

Or thus: In Cafe the Number of Years or Chances of Vitality for any Age of Life be ftated and known; if we want to know what be the Number of Years or Chances of Vitality due and belonging to a Perfon of the next or any other fubfequent Age of Life, we need only to take that Number of Years or Chances of Vitality, to which the Life of the Perfon of the Age next precedent to the Age of the Perfon after whofe Chances of Vitality we are inquiring; and then to inveftigate what is the Proportion in fuch Years or Chances of Vitality, which the Number of Perfons living at the Beginning of fuch Year or Term of

of Years bears to the Number of thofe dead at the End of the Year or Term of Years, and then to proceed as before directed.

I fhoud obferve here, whether we take a Number of Years or Chances, according to all the poffible Years or Chances fubfifting on the Life given, which in a whole Life may be fuppofed to be an 100, it being poffible that a Perfon juft born may live to be an 100 Years old; or whether we take fuch a Number of Years or Chances, as anfwers to the even Chance of Vitality on the Life of a Perfon of the Age next precedent to the Age of the Perfon of whom the Inquiry is made, where fuch even Chance is already ftated and known, the Decreafe of the Years or Chances in each Life or Term of Years muft be and will be exactly in the fame Proportion. So that either of thofe two Numbers may be ufed with equal Succefs.

As this Way of Reafoning and the Rule may feem a little abftrufe, I will apply it in an Inftance or two on the Perfons named in the Government Annuities on Survivorfhip. The Nominees in thefe Annuities were named at or before *Midfummer* 1693, and at *Chriftmas* 1693, when my Account begins, the Number of them then living was 1009, and I did fuppofe that the Age of each Nominee, at an Average, might be about nine or ten. We want to know then to what Age any one of thefe Nominees might probably live, or what were the Chances of Vitality due and belonging to any one of them, fuppofing him

to be ten Years old, and as of fuch Age to have 90 Years or Chances of poffible Life. The Number of Perfons living at the Beginning of this Year, *viz.* at *Chriftmas* 1693, was 1009: At the End of the Year 1728, there were living 504, and, if we deduct 504 out of 1009, the Number remaining will be 505, that is, in that Compafs of Time died 505. The Age to which any one of thefe Nominees might poffibly live, being all fuppofed to be ten Years old, is 90, Then under the Direction of the Rule we multiply 505, the Number of the dead, by 90, which is the Number of Years or Chances through the whole poffible, and the Produce is 45450: And if we divide this Produce by 1009, which is the Number of Perfons living at the Beginning of the Term, the Quotient comes out to be 45, a fmall Matter over. From hence, we find that between *Chriftmas* 1693 and *Chriftmas* 1728, that is, in 35 Years or very near it, one Half of thefe 90 poffible Chances were loft, and confequently the other Half left: Therefore the Life of one of thefe Nominees, on the even Chance, was equal to a Term of 35 Years, or might probably continue in Being for fuch a Term; and on this Computation, and fom others before mentioned, I ftated the even Chance of a Life of ten Years Age in general at 35 Years, as I found it to be in this Particular.

To give another Inftance: We will compute the Chances of a Life, where we will fuppofe that thofe Chances of Vitality for a Life of ten Years Age

Age is fixed and known; and there will make Ufe of the fecond Part of the Direction of the Rule. The Number of Nominees in the Annuities on Survivorfhip living at *Chriftmas* 1693 was 1009: The Number dying between *Chriftmas* 1693 and *Chriftmas* 1703 was 117. The Age of each Nominee, at an Average, we have fuppofed to be ten Years or fomething near it: A Life of ten Years Age, on the even Chance, we have ftated as equal to 35 Years: Now, if we multiply 117 by 35, the Produce will be 4095, and, if we divide that Number by a 1009, the Quotient will be 4, and a little over: So that the Decreafe of the 35 Years or Chances of Vitality in thofe ten Years will be 4 Years and a fmall Matter over; and, if we deduct 4 out of 35, the Years or Chances of Vitality remaining will be 31, and this will be the Number of Years or Chances for a Life of 20 Years Age. If we proceed in like manner on a Life of 30 Years Age: The Number of Perfons living at *Chriftmas* 1703, fuppofed then to be of the Age of 20 or near it, was 892: A Life of 20 Yeas Age, on the even Chances, has been computed and ftated as equal to a Term of 31 Years: The Number of Perfons dying between *Chriftmas* 1703 and *Chriftmas* 1713 was 169: And, on the like Operations as before, the Decreafe of the Years or Chances in that Time will be 5 Years and $\frac{7}{8}$, or near fix Years; and, if we deduct fix Years out of 31, the Years or Chances then remaining will be 25, and

and will be the Number of Years or Chances for a Life of 30 Years Age.

By this Rule, and in this Manner I computed the Decreafe of thefe 35 Years or Chances, by Periods of ten Years, from the Age of 10 to an 100; and made Ufe of the Bills of Mortality, in the Annuities on Survivorfhip from the 10th to the 68th Year of Life, which was as far as thofe Bills coud conduct me, and from thence, being deftitute of this fure Guide, for the four Periods of remaining Life, I made Ufe of the *London* Bills of Mortality, obferving ftill my own Rule: And by thefe Decreafes I conftructed the Value of Annuities exhibited in the Table annexd. I have before computed, on the Bills of Mortality for *London*, the Decreafe of thefe 35 Years or Chances of Vitality, for each Period of ten Years, from 10 to an 100 Years Age, from the Account given of them by Mr. *Hodgfon*, and have fet out the Proportions of them for each Period of ten Years in Scheme N. I. — Here I have made a Computation likewife, in my own Method, of the Decreafes of the fame Chances on the fame Bills of Mortality, for the fame Periods of Life, and the Proportions of them, as ftated by me, ftand as in Scheme N. II; and I believe it may be fafely left to the Reader, on a near Infpection on thofe two Schemes to difcern which is neareft a regular and proportionate Decreafe, and deferves the Preference; efpecially if this Confideration be taken along with it, *viz.* that

that this Writer does in exprefs Terms fay, that, *after Perfons arrive at ten Years, the Decrements of Life are little in Comparifon with the former Years, and decreafe regularly.*

I affert then, on computing by my Rule, and on the *London* Bills of Mortality, that the Years or Chances of Vitality loft in the ten Years, between the Age of ten and twenty, are no more than $2\text{-}\frac{7}{8}$, as ftated in the Scheme, N. II; and that a Life of 20 Years Age is equal to $32\text{-}\frac{1}{8}$. Nay, I may go farther, and affert, that the Cafe is exactly the fame, both with Regard to the Years or Chances loft in that Time, and the Years or Chances remaining at the End of it, even where we ufe Dr. *Halley's*, or this Author's Rule for computing the Odds or Chances that a Perfon of ten Years Age does or does not live to the Age of 20 Years, and this too, whether we make fuch Computation by the whole Period of ten Years collectively, or by each Year fingly and feparately. Confequently, if a Perfon of ten Years Age has at that Time an even Chance to a Term of 35 Years, which is a Point admitted on all Hands, and in living ten Years lofes no more than $2\frac{7}{8}$ of thofe Years, or Chances, the Years or Chances remaining muft of Neceffity be $32\frac{1}{8}$, and his Life at 20 Years Age be equal to that Term: And therefore the even Chance of Vitality for a Life of 20, computed by Dr. *Halley's* and this Author's Rule, and ftated as equal to 27 Years only, is very far wide of the Mark.—

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The Upſhot of the Matter is this; that in ſtating theſe Chances by Dr. *Halley*'s, or this Writer's Rule, which is the ſame, the Decreaſe of them, in moſt Inſtances, eſpecially in the younger Part of Life, is the greateſt when the Number of Perſons dying is proportionably loweſt, and the Decreaſe the leaſt, when the Number dying is higheſt; which is directly contrary to the Truth, for no Man, ſurely, can conceive that theſe Chances ſhoud ſink 8 Years in the Period between 10 and 20, in which die only 31, out of 490, that is, at an Average, three only in each Year, and one over in the Whole; and yet, that they ſhould ſink no more than three Years in the Period between 40 and 50, in which there die 90 out of 294, that is, at an Average nine in each Year; and at the ſame Time admit, and it muſt be admitted, that the Number of Perſons dying is the Meaſure of ſuch Decreaſe.

In conſtructing the Tables for theſe Chances of Vitality, and the Value of Annuities on a ſingle Life depending thereon, I made Uſe both of the Government Annuities, and *London* Bills of Mortality, as beforementiond but by the Method laid down by myſelf; and choſe here to compute thoſe Chances by Periods of ten Yeas firſt, and then divide each Period into ſingle Years at an Average, rather than to compute them by ſingle Years, in Regard that in ſom one Year or other the Number of Perſons dying in that Year might much exceed or fall ſhort of the Number dying in the precedent or ſubſequent Year, merely by

by Accident, from the greater or leſs Salubrity of the Air in one Year than another, and this appears to be frequently the Caſe in the Nominees in the Government Annuities. As for the Chances of the intermediate Years within each Period, I interpolated them by an Equation, as near as coud be done, by keeping within the Compaſs of an eighth Part of a Year, and that there I ſuppoſe is as near the Mark as is neceſſary in theſe Matters, or can well be put in Practice. To this I have added, and inſerted in Table N. IX. a Scheme of the Chances of Vitality for the City of *London*, conſtructed, according to my own Method, from the Bills of Mortality there; and, on comparing it with the Chances in the general Tables, the Difference does not ſeem ſo great, but that in Practice, in moſt Caſes, a Man may ſafely enough make Uſe of either of them as he likes beſt: And, tho' I have not here added the Value of an Annuity on any of theſe Chances of Life for *London*, yet that Defect is eaſily ſupplied by one or other of the precedent Tables.——

The Value of Annuities on a Life given in the general Tables, and that which will ariſe on a Computation made by Table, N. IX, will at firſt View and to ſom Perſons ſeem very high; but, for my own Part, I am fully perſuaded, and others, perhaps, when they have well conſiderd what has been here offerd to prove it, may be induced to think ſo too; eſpecially if they admit that the Chances or Probabilities of the Duration of Life are in the Tables rightly eſtimated. Where

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a great Number of Annuities is granted to Variety of Perfons on their Lives, a Variety of Chances muſt in the Event neceſſarily come out, and I believe it will be found ; that the Eſtimate here made, at an Average, is a fair and reaſonable one, on the Whole : And on ſingle Grants to ſingle Perfons there's no Way to form a Judgment of the Value of them, but by accumulating a great many Grants together, and taking a Medium. And in this my Opinion, that I have not overvalued theſe Annuities for a Life, I am very much confirmd by a late and recent Inſtance, and which will not ſoon be forgot by ſom People, ſince they have terribly ſufferd by not well conſidering or ill calculating the Value of ſuch ſort of Eſtates ; and will be a Caution, at leaſt, to Perfons to be on their Guard in Tranſactions of this Nature : *Felix quem faciunt aliena Pericula cautum.*

ADDENDA.

ADDENDA.

AFTER writing the preceding Diſcourſe, and indeed ſince it was prepared and ready for the Preſs, was put into my Hands Mr. *De Moivre*'s Treatiſe on this Subject, the third Edition ; and having no convenient Opportunity of inſerting my Obſervations upon it in the Body of my Book, and as his Character challenges a ſpecial Attention, I hope I ſhall be excuſed, if I give my Thoughts upon it in this Place ; and they are, in ſhort, that the Rule which he propoſes for inveſtigating the Value of Annuities for a ſingle Life is a wrong one, and all the Tables which he has conſtructed by that Rule, in every Inſtance, erroneous. My great Objection to this Rule, which I think quite unneceſſary to be inſerted here, is the ſame as I made to Dr. *Halley*'s Rule, and is this ; *viz.* that in Fact it does, and neceſſarily muſt, produce ſuch a Sum as the Value of an Annuity for one and the ſame individual Life, as that, on the Variation of Intereſt, the Term to which ſuch Value correſponds ſhall be a different one.

Now the Defect in the Doctor's Rule which produces theſe abſurd Values ariſes from hence, that it directs, that out of the Value of an Annuity

nuity

nuity certain for a Year, in each Year of Life and through the whole Life, we are to deduct a Value corresponding to the Chance of Mortality arising in each Year respectively, or, which is much the same Thing, a Value in Proportion to the Degree of Probability that the Life may determin within the Year. Mr. *De Moivre*, in the first Edition of his Treatise on this Subject, P. 5, *&c.* gives the same Direction in Substance, tho' in Expression somthing variant; and in Persuance and under this Direction, P. 11, *&c.* he computes the Value of an Annuity for the Life of a Person aged 50, and at an Interest of 5*l. per Cent.* and concludes that 'tis in Value 10-15. In this his third Edition, which he calls a more full one, he has thought fit to leave out the Whole of this Direction, and enters immediately upon the Solution of that Problem, *viz.* what may be the Value of an Annuity for the Life of a Person aged 50, and, after a Calculation in the same Method as in his first Edition, concludes that 'tis in Value 10-35. But 'tis impossible that 10-15 and 10-35 should both be the Value of this Annuity; and it happens very unluckily that neither of them are right, for the exact Value is 10-34. But to let this pass, for 'tis not material which of the three, or whether any of them, be the real Value; but it may be a proper Question, what might be the Reason of omitting, in this his fuller Edition, the Whole of this Direction. Som People, perhaps, will solve this Problem by saying, that, since his first Edition, he had discover'd

coverd that this Direction produced the preposterous Values of Annuities before-mentioned; but that, to be sure, must be an idle Surmise, becaufe, even in this more correct Edition, and in his Tables inserted in it, he has perfued this very same Direction. Others will suggest, that on finding this Objection made to his Rule, and this Direction in particular, to which they are confident he was no Stranger; and that it was too clear and strong for him to grapple with, and most certainly it has never been answerd, and probably never will be; he thought it the best Courfe to remove it out of Sight, as much and as far as he coud.

But be that as it will; since I have already, in my *Essay*, treated of this Matter, *viz.* of the absurd Values produced by the Rule, and have farther reinforced my Objections to it in the preceding Treatise; I shall here content myself with afferting, which indeed is very little more than repeating, that 'tis in Nature impossible that one and the fame Life can have one and the same Chance or Probability of Duration for different Terms of Years; for Instance, for 25, 26, or 28 Years, as the Rate of Interest, at which the Value of the Annuity is computed, is or may be varied; or indeed on any other Account, or by any other Means whatfoever. I need therefore only to show that the Rule does in Fact produce fuch Values; and that is feen by a mere Infpection into the Tables conftructed by the Author himfelf, by the Rule, and there they ftand thus.

The

The Annuity for the Life of a Perfon of 10 Years Age, at 4 *l. per Cent.* is in Value 16-88, which is equal to a Term of 28 $\frac{1}{8}$—of the fame Age, at 5 *l. per Cent.* is 14-60, which is equal to a Term of 26 $\frac{7}{8}$—and of the fame Age, at 6 *l. per Cent.* is 12-84, which is equal to a Term of 25 $\frac{1}{4}$. Now I fay, and I believe moft of Mankind will concur with me, if a Perfon of 10 Years Age has a Chance or Probability to live only 25 Years and $\frac{1}{4}$, that 'tis not poffible he fhould have the fame Chance or Probability to live to the Age of 26 $\frac{7}{8}$, or of 28 $\frac{1}{8}$.

If we carry this Matter a Step farther, and compute by this Author's Rule what is the Value and Term correfponding on a like Life, at 3 *l. per Cent.* we fhall find it to be 19-87, which is equal to a Term of 30 $\frac{3}{4}$; and 'tis Matter of fom Wonderment that this Author, in this his Edition of the Year 1750, has not inferted in his Tables the Value of Annuities for a Life at that Rate of Intereft. Since the Government has reduced the Intereft on public Securities, which in Stock and Annuities amount to more than fifty Millions, from 4 to 3 $\frac{1}{2}$ for feven Years, and from thence to 3; and as the Profpect that the common Price of Money will be at that Rate feems to be no very diftant one; I fuppofe, if private Perfons, public Companies, or the Government, were difpofed to grant Annuities for Life to any Number of Perfons, that they woud hardly think it prudent to charge their Eftates or Income with an Intereft exceeding 3 *l. per Cent.* efpecially on Securities

Securities not redeemable. On this Account I fhoud have thought that fuch a Table, now at leaft, woud have been very proper, and fo I have practifed myfelf; but this Author, perhaps, might be apprehenfive that fuch a Table woud have expofed the Error of his Rule too nakedly; for in Truth the Difference between the Term on a Life of 10 Years Age at an Intereft of 3 *l.* and the Term on the fame Life Intereft at 6 *l.* is five Years and an Half: However, the Tables which he has exhibited have done it fufficiently.

To the Evidence from his Tables we muft add, that the Author himfelf in the Body of the Book, in this laft Edition, has inferted fomthing which puts this Matter beyond all Doubt: And, fince he has been fo good as to have brought in fo ftrong a Reinforcement to fupport my main Objection, I fhoud be uncivil, if I did not acknowledge and take Notice of his Supply. Let us read them. In *p.* the 22*d,* he gives us this Problem. *Suppofing three equal Lives of any given Age, for Inftance 30, and that, upon the Failing of any one of them, that Life fhall be immediately replaced* (which by the Way has fomthing of the impoffible in it) *and I then receive a Sum I agreed upon, and that to Perpetuity for me and my Heirs: What is the prefent Value of that Expectation, and at what Intervals of Time, one with another, may I expect to receive that Sum.*— I have fet forth the Problem at large, becaufe the fubfequent Remark which he makes, on which I lay my Finger, becomes thereby more

<div align="right">plain</div>

plain and intelligible.—The firſt Queſtion is, *viz.* what is the Value of ſuch Expeſtancy, after having ſuppoſed that the Sum to be paid on theſe Contingencies was an 100 *l.* he reſolves that the preſent Value is—162 *l.*—Now I very much doubt the Juſtice of this Solution; for the Sum produced differs conſiderably from the Sum produced by the Method which I uſe to compute the Value of ſuch Expeſtancies, and I may be confident that my Method is a right one, becauſe 'tis founded on the Reaſon of the Thing, and might be confirmd by Demonſtration, and on a Computation in my Manner, if we ſuppoſed theſe Inſtalments to incur once in ten Years for ever, and he ſuppoſes them to incur in 9¼, the Value of this Expeſtancy comes out to be 181-16-3-1, and near a Farthing more.

But let the true Value of it be what it will: We will now read his Remark on the ſecond Queſtion, *viz.* at what Intervals of Time, one with another, he might expeſt to receive that Sum: And after ſolving that Queſtion, in *p.* the 28*th,* he adds—Remark—*It is not to be expeſted, if the Intereſt of Money was higher than* 5 *per Cent.* which was the Intereſt made made Uſe of in the preceding Calculations, *the Intervals of Time, after which the Renewals are made, woud be the ſame as now; for it will be found, that, as Intereſt is higher, the Intervals will be ſhorter, and, as it is lower, the Intervals will be longer: Yet one might make it an Objeſtion to our Rules, that the Length of Life woud thereby ſeem to depend upon the Rate*

of

of Intereſt. The Author admits then, that 'tis an Objeſtion to his Rules, if they do in Faſt make the Length of Life to depend on the Rate of Intereſt; and, whether he admits it or not, moſt certainly it is ſuch—The ſole Queſtion then will be whether a Variation of the Rate of Intereſt in this Caſe does not likewiſe cauſe a Variation in the Length of the Lives, as it does in the Length of the Intervals. He lays it down, that the Intervals of Renewal, or the Space of Time between one Renewal and another, will and muſt vary as the Intereſt of Money varies, on the one Side or the other. From thence I argue thus: If the Intervals of Renewal are made ſhorter or longer (as the Caſe may happen to be) by a Variation of the Rate of Intereſt, then the Duration of the Lives, on which the Intervals depend, muſt be ſhorter or longer in the like Proportion, becauſe the Duration of the Lives is made, and by this Author himſelf is taken for, the Meaſure of theſe Intervals. Or take the Reaſoning thus: The Rate of Intereſt governs the Intervals, the greater or ſhorter Length of the Lives conſtitutes, or rather is one and the ſame Thing as thoſe Intervals: The Rate of Intereſt therefore governs the Length of the Lives. This Way of Reaſoning is ſure enough right and concluſive; but it may be further added, that this Writer's Tables, drawn by himſelf, and by his own Rule, in the Inſtances beforementioned, do demonſtrate that, in the Caſe of Annuities for a ſingle Life, the Rate of Intereſt governs the Length of Life,

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for, as one is varied, the Length of the other is varied with it and by it.

However, fince this Writer offers at fomthing which he calls an Anfwer to this Difficulty, let us read ; and I choofe to give it in his own Words, becaufe it has fomething of the extraordinary in it, *viz.* a Defire to feem to fay fomthing, a Study to involve that feeming fomething in obfcure and perplexd Language, and a Care at the fame Time to fay nothing to the Purpofe ; which laft may indeed very eafily be done in a Matter to which nothing can be faid to the Purpofe. The Words are thefe. *It muft be obferved, that the calculating Time imports no more, than that (confidering the Circumftances of the Purchafer and Proprietor of the Lives, in Refpect to the Rate of Intereft agreed upon, and the Sum to be given upon the Renewal of a Life or Lives) the Proprietor makes the fame Advantage of his Money, as if he had agreed with the Purchafer, that he fhould pay him a certain Sum of Money at equal Intervals of Time, for redeeming the Rifque, which he the Purchafer* (read rather *the Proprietor*) *runs of paying that Sum when the Life or Lives drop ; but the real Intervals of Time will be fhown afterwards.* Part of this Sentence I have included in a Parenthefis, fo that thofe Words being omitted in the reading, as they may very well be, it may more eafily be underftood ; and for the Word *Purchafer* we muft read *Proprietor* : And then the Subftance of his Anfwer will be, that the Proprietor of the perpetual Leafe, as I woud call it, will make the fame

Advan-

Advantage of his Money, if he continues to make his Payments on the Contingency of Lives dropping, as if he made an Agreement to pay the fame Sum of Money at the End of a determinate Number of Years, equal to the computed Intervals. But is not the Calculation of the Time a juft one, and the very fame, whatever was the Import or Views in making it ? Do not the Words at the Clofe of the Sentence, that the real Intervals of Time fhall be fhown afterwards, ftrongly intimate, and fuppofe that there is a real Difference in thofe Intervals, where there is an Alteration in the Intereft, otherwife no Occafion to fhow them, nor coud any be fhown ? Does not the Author himfelf affert as much, and is not the Fact notorioufly true ? And is it not evident, that thefe Intervals are created by, or are one and the fame Thing as, the Duration of the Lives, and this Author himfelf meafures and demonftrates the Length of the one by the Length of the other ? To thefe Queftions if the Anfwer be in the Affirmative, and in the Affirmative the Anfwer muft be ; it will follow, as clearly and certainly as any Demonftration in this Writer's Book, that the Length of the Lives as well as of the Intervals is by his Rule made to vary and depend upon the Rate of Intereft : And thus by one Breath of his Mouth has he blown in Pieces all his fine Piece of Machinery.——

If we carry our Inquiry farther and examine what Effect his Rule has on the Value of Annuities for joint and furviving Lives, and whe-

ther

ther it does not there likewise make the Duration of Lives depend on the Rate of Intereſt, we ſhall find the Caſe to be the very ſame: And indeed 'tis impoſſible but that it ſhoud have the ſame Influence there as in Annuities for a ſingle Life, for the Values of Annuities for two or more Lives, either joint or ſurviving, are all of them, by an ill-concerted Jumble of Values, form'd from the Values of Annuities for a ſingle Life. If this Author had been ſo good as to have given us Tables of the Values of Annuities for theſe Lives, as he has done of thoſe for a ſingle Life, I doubt not but he woud have enabled us to have ſhown in like Manner, that is, out of his own Mouth, that his Values of Annuities for theſe Lives likewiſe had the very ſame Fault: But, ſince we are deprived of an Opportunity of doing it in that Way, I will try what may be done without that Aſſiſtance.

In *p.* the 54*th*, third Edition, he gives us this Problem, *viz. to find the Expectation of two Joint Lives, that is, the Time which two Lives may expect to continue together in Being*; and then having taken two Lives of the ſame Age with thoſe given in the Solution of his ſecond Problem, *viz.* one of 40 and the other of 50, he ſolves the Queſtion, and ſays, that 13-31 is the Number of Years due to the two Joint Lives. Now if the Expectation or Probability be, that one or other of theſe two Lives will have a Being for that Term, I humbly apprehend that the Annuity too will ſubſiſt for that Term, and at 5 *l. per Cent.* be of the Value

Value 9-54. No, by no Means ſays a Learned Advocate of Mr. *De Moivre*; for he has demonſtrated, in his Solution of the ſecond Problem, and the Demonſtration which follows, that the Sum 7-62 is the true Value of the Annuity for theſe two Joint Lives. The Matter under Conſideration here is, whether the Rate of Intereſt does not govern this Value, and this Value govern the Duration of the two Lives, and not whether the Value be the right one or there be a Demonſtration of it, tho' if there were Room for it here, and it were pertinent to the preſent Purpoſe, I coud ſhow there was none; let us ſee to what Term of Years this Value correſponds. If we look into this Author's third Table, which is conſtructed for the Value of Terms for Years at 5 *l. per Cent.* and that is the Intereſt which is made Uſe of in computing the Value of the Annuity for theſe two Joint Lives; we find 7-72 to be the Value of an Annuity certain for a Term of ten Years; ſo that this Annuity is not equal to a Term of ten Years, but muſt determin before that Time, tho' both the two Lives will ſubſiſt and continue for thirteen Years and above.——

As I do not deſign, here, to deprive this Gentleman and his learned Friend of the Pleaſure of their Conceit (and I can call it by no other Name) that there is Demonſtration of this Value, I will admit it to be ſo for the preſent, on a Proviſo neverthelefs and Condition that they return me the like Civility, and admit that there is Demonſtra-

tion

tion likewife of the 18th Problem beforemen- tiond, and then we fhall have Demonftration fighting againft Demonftration ; for one fays that the Annuity is in Value 7·62, and the other fays that it is 9·54, as before computed. Now, if it may be demonftrated, nay, whether demon- ftrated or not, if it be a reafonable Expectation or Probability, that one or other of thefe two Lives will fubfift for thirteen Years and above, I very much doubt, whether the Learning of Mr. *De Moivre* and his Friends, both put together, will ever convince any Man, unlefs it be a very fpecial Friend or fo, that 'tis probable that this Annuity, which determines, when one of the two Lives fails, and not before, will determin in ten Years Time or lefs, and at the fame be equally pro- bable, that both the Lives will continue in Being thirteen Years and above. *Credat Judæus appella non ego.*

To apply this. Let then the Value of this An- nuity be 7·62, or any other, except 9·54, which is the Value when computed on the Probability of the Duration of the Lives ; fince the Rate of Intereft creates this Value, and this Value deter- mins the Duration of thefe Lives, if that fame Rate of Intereft does not govern the Length of the Lives, I fhoud be glad to be informd what does; fince 'tis evident that the Expectation or Probabi- lity of the Lives does not ; for then the Value woud be 9·54, as before computed. But that which will put the Matter beyond all Difpute will be to vary the Rate of Intereft, and to calculate the Value of Annuities on two Joint Lives at the

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feveral Rates of 4, 5, and 6 *per Cent*, and then examining to what Terms thofe Values do ref- fpectively correfpond—On an Annuity for two Joint Lives, each of 45 Years, the Values, and the Terms to which they refpectively correfpond, as I compute them, ftand thus—

At 4 *per Cent.* Value 8-21, equal to a Term of $10\frac{1}{8}$
At 5 *per Cent.* Value 7-42, equal to a Term of $9\frac{1}{2}$
At 6 *per Cent.* Value 6-76, equal to a Term of $8\frac{7}{8}$

In Mr. *De Moivre's* Way of computing thefe Va- lues, they differ a little, and may ftand thus——

At 4 *per Cent.* Value 8-34, equal to a Term of $10\frac{3}{4}$
At 5 *per Cent.* Value 7-72, equal to a Term of 10
At 6 *per Cent.* Value 7-06, equal to a Term of $9\frac{1}{2}$

Now let us take which of the two Valuations we pleafe, 'tis plain and obvious that the Diffe- rence in the Rate of Intereft produces fuch a Difference in the Value as creates a Difference in the Term to which the Lives are equal, and this is no other than governing the Length of the Durati- on of the Lives. It will be faid, perhaps, that my Va- luation is a wrong one, but, if it fhoud be fo, yet the Valuation made in Mr. *De Moivre's* Method juftifies the Conclufion ; and, if Mr. *De Moivre* has demonftrated that thefe are the true Values of this Annuity at thefe three feveral Rates of Intereft, he has then demonftrated the Point in Queftion, *viz.* that by his Rule the Length of the Lives is made to depend on the Rate of Intereft— I fay, however, that my Valuation, where 'tis

made

made by a Computation on the Values, is a right one, as founded on the Reafon of the Thing, and confirmd by this Author's Solution of his 18th Problem, where he fays, that on two Lives of equal Age, the Probability is ⅓ of the Complement of any one of the two Lives. Now the Method by which I attaind the Values before given by me is a very eafy and a very obvious one, *viz.* by taking two thirds of the Value of one of the Lives fingly taken, or, which is the fame Thing, by taking one third of the Values of both put together. On this Account and in this View, fom People will be apt to furmife that Mr. *De Moivre* had the fame Thing in his View, *viz.* to take two thirds of the Value of one Life, or one third of the Value of both put together, becaufe 'tis confonant to his Reafoning on this 18th Problem ; but that he has inveloped the Matter in fuch Clouds of Multiplications, Subtractions, Divifions, and of Things not fubject to the fame Rules of increafing or decreafing, that he has made a fhift to mifs his own Mark : And this moft certainly he has done, unlefs he difclaims his own Problem.

But fom will fay that on Mr. *De Moivre*'s Side there is Demonftration of the Value, but I fay there is no fuch Thing. His Solution of his fecond Problem, which produces this Value 7.72, contains no more than this. The Value of an Annuity for two Lives fingly taken, being given ; then follows a Multiplication of thefe two Values together, then a Multiplication of this Product by the fuppofed Rate of Intereft, afterwards a

Subtraction

Subtraction, and laftly a Divifion; and the Quotient is faid to be the Value of the Annuity. Now this Procefs does indeed demonftrate that the laft remaining Quantity is 7.72 : But does no more prove that 'tis the Value of an Annuity for thefe two Joint Lives, than it does, that 'tis the Value of a Mare or a Mule. The Thing which ought to be proved is, that this Procefs on the given Suppofitions will produce fuch a Sum as is the Value of the Annuity, and not merely what is the Sum which the Procefs will produce, and the Procefs carries us no farther. Ay, but then follows a Demonftration ; very true, but that ftill is no more than this, that Letters and Lines are fubftituted in the room of Numbers or Figures, and upon an Operation on them that the Value produced on fuch Operation is one and the fame, as that produced by the Procefs, and it woud be very ftrange if it were otherwife; but no Proof yet that this Value is the Value of the Annuity, or that the Procefs in one or the Operation in the other does produce fuch a Value. But that's not all : For there is, or feems to be, a Defect running both through the Solution and Demonftration, which muft neceffarily make them produce a Sum not exactly the true Value. It may indeed produce a Sum which fhall come fomthing near the Truth, and he muft be a very bungling Operator who coud not contrive Matters, fo as not to be wide of the Mark ; but 'tis a Rule in Mathemetics, I think, that if a Man does not hit the Bird in the Eye, as the common faying is,

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he is as much in the wrong, as if he were hundred Miles off.

The Defect I meant is this. In computing and fixing the Value of thefe two Annuities, when fingly taken, he lays it down that the Decrements of Life, or of the Probabilities of the Duration of Life, are in Arithmetic Progreffion: and then proceeds to multiply thefe two Values thus fixd, and then multiplies the Product by the given Rate of Intereft, *viz.* by 5. The Years or Chances of Life he calls the Complement of Life, and on the whole poffible Life ftates them at 86, afferting that Life cannot reafonably be extended beyond 86; and then, thefe two Lives being each of the Age of 45, the Complement or Chances of Life refting on each of them will be 41: And then fays, that thefe 41 Chances will decreafe in Arithmetic Progreffion, as the Life proceeds in Age. I will admit that the Chances of Life do decreafe in that Proportion, under the fame Limitations as he puts upon them; *viz.* that it holds for fmall Portions of Life, and when the Years are near one to another, but not abfolutely for the whole Life, when taken intire. But then, as to the fixd Values of the Annuities on thefe two Lives, which are each ftated at 11-14, there can be no Pretence that they likewife decreafe in the like Arithmetic Progreffion; for, if they did, then at the End of 20 Years and an Half, for Inftance, when the Complement or Chances of Life are reduced to Half, then the Value of the Annuity for the Life fhoud be reduced to Half in like Manner; but this

this is notorioufly untrue and vifible by a meer Infpection into his Tables of thefe Values, and comparing them, by which it appears that one is 11-14, and the other 7-05; and the Truth of the Cafe is, that the Decreafe in the Values is in geometric Progreffion. And then as to the Number 5, the fuppofed Rate of Intereft, in that there's no Decreafe at all, it being invariably the fame through the whole poffible Life; what fhall we do with this? Why truly I was thinking, fince we are upon the decreafing Pin, that the Show, perhaps, woud have a more oftentatious Appearance, if, inftead of 5, we fhoud take 4-9524, which is the prefent Value of 5 *l.* to be paid at the End of the Year; and we know Intereft is not payable till the End of each Year, and in Fact is feldom paid fo foon, nor do the Probabilities of Life decreafe in their arithmetic Progreffion, nor the Values in their geometric Proportion, till the End of the Year; and therefore I hold it advifable, in the next Edition, by all Means to make them fink, one and all, and all at one Time. If then we fubftitute 4-9524 in the Place of 5, there will be an Opportunity of adding fom new Numerical and Mathematical Decorations; and befides upon my Word, *aut video, aut vidiffe puto,* I fee, or think I have feen, and I muft not fay more, becaufe I have not computed it, nor defign ever to compute it, that the Sum forth coming will be nearer the Mark, *viz.* 7-42, for that moft certainly is the Value when the Computation is made by Values, as is done by him in this Cafe.

Now to blend and mix together, as it were in Hotchpot, two Things which decreafe in one Proportion with two other Things which decreafe in another Proportion, and then to throw in a third Thing, which has no Decreafe at all; and from fuch heterogenous Ingredients to hope that we may extract the Value of a third Thing, is furely an idle and vain Imagination. But if Mr. *De Moivre* thinks otherwife, and will adhere to his Demonftration, he may, for me, and, for the prefent at leaft, continue in the peaceable Enjoyment of his beloved Mathematical Proofs, and I will content myfelf with the humble Productions of plain common Senfe and Reafon. I have faid at prefent, becaufe the Queftion here is not fo much, whether 7-72 or 7-42 be the true Value of the Annuity for thefe two Joint Lives, and for my own Part I am fatisfied that neither of them be fo, as whether on a Variation of the Rate of Intereft, in this Method of computing their Values, there be not fuch Values produced as make a Variance likewife in the Term to which the Lives correfpond, and in the Length of the Lives. And, fince that Point is evident, I fhall not purfue the other any farther here, becaufe it tends rather to fhow that this Author's Method of inveftigating the Value of Annuities for Joint Lives is erroneous, and the preceding Difcourfe treats only of the Value of Annuities for a fingle Life: And one Time or other, if Health and Leifure will give me Leave, tho' at my Age of Life neither of them are much

to

to be expected, I may poffibly refume the Difcuffion of the fecond Matter.

I muft fay a few Words on the Subject of Annuities for two or more Lives and the Survivor, and I need to fay very few, becaufe the Point here is clear, that the Length of the Lives is by the Rule made to depend on the Rate of Intereft. On computing, by the Rule, the Value of an Annuity for three Lives and the Survivor, all of the Age of 12, and at different Rates of Intereft, the Values, and the Term to which fuch Values correfpond, ftand thus—

At 4 *per Cent.* Value 23-01, equal to a Term of 65
At 5 *per Cent.* Value 19-05, equal to a Term of 63
At 6 *per Cent.* Value 16-19, equal to a Term of 61
At 7 *per Cent.* Value 13-92, equal to a Term of 58-2
At 8 *per Cent.* Value 12-29, equal to a Term of 56

I do not affirm that thefe Values, all of them, are exactly true; however, they are near enough the Truth to anfwer the Purpofe for which they are here inferted, *viz.* to fhow that, whenever we vary the Rate of Intereft, we find a Variance likewife in the Term to which the Lives are equal; that is, that the Rate of Intereft governs and determins the Length of the Lives; and it appears here that, between the higheft and loweft Intereft, the Difference in the Term is nine Years. This wants no farther Comment, fo I fhall add only a fhort Obfervation, tho', fhort as it is, it trenches deep into the Scheme, and all the Rules which Mr. *De Moivre* has contrived for inveftigating the

Value

Value of Annuities for one or more Lives, of any Kind, or in any Shape ; and my Remark is this. On computing, by his Scheme and his Rules, the Value of Annuities for a ſingle Life, and for two or more Joint Lives, the Values come out to be ſuch as correſpond to a Term below the even Chance, or Probability of the Duration of the Life or Lives : And on the contrary, on computing the Value of Annuities for two or more Lives and the Survivor, the Values are always ſuch as correſpond to a Term above the even Chance, or Probability of the Duration of the Lives, or the Survivor of them. And I will ſay here, as I ſaid in my *Eſſay,* on much the like Occurrence, that this is a Contradiction to common Senſe and Reaſon, and leave it to Mr. *De Moivre* to reconcile it if he can ; but I bar all Proofs from Mathematical Schemes, unleſs he take ſpecial Care that his Data and Poſtulate are all right and clear.
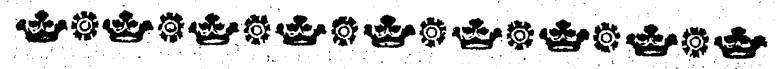
TABLES

Of the VALUE of

ANNUITIES

A N D

LEASES certain,

For Years, and for a ſingle Life, at every Age of Life.

At the the Rates of 3, 4, 5, and 6 *per Cent.* Intereſt.
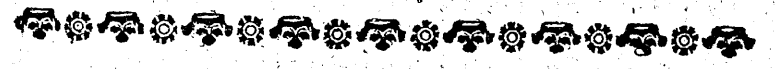
TABLE I.

Showing the Value of Annuities and Leafes certain, for any Term of Years, in Years, and in the eighth Part, and hundredth Part of a Year. Intereft, three *per Cent.*

Term of Years.	Value in Y. 8 th. 100 th.	Term.	Value in Y. 8 th. 100 th.	Term.	Value in Y. 8 th. 100 th.
I	00-7-97	26	17-6-86	51	25-6-86
2	01-7-91	27	18-2-31	60	27-5-68
3	02-6-83	28	18-6-75	70	29-1-12
4	03-5-61	29	19-1-17	80	30-1-21
5	04-4-58	30	19-4-58	90	31——
6	05-3-42	31	19-7-98	100	31-4-60
7	06-2-24	32	20-3-37	Fee	33-2-33
8	07---03	33	20-6-75		
9	07-6-81	34	21-1-12		
10	08-4-56	35	21-3-48		
11	09-2-29	36	21-6-83		
12	10——	37	22-1-17		
13	10-5-67	38	22-4-19		
14	11-2-33	39	22-6-79		
15	11-7-97	40	23——11		
16	12-4-59	41	23-3-42		
17	13-1-19	42	23-5-71		
18	13-6-78	43	24——		
19	14-2-35	44	24-2-27		
20	14-7-89	45	24-4-53		
21	15-3-42	46	24-6-78		
22	15-7-94	47	25——02		
23	16-3-44	48	25-2-25		
24	16-7-92	49	25-3-47		
25	17-3-40	50	25-5-67		

TABLE II.

Showing the Term to which Annuities and Leafes certain, for a fingle Life, are computed to be equal; and the Value of fuch Annuities and Leafes, in Years, and the eighth Part of a Year. Intereft, three *per Cent.*

Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.
I	25--	17-3	26	27-4	18-4	51	15-3	12-1
2	28--	18-6	27	26-7	18-2	52	15--	11-7
3	31--	19-7	28	26-2	17-7	53	14-4	11-5
4	33--	20-6	29	25-5	17-5	54	14--	11-2
5	34-4	21-2	30	25--	17-3	55	13-4	11--
6	35--	21-3	31	24-4	17-1	56	13--	10-5
7	35--	21-3	32	24--	16-7	57	12-4	10-3
8	35--	21-3	33	23-4	16-5	58	12--	10--
9	35--	21-3	34	23--	16-3	59	11-4	9-5
10	35--	21-3	35	22-4	16-1	60	11--	9-2
11	34-4	21-2	36	22--	15-7	61	10-4	8-7
12	34--	21-1	37	21-4	15-5	62	10--	8-4
13	33-5	20-7	38	21--	15-3	63	9-4	8-1
14	33-2	20-6	39	20-4	15-1	64	9--	7-6
15	32-7	20-5	40	20--	14-7	65	8-4	7-3
16	32-4	20-4	41	19-4	14-5	66	8--	7--
17	32-1	20-3	42	19--	14-3	67	7-3	6-4
18	31-6	20-2	43	18-4	14--	68	6-6	6--
19	31-3	20-1	44	18--	13-6	69	6-1	5-4
20	31--	19-7	45	17-5	13-4	70	5-4	5--
21	30-4	19-6	46	17-2	13-2	75	4-2	3-7
22	30--	19-5	47	16-7	13--	80	3--	2-6
23	29-3	19-3	48	16-4	12-7	85	2--	1-7
24	28-6	19--	49	16-1	12-5	90	1--	0-7
25	28-1	18-6	50	15-6	12-3	&c.		

TABLE III.

Showing the Value of Annuities and Leases certain, for any Term of Years; in Years, and in the eighth Part, and hundredth Part of a Year. Interest, four *per Cent.*

Term of Years.	Value in Y. 8 100	Term.	Value in Y. 8 100	Term.	Value in Y. 8 100
1	00-7-96	26	15-7-97	51	21-4-60
2	01-7-88	27	16-2-31	60	22-4-61
3	02-6-78	28	16-5-64	70	23-3-39
4	03-5-62	29	16-7-97	80	23-7-90
5	04-3-44	30	17-2-29	90	24-2-30
6	05-1-23	31	17-4-57	100	24-4-50
7	06———	32	17-6-85	Fee	25———
8	06-5-72	33	18—11		
9	07-3-42	34	18-3-37		
10	08—10	35	18-5-62		
11	08-6-75	36	18-6-86		
12	09-3-37	37	19—10		
13	09-7-97	38	19-2-33		
14	10-4-55	39	19-4-56		
15	11—10	40	19-6-78		
16	11-5-64	41	20———		
17	12-1-15	42	20-1-20		
18	12-5-64	43	20-3-39		
19	13-1-12	44	20-4-57		
20	13-4-59	45	20-5-73		
21	14—01	46	20-7-90		
22	14-3-46	47	21—05		
23	14-6-84	48	21-1-20		
24	15-2-25	49	21-2-34		
25	15-4-61	50	21-3-47		

TABLE IV.

Showing the Term to which Annuities and Leases certain, for a single Life, are computed to be equal; and the Value of such Annuities and Leases, in Years, and the eighth Part of a Year. Interest, four *per Cent.*

Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.
1	25--	15-3	26	27-4	16-3	51	15-3	11-2
2	28--	16-5	27	26-7	16-2	52	15---	11--
3	31--	17-4	28	26-2	16--	53	14-4	10-6
4	33--	18--	29	25-5	15-6	54	14--	10-4
5	34-4	18-4	30	25--	15-4	55	13-4	10-2
6	35--	18-5	31	24-4	15-3	56	13--	9-7
7	35--	18-5	32	24--	15-2	57	12-4	9-5
8	35--	18-5	33	23-4	15--	58	12--	9-3
9	35--	18-5	34	23--	14-6	59	11-4	9--
10	35--	18-5	35	22-4	14-5	60	11--	8-6
11	34-4	18-4	36	22--	14-3	61	10-4	8-3
12	34--	18-3	37	21-4	14-2	62	10--	8--
13	33-5	18-2	38	21--	14--	63	9-4	7-6
14	33-2	18-1	39	20-4	13-6	64	9--	7-3
15	32-7	18--	40	20--	13-4	65	8-4	7--
16	32-4	18--	41	19-4	13-2	66	8--	6-5
17	32-1	17-7	42	19--	13-1	67	7-3	6-2
18	31-6	17-6	43	18-4	12-7	68	6-6	5-6
19	31-3	17-5	44	18--	12-5	69	6-1	5-2
20	31--	17-4	45	17-5	12-3	70	5-4	4-6
21	30-4	17-3	46	17-2	12-2	75	4-2	3-6
22	30--	17-2	47	16-7	12--	80	3--	2-6
23	29-3	17-1	48	16-4	11-7	85	2--	1-7
24	28-6	16-7	49	16-1	11-5	90	1--	0-7
25	28-1	16-5	50	15-6	11-4	&c.		

N 2

TABLE V.

Showing the Value of Annuities and Leases certain, for any Term of Years; in Years, and in the eighth Part, and hundredth Part of a Year. Interest, five *per Cent*.

Term of Years.	Value in th. th. Y. 8 100	Term	Value in th th. Y. 8 100	Term	Value in th th. Y. 8 100
1	00-7-95	26	14-3-37	51	18-2-33
2	01-6-86	27	14-5-64	60	18-7-93
3	02-5-72	28	14-7-89	70	19-2-34
4	03-4-54	29	15-1-13	80	19-4-59
5	04-2-33	30	15-2-36	90	19-6-75
6	05—07	31	15-4-59	100	19-6-85
7	05-6-79	32	15-6-80	Fee	20——
8	06-3-46	33	16——		
9	07—11	34	16-1-19		
10	07-5-71	35	16-3-37		
11	08-2-31	36	16-4-54		
12	08-7-87	37	16-5-70		
13	09-3-39	38	16-6-86		
14	09-7-89	39	17—01		
15	10-3-38	40	17-1-15		
16	10-6-84	41	17-2-30		
17	11-2-27	42	17-3-43		
18	11-5-68	43	17-4-53		
19	12——	44	17-5-60		
20	12-3-46	45	17-6-77		
21	12-6-82	46	17-7-88		
22	13-1-16	47	17-7-98		
23	13-3-48	48	18—08		
24	13-6-79	49	18-1-17		
25	14—09	50	18-2-25		

TABLE VI.

Showing the Term to which Annuities and Leases certain, for a single Life, are computed to be equal; and the Value of such Annuities and Leases, in Years, and the eighth Part of a Year. Interest, five *per Cent.*

Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.
1	25--	14--	26	27-4	14-6	51	15-3	10-4
2	28--	14-7	27	26-7	14-5	52	15--	10-3
3	31--	15-4	28	26-2	14-3	53	14-4	10-1
4	33--	16--	29	25-5	14-2	54	14--	9-7
5	34-4	16-2	30	25--	14--	55	13-4	9-5
6	35--	16-3	31	24-4	13-7	56	13--	9-3
7	35--	16-3	32	24--	13-6	57	12-4	9-1
8	35--	16-3	33	23-4	13-5	58	12--	8-7
9	35--	16-3	34	23--	13-4	59	11-4	8-4
10	35--	16-3	35	22-4	13-3	60	11--	8-2
11	34-4	16-2	36	22--	13-1	61	10-4	8--
12	34--	16-1	37	21-4	12-7	62	10--	7-5
13	33-5	16--	38	21--	12-6	63	9-4	7-3
14	33-2	16--	39	20-4	12-5	64	9--	7--
15	34-7	15-7	40	20--	12-3	65	8-4	6-6
16	32-4	15-7	41	19-4	12-2	66	8--	6-3
17	32-1	15-6	42	19--	12--	67	7-3	6--
18	31-6	15-5	43	18-4	11-7	68	6-6	5-4
19	31-3	15-5	44	18--	11-5	69	6-1	5-1
20	31--	15-4	45	17-5	11-4	70	5-4	4-5
21	30-4	15-3	46	17-2	11-3	75	4-2	3-5
22	30--	15-2	47	16-7	11-1	80	3--	2-5
23	29-3	15-1	48	16-4	11--	85	2--	1-6
24	28-6	15--	49	16-1	10-7	90	1--	0-7
25	28-1	14-7	50	15-6	10-5	&c.		

TABLE VII.

Showing the Value of Annuities and Leases certain, for any Term of Years, in Years, and in the eighth Part, and hundredth Part of a Year. Interest, six *per Cent*.

Term of Years.	Value in th th. Y. 8 100	Term.	Value in th th. Y. 8 100	Term.	Value in th th. Y. 8 100
1	00-7-94	26	13———	51	15-6-79
2	01-6-83	27	13-1-21	60	16-1-17
3	02-5-67	28	13-3-40	70	16-3-39
4	03-3-46	29	13-4-58	80	16-4-50
5	04-1-21	30	13-6-76	90	16-4-58
6	04-7-92	31	13-7-93	100	16-4-61
7	05-4-58	32	14——09	Fee	16-5-67
8	06-1-21	33	14-1-23		
9	06-6-80	34	14-3-38		
10	07-2-36	35	14-4-51		
11	07-7-88	36	14-5-63		
12	08-3-38	37	14-6-74		
13	08-6-85	38	14-6-84		
14	09-2-29	39	14-7-94		
15	09-5-71	40	15——03		
16	10——10	41	15-1-12		
17	10-3-47	42	15-1-20		
18	10-6-83	43	15-2-28		
19	11-1-16	44	15-2-36		
20	11-3-47	45	15-3-43		
21	11-6-76	46	15-4-50		
22	12——03	47	15-4-56		
23	12-2-30	48	15-5-62		
24	12-4-55	49	15-5-68		
25	12-6-78	50	15-6-74		

TABLE VIII.

Showing the Term to which Annuities and Leases certain, for a single Life, are computed to be equal; and the Value of such Annuities and Leases, in Years, and the eighth Part of a Year. Interest, six *per Cent*.

Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.
1	25--	12-6	26	27-4	13-2	51	15-3	9-6
2	28--	13-3	27	26-7	13-1	52	15--	9-5
3	31--	13-7	28	26-2	13--	53	14-4	9-4
4	33--	14-1	29	25-5	12-7	54	14--	9-2
5	34-4	14-3	30	25--	12-6	55	13-4	9--
6	35--	14-4	31	24-4	12-5	56	13--	8-6
7	35--	14-4	32	24--	12-4	57	12-4	8-4
8	35--	14-4	33	23-4	12-3	58	12--	8-3
9	35--	14-4	34	23--	12-2	59	11-4	8-1
10	35--	14-4	35	22-4	12-1	60	11--	7-7
11	34-4	14-3	36	22--	12--	61	10-4	7-5
12	34--	14-3	37	21-4	11-7	62	10--	7-2
13	33-5	14-2	38	21--	11-6	63	9-4	7--
14	33-2	14-2	39	20-4	11-5	64	9--	6-6
15	32-7	14-1	40	20--	11-3	65	8-4	6-4
16	32-4	14-1	41	19-4	11-2	66	8--	6-1
17	32-1	14--	42	19--	11--	67	7-3	5-6
18	31-6	14--	43	18-4	10-7	68	6-6	5-3
19	31-3	13-7	44	18--	10-6	69	6-1	5--
20	31--	13-7	45	17-5	10-5	70	5-4	4-5
21	30-4	13-6	46	17-2	10-4	75	4-2	3-4
22	30--	13-6	47	16-7	10-3	80	3--	2-5
23	29-3	13-5	48	16-4	10-2	85	2--	1-6
24	28-6	13-4	49	16-1	10-1	90	1--	0-7
25	28-1	13-3	50	15-6	10--	&c.		

TABLE IX.

Showing the Term in Years, and the eighth Part of a Year, to which Lives of all Ages in the City of *London* are computed to be equal.

Ages.	Y.	8th.	Ages.	Y.	8th.	Ages.	Y.	8th.
1	25	—	26	30	—	51	14	2
2	28	—	27	29	4	52	13	6
3	31	—	28	28	7	53	13	2
4	33	—	29	28	2	54	12	6
5	34	4	30	27	5	55	12	2
6	35	—	31	27	—	56	11	6
7	35	—	32	26	3	57	11	2
8	35	—	33	25	6	58	10	6
9	35	—	34	25	1	59	10	2
10	35	—	35	24	4	60	9	6
11	34	5	36	23	7	61	9	2
12	34	2	37	23	2	62	8	6
13	34	—	38	22	5	63	8	2
14	33	6	39	21	7	64	7	6
15	33	5	40	21	1	65	7	3
16	33	4	41	20	3	66	7	—
17	33	3	42	19	6	67	6	5
18	33	2	43	19	1	68	6	2
19	33	1	44	18	4	69	5	7
20	32	7	45	17	7	70	5	4
21	32	4	46	17	2	80	2	7
22	32	—	47	16	5	90	1	1
23	31	4	48	16	—	&c.		
24	31	—	49	15	3			
25	30	4	50	14	6			

TWO SCHEMES:

The FIRST,

Showing the Proportion in which the supposed 35 Years or Chances of Vitality, on the Life of a Person of ten Years Age, decrease in each Period of ten Years; as stated by Mr. *Hodgson* on the *London* Bills of Mortality, *p. 9, &c.*

The SECOND,

As computed by Mr. *Lee* on the same *London* Bills of Mortality.

O

SCHEME I.

Showing the Proportion in which the supposed 35 Years or Chances of Vitality, on the Life of a Person of ten Years Age, decrease in each Period of ten Years; as stated by Mr. *Hodgson* on the *London* Bills of Mortality, *p.* 9, *&c.*

From			Y.	8th.
10	to	20	8	-
20	to	30	5	4
30	to	40	4	-
40	to	50	3	-
50	to	60	3	4
60	to	70	2	4
70	to	80	3	4
80	to	90	2	-
90	and	upwards	3	-
			35	-

SCHEME II.

Showing the Proportion in which the supposed 35 Years or Chances of Vitality, on the Life of a Person of ten Years Age, decrease in each Period of ten Years; as computed by Mr. *Lee* on the same *London* Bills of Mortality.

From			Y.	8th.
10	to	20	2	1
20	to	30	5	2
30	to	40	6	4
40	to	50	6	3
50	to	60	5	-
60	to	70	4	3
70	to	80	2	6
80	to	90	1	4
90	and	upwards	1	1
			35	-