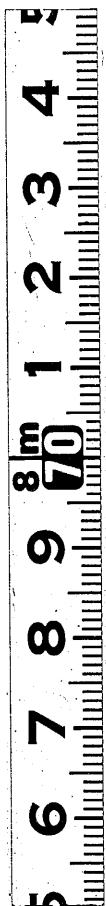


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OF  
ANNUITIES  
AND  
LEASES certain,  
FOR  
A SINGLE LIFE.  
WITH  
TABLES,

SHOWING  
At one View the Value of them, in Years,  
and the eighth Part of a Year, for every  
Year of Life, from the Age of 1 to 70, and  
from thence to the End of Life, for every  
Period of five Years.

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To the HONORABLE  
*Alexander Hume Campbell,*  
T R E A S U R E R,  
And the rest of the  
M A S T E R S of the BENCH,  
OF THE  
A N C I E N T S O C I E T Y  
OF THE  
I N N E R T E M P L E,  
THIS  
T R E A T I S E,

With all due Deference,

i s  
I N S C R I B E D,

*By their most obedient*

*Humble Servant*

*And Brother,*

**Weyman Lee.**



A  
VALUATION  
OF  
ANNUITIES  
FOR  
LIVES, &c.

**S**OME Time ago, I publish'd an  
Essay on the Method of ascertain-  
ing the Value of Annuities, and of  
Leases reduced to Annuities cer-  
tain, for one or more Lives; and  
therein I laid down this Position as the Ground-  
work, *viz.* that the even Chance of the Duration of  
a Life, computed once only for the whole Life, was  
the sole true Measure of the Value of an Annuity  
depending

depending on that Life. But since some Persons have made Observations on this my Method, and others have formd Valuations of these Estates, and have defended and still adhere to the Method prescribed by Dr. Halley; I think it will not be amiss to make some Remarks upon them, partly to answer and remove the Objections which have been offerd, but chiefly, because it gives me an Opportunity to maintain and further confirm the fundamental Position.

Amongst my Adversaries Mr. H. B. appears to be most potent, and indeed the only one who deserves any serious Consideration; and the Question between him and me is, Whether Dr. Halley's Rule or mine be the righter, for forming an Estimate of the Value of Annuities on a single given Life. The Dr's Rule is to find the Value of the Annuity for each Year of the given Life to an 100; which is done, and can be no otherwise done, but by computing the Chances of Mortality arising in each Year, and deducting the Value thereof out of the Value of an Annuity absolute for that Year, and then to put together all the Values so collected, and the Sum total is the Value of the Annuity for the given Life. Mine is, to find to what Term of Years any Life is equal, or an even Chance may probably continue; and the Value of an Annuity for the given Life will be the same as the Value of an Annuity for that Term of Years.

Now if my Rule be a right one, and from the slender Objections made to it, and the weak

Defence

Defence of the other by this Advocate, and, if this other could be defended, by this Hand it would have been defended, I am fully satisfied that it is a right one; this my Rule, and the Tables formd upon it, will have a manifest Advantage over any other Rule, and any Tables that have been or can be planned from thence. By the other Rule, if the Value of an Annuity for a given Life, is to be computed, the Computer, on every Variation of the Rate of Interest, whether real or on Supposition only, must, of Necessity, calculate the Value of the Annuity for every individual Year, to which the Life of the Person nominated may possibly continue, for each Year separately and distinctly, tho' it should happen to be 80, 90, or an 100 Years; and believe me, those who have tried it, have found this not only a tedious, but very intricate Affair: Nay, this elaborate and perplex Calculation does not attend only on every Change of the Rate of Interest, but occurs again in some Measure on every Age of Life, tho' it differs one Year only, and even tho' the Rate of Interest is, or is supposed to be, the same. On the other Side, and upon my Rule, let the Rate of Interest vary as often or as much as you will, either in Practice or Imagination, yet the Chance of the Duration of a Life of one and the same Age, in one and the same Country, will not vary, but perpetually be the same, and equal to the same Term of Years, and such Term of Years appears in my Tables on the first View.

These Tables go farther, and show what is the Value of such Term, that is, what is the Value

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of



4      *A VALUATION of*  
of the Annuity for the given Life at the Rates of 3, 4, 5, and 6 *per Cent.* Interest. And if we could suppose that the Interest of Money should fall from 3 to  $2\frac{1}{2}$ , or to 2, which I do not suspect will ever be the Case, at least not so soon as some People vulgarly imagin; yet the Term, to which the given Life is equal, will be perpetually the same, and will appear in these Tables; and, such Term being once known, the Value of such Term will be easily found out by the common Tables, which shall then be in Use, for the Value of Terms for Years, at the Rates of 2 or  $2\frac{1}{2}$ ; for I make no Doubt, but that, if and when those shall become the common Rates of Interest, such Tables will be constructed, for they are very easily constructed, and will be in every body's Hand.

Mr. *H. B.* begins his Attack on the Essay, with some Observations on the Table for Annuities on a single Life; but since I have thought fit, for the Reasons which I shall hereafter mention to alter it, there is no Need to take any Notice of them. From my Table he proceeds to consider the Rule which I just now mentiond, and says, he will give a few Reasons why he thinks the very Foundation I go upon is wrong, and the Rule I first lay down not a right one. I am very much obliged to the Gentleman, that he will give but a few Reasons against the Rule; and more so, that he has reduced those few to one; but he is Master of his own Reason. The Rule I have already set forth; and the Instance I gave of it, which 'tis necessary to repeat here, because of frequent References

ANNUITIES for LIVES, &c. 5

ferences to it hereafter, was this: The Number of Persons of the Age of 10, of which we will suppose *A* to be, by a Calculation made on the *Breslaw* Table, is 661; the Half of this is  $330\frac{1}{2}$ ; and this Number we find between 51 and 52, so that *A* has an even Chance to live 41 Years and something over. A Term for 41 Years Interest computed at 6l. *per Cent.* is in Value 15—12 Centesimals; therefore an Annuity for the Life of *A*, who has an even Chance to live 41 Years and beyond, must be worth 15—12 Centesimals and above. This Author admits the Premises to be true; *viz.* that *A* has an even Chance to live 41 Years; that a Term for 41 Years is in Value 15—12; but denies the Conclusion, *viz.* that an Annuity for his Life is worth 15—12. His Reason for denying this Conclusion runs thus: *That, tho' the Chances of A's dying before 51 or after 51 are equal in Number, yet they are not equal in Value.* He then establishes the Truth of this Position by a Demonstration, in two several Instances, and from thence, without further Proof or Allegation, concludes: *Consequently, an Annuity for A's Life, who has an even Chance to live 41 Years and beyond, is not worth as much as an Annuity for 41 Years absolute, that is, is not worth 15—12, Q. E. D.*

When he says, that these Chances are of unequal Value, I suppose he means, that the Annuities, attendant on those Chances for each Year, are of unequal Value; and he cannot well mean any Thing else, because in any other Sense the Position is not true; he might have saved himself the  

Trouble

Trouble of a Demonstration, that these Chances are of unequal Value, since the Annuities attending them are Part in Possession, and Part in Reversion; and, to be sure, there could not be Occasion for a Demonstration, at least, not in two Instances, to have shown that the Annuities for a Year of a Life must be of more or less Value, as they are more or less removed from the first Year's Annuity; for Instance, that the present Value of an Annuity for the seventh Year of a Man's Life is greater than the Value of the like Annuity for his twentieth Year. Admitting then, that these Chances, or the Annuities attending them, are of unequal Value, to which, after such an irrefragable Proof as two Demonstrations, I shall certainly submit; Does it follow from thence, that the Rule and the Values produced by it are wrong? This is the Conclusion which he draws; but, as I apprehend, long before he is come at it, for these Premises and these Demonstrations do no more prove it, than they do, that the three Angles of an equilateral Triangle are equal to two right ones. He admits that the Rule does not direct to put in more Chances in Number, than the Annuitant has a Right to; and he does not so much as assert, that it takes in any Chances of greater Value than he has a Right to; and this ought not only to have been asserted, but proved too, otherwise the Consequence he draws is a strange one; for the Argument, as it now stands, is no better than this: An Annuitant, by this Rule, has no more Chances in Number than he has a Right to;

but,

but, in Fact, there are some Chances of greater Value than others, without proving or even asserting, that any such Chances of greater Value are inserted in the Computation. Now since the Matter in Dispute is not, whether such Annuitant, by Virtue of the Rule, has more Chances in Number, but whether he has some better in Value, than he has a Right to: This is evidently to beg the Question, and not to prove it, notwithstanding the terrible Sting at the Tail of it; for the Demonstrations plainly go to one Proposition, *viz.* to prove this or that is the Value of such Chances; and the Argument concludes to another, *viz.* to such and such Chances the Annuitant has a Right, and no other.

However, to help out a lame Argument, for such I must call it, as here form'd, I will admit, that the Rule does direct to put in, and that the Value produced thereby does include, all those Chances which arise on the Life of *A*, before he arrives at the Age of 41; and that the Chances, so inserted, are of greater Value, than those which arise after he is past 41 Years. But not one Word of Proof is here so much as offer'd, that these greater Values are not the real Values, which an Annuitant on the Life of *A* has the Chance to receive; and in the Essay it is asserted, and so strongly and fully proved, that these greater Values, and no other, are the very Values which such an Annuitant has a Chance to receive, that I cannot well add any thing to confirm it.

But

But this Author, instead of attempting to invalidate these Arguments, or to prove that the Annuitant on the Life of *A*, who has an even Chance to live 41 Years, has not an even Chance to enjoy his Annuity for as many Years as he has an even Chance to live, that is, 41 Years, or to enjoy those Chances which arise in those first 41 Years of his Life; which he must do, before he destroys the Foundation of my Rule, and yet I may be confident he never will be able to do; instead of this, he very valiantly demonstrates something, which, to be sure no Man did ever make any Doubt of, that the Chances, or the Values attending them, where some are in Possession, and some are in Reversion, are not all of equal Value.

Since this Author is so full of, and so learned in, the Doctrine of Chances, as he calls it, I will take the Liberty of offering to his Consideration an Argument or two, in Behalf of my Rule, according to his Way of Thinking. The Value of the Chances in this Case, depends on, or is one and the same Thing, as, the Value of the Annuities dependant on those Chances. All the possible Chances, which an Annuitant has on the Life of *A*, are agreed to be 661, and of those he has an even Chance to enjoy a Moiety, that is, 330  $\frac{1}{2}$ . The Annuities which attend those Chances for the whole Life, are Annuities for a Term of 90 Years, it being supposed possible that he may live for so many Years; the total Value of them for 90 Years, as I compute, is

16-58 Centesimals, and the Value of those in Possession for 41 Years is 15-12, and of those in Reversion 1-46. Since then the Annuitant on the Life of *A*, has an even Chance to 330  $\frac{1}{2}$  Chances on his Life, and the Annuities will attend those Chances; and since these 330  $\frac{1}{2}$  Chances will, and from the Nature of the Life of Man of Necessity must, arise in the first Part of those 90 Years, if they arise at all, and 'tis supposed in the Case to be an even Chance that so many Chances will arise in his Life: He must then of Necessity have the Annuities attending those very Chances, that is, he must have the Annuities for the first 41 Years of Life. This Author, or whoever will undertake to answer this Argument, must show that 'tis possible for an Annuitant on the Life of *A* to have the Benefit of all these 330  $\frac{1}{2}$  Chances, to which Number of Chances he has without Dispute a Right or an even Chance to enjoy, unless he does enjoy those which arise in the first Part of Life; and in immediate Possession, that is, those of the greatest Value; and this I am very sure he cannot show.

Again, These Positions are not contested: That an Annuity for a Term of 41 Years is in Value 15-12; and that an Annuitant for the Life of *A* has an even Chance that *A* lives for 41 Years. 'Tis a Maxim made use of by this Author, and is certainly a true one, that in estimating the Value of Annuities for a Life all the possible Chances of Life must be computed. From hence I argue thus: To estimate the Value

10 A VALUATION of

of an Annuity for the Life of *A*, we must compute all the possible Chances on the Life of *A*; the Chance that *A* lives for 41 Years is an even Chance, and consequently is one of the possible Chances on the Life of *A*; therefore to estimate the Value of an Annuity for his Life we must compute this even Chance. The even Chance on this Life is to a Term for 41 Years. A Term for 41 is in Value 15-12. Therefore the Value of an Annuity for this Life is 15-12.

This Author, having by this and by no other Argument demolishd my Rule, even to the very Foundations, as he expresses himself, proceeds to consider Dr. Halley's Rule; and, having given it us, pronounces that 'tis most certainly right, my Objections notwithstanding; and, to be even with him, I say, notwithstanding his Defence, that 'tis certainly wrong. One of my Exceptions to his Rule, and which this Writer's Observations on it make necessary to repeat, is this: That the Value produced by his Rule is not the Value of an Annuity for the Life of *A*, but will be the Value of 90 several Annuities or Grants made to 90 several Persons, suppose to N. 1, to N. 2, and so on to N. 90, formd in this Manner, viz. to N. 1 for a Year, if he shall live from 10 to 11 Years of Age; to N. 2 for a Year, if he shall live from 11 to 12 Years Age; and so on successively to N. 90 for a Year, if he shall live from 99 to an 100 Years Age. I then affirm, that the Value of an Annuity for the Life of *A* absolutely, or for 90 Years, if *A* so long lives,

is

ANNUITIES for LIVES, &c. II

is of a different and much greater Value than the Interest of N. 1, N. 2. and so on to 90, all put together. To this he says, that it would take up too much Time to answer all the Arguments I have brought to support this Assertion. This surely is a very small Reason for not attempting to answer any of them, or the most material of them; and som People will be apt to surmise that he wanted something else rather than Time to answer them. However, to be sure it will be necessary for me to spend my Time in repeating and defending them; so that, since he has left them quiet and unanswerd, I shall e'en leave them, in *statu quo*, undefended.

He does indeed afterwards make an Attempt to prove, that an absolute Annuity for this Life, which I contend to be in Value 15-12, is of no greater Value than these several Annuities or Grants, which I suppose to be made to 90 Persons for 90 Years, when put together, that is, not more than 13-44. He says then, that 90 several Annuities granted to *A*, in this manner; the first Annuity for one Year, if he shall live from 10 to 11 Years Age; the second for one Year, if he shall live from 11 to 12 Years Age, and so on successively to a 90th Annuity for one Year, if the Nominee shall live to be an 100 Years old; will be of equal Value with one Annuity granted to him for his Life absolutely: For the Chances of receiving the Annuity will be the same in both Cases, because the Chances of receiving it depend on the Chances of his Vitality. Here he alters

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the

the State of the Case intirely from what I gave; for whereas I suppose these several Annuities or Grants for 90 Years, determinable at the End of each Year, to be made to 90 several Persons on their several Lives, he supposes these several determinable Annuities to be granted to one and the same Person, and on one and the same Life; which Alteration manifestly varies the State of the Chances. The Difference between the two Cases is evidently this; that in the Manner I stated it there are as many Chances of Mortality as there are Lives, viz. 90; but, in the Case as he states it, there is but one Chance of Mortality, for there is but one Life in the whole, viz. the Life of *A*.

He asserts farther, that these several Annuities, as he calls them, *will be of equal Value, whether they are granted by one Person to one Person, or by 90 several Persons to one Person; and all equal to an absolute Annuity for a Life.* That they are so in the first Case I admit, because the Grants amount to an absolute Annuity for a Life; but that they are so in the latter Case, as well on the Part of the several Grantors as of the Annuitant, is a Point at least doubtful; and I shall go farther and affirm, that they are not in all Instances equal, and that the Reason he gives to prove their Equality, viz. *That the Chances for paying and receiving it must in both Cases be the same,* is a Fact not in all Cases true, and therefore does not make good his Assertion. In the first Place, if several Grants were made to one and the same Person, viz. *A*.  
by

by 90 distinct Persons in the manner here supposed, it might be a Question, whether such Grants were properly an Annuity, or a Grant only of so many several Sums in Gros made by so many distinct Persons. With Regard to *A*, the several Sums so granted, when put together, woud amount to something which woud be equivalent to an Annuity for his Life; but *quoad* each separate Grant, they seem to be no other than a Grant of so many several Sums in Gros. If we construe them as an Annuity for the Life of *A*, a Dispute woud then arise between *A* and the several Grantors, whether the greater Sum, viz. 15-12, or the less Sum, viz. 13-44, was to be paid to the Grantors in proportionable Parts. *A* on his Side may allege, that the Grantors, on their Engagements for the respective Sums to be paid *in futuro*, are to receive no more than 13-44 in the whole, and all put together, because *quoad* them the present Value of the Chances to pay them, severally and distinctly considerd, do amount to no more. The several Grantors on the other Side may allege, that *A* on the Purchase of such Grants is to pay for them in the whole, and all put together, 15-12, because *quoad* him the present Value of all the Sums put together which he has a Chance to receive, by the Combination of their several Grants, amounts to that greater Sum: And, believe me, such their Allegation is founded on very just and good Reasons. If these Grants, supposed to be made by 90 several Persons, are only so many Sums in Gros,

## 14 A VALUATION of

Gross, they are not pertinent to the present Purpose; and, if we construe them to amount to an Annuity for Life, 'tis disputable at least which of the two Sums is to be paid, the greater or the less; and it may be Time enough to discuss that Controversy when any such Grants shall be made, which is not very likely ever to come to pass; this imaginary Case therefore will avail nothing to determine the Point in Dispute, but it must be left to be decided by other Arguments and upon other Topics.

I affirm then, in the second Place, that this Author's Reason, *viz.* that the Chances of paying or receiving in both these Cases are the same, do not prove that the Annuities in each are of equal Value. In this Matter, the Chances of not paying, or not receiving, are to be taken into the Consideration as well as those of paying and receiving. This being premised, I say, that in the Case of a single Grantor of an Annuity for the Life of *A*, he has only one Chance during the whole 90 Years that he shall be exempted from paying, but the several Grantors have, each in their respective Years, a Chance for such Exemption; consequently the Value of the single Chance of not paying in the first Case is not the same as the Value of the accumulated Chances in the latter; therefore the Value of the Annuities attending is not equal. Each of these Grantors of the Sums in Gross or annual Payments, on their Grants, must be allowed and have a Deduction for so many Chances as each of them has of not paying

## ANNUITIES for LIVES, &amp;c. 15

paying the Sum at the End of each Year, that is, for so many Chances of Mortality as arise on the Life of the Annuitant in each Year respectively. And why is this done? Because the Chance of Mortality *quoad* each of them arises in each Year, and is confined to each Year respectively. But in the Case of a single Grantor, a Grantor of an Annuity for a Life absolutely, or a Combination of Grantors who shall make so many Grants as to be equal to an Annuity for a Life, this Chance of Mortality, and consequently of not paying, does not arise in that Manner, but does, and from the Nature of the Life of Man of Necessity must arise at once, and once only; and consequently on the Purchase an Allowance and Deduction can be made for one Chance only, since it can arise but once. Now that these Chances of Mortality, for which an Allowance and Deduction is to be made, in the Case of the several Grantors, when all put together, are of greater Value than the single Chance of Mortality in the Case of the single Grantor is evident from hence; because the Chances in the first Case arise, some in the first Year, some in the second, and so on successively for 90 Years; whereas the one Chance of Mortality in the latter Case does not arise till the even Chance of the Mortality of *A*, that is, not till the End of 41 Years; and Chances in immediate Possession are of much greater Value than one or the like Number of Chances in Reversion after 41 Years, as this Author has very learnedly demonstrated.

I have



I have supposed here, that a single Grantor of an Annuity for a Life has but one Chance of the Mortality of his Annuitant throughout the whole 90 Years, and most certainly he has not. That every Man shall die once, and no Man more than once, must be admitted; but 'tis absurd to say that a Person has more than one Chance to suffer that which 'tis impossible he should suffer more than once. However, if it be insisted on, in the Case put of 90 several Grants to *A*, that the Chance of receiving the Annuity is the same as in the Case of an Annuity absolutely for his Life, and it seems to be the better Opinion, then *A* has in both Cases the same Chance of Mortality against him, and that is one only: Then it will be insisted on by the other Side, that *A* having by such a Combination of Grants put himself into such a Condition, that he has only a single Chance of Mortality against him, and of not receiving his Annuity, that then for these accumulated Grants he must pay the same Sum as he should do for an absolute Annuity for his Life, that is, the greater Sum.

Hitherto we have been speaking of and considering what would be the Case of *A*, whom we have supposed to be 10 Years old only, on a single Grant of an Annuity for his Life absolutely, and on 90 several Grants to him by 90 Persons, determinable by his Death within any one Year of 90 Years: And, if the Point in Dispute remains there doubtful, the Instance which follows will put the Matter beyond all Doubt. We will suppose

suppose then that *B* is 80 Years old, that there are 20 Years of possible Life remaining to him, that he has from one Person an absolute Annuity for his Life, and has likewise 20 several Grants made to him by 20 several Persons determinable in the same Manner as in the former Case. The even Chance of the Duration of the Life of *B*, Computation being made of such Chance by the *Breslaw* Table, is 4 Years; or hardly so much; and an Annuity for 4 Years, Interest stated at 6 *l. per Cent.* is 3-46, an Annuity for the Life of *B* absolutely is therefore in Value 3-46. An Annuity or Grant for the Life of *B*, when made by 20 several Persons for 20 Years, determinable in the Manner aforesaid, is equal to a Term for four Years and an Half, and is in Value 3-84, all which Facts are admitted by this Writer: From whence 'tis evident, as evident as Numbers or Figures can make it, that the Annuities in both Cases are not equal. 'Tis true likewise, and as demonstrable, in the Case of *B*, that the Chances of receiving or paying are not the same in both Instances, unless it be the same Thing to have a Chance of receiving 3-46, as 3-84; or *B* have a Chance of receiving the Annuity for his Life, after he is dead and gone.

He adds farther, that the Chances of Vitality cannot be increased or decreased by the Manner of granting the Annuity: But, I think, he should rather have said, that they ought not, or should not be varied by the Manner of granting, for

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'tis

'tis visible, that in Fact they are varied in the Case of *B*; for the Chance of Vitality on the single Annuity for his Life is only four Years, and very barely that; but on the Annuity by the accumulated Grants the Chance is four Years and an Half; and 'tis a Contradiction to common Sense to say he has any Chances of Vitality left on the accumulated Annuity, when on the single absolute Annuity they are all determin'd and gone.

I woud here recommend it to this Writer to reconcile these Matters, but that I am afraid he will be sick of his reconciling Function, when he finds, as he will do in an Instance herein after mentioned, how little Success he meets with in such Undertakings. I shall choose rather to observe that this Argument, as weak as it is, is the only one which this Author has offer'd to maintain the Justice of Dr. *Halley's* Rule, for making a Valuation of these Annuities; and proceed to consider what may seem to be alleged by him, in Answer to my main Exception to the Dr's. Rule.

The Exception I mean, and which is the plainest and strongest against the Dr's. Rule, may be reduced to this short Syllogism: Any Rule for computing the Value of Annuities for a Life, which produces such a Value as that one and the same Life shall be equal to a different Term, as the Rate of Interest varies, is a false Rule: This Rule does necessarily produce such a Value: Therefore is a false one. The first Proposition is so clear and obvious a Truth, that this Author does

does not offer to contradict it, nay, he goes farther, and by his own Position just mention'd, viz. That the Chances of Vitality cannot be increased or decreased by the Manner of granting an Annuity for a Life, he does effectually confirm it; for if, under the Direction of the Rule, such Sums are necessarily produced as the Value of an Annuity on a Life, as that the Term to which such Life is equal, or the Chance of the Duration of such Life, shall vary as the Rate of Interest varies; most certainly this is to make the Manner of granting the Annuities, and the Rate of Interest at which they are granted, to govern and to increase or decrease the Chance of Vitality on that Life. My second Premise I demonstrate by showing that Mr. *Richards's* Tables, which were constructed by that Rule, do in Fact exhibit such a Value of these Annuities. This appears by an Inspection into his Tables, where the Value of an Annuity for a Life of 12 Years old, when Interest is computed at 4 *l. per Cent.* is 17-20 Centesimals, which is equal to a Term of 29-3-00; and when computed at 8 *l. per Cent.* is 10 67 Centesimals, which is equal only to a Term of 25—. To show that these Values do not proceed from any Error or Defect in the Operation made by Mr. *Richards*, but are a necessary Result from the Rule, and consequently the Rule a false one, I advance these two Propositions: That if a certain Value and Term corresponding are given, and out of that Value a proportionate Part be deducted; first, that the Term correspond-



ing to the Value remaining will not bear the same Proportion to the Value remaining, as the original Term does to the original Value; secondly, that the Term corresponding to the Value remaining, after a Deduction at one Rate of Interest, will greatly differ from the Term corresponding to the Value remaining, after a Deduction made at another Rate of Interest: To which I add, that at 4 *l. per Cent.* the Difference of the Term is the least, and at 8 *l. per Cent.* the greater of the two. To prove these Positions I have subjoind a Calculation which demonstrates them. These Positions and the Calculation this Author has inserted at length in his Observations, p. the 26th, without taking Notice what introduced them, or how they are applied; however, he admits the Truth of them, and that from thence it necessarily follows, if the proportionate Part deducted out of a given Value be an 100th Part only (my Calculation being made on an half Part of the Value) yet that the Term corresponding to the Value remaining, after the Deduction, must be different at different Rates of Interest. But he cannot agree that this is a Demonstration that the Rule is wrong. I did not lay down these Positions immediately and directly to prove that the Rule is wrong, but to prove that the Values given in the Tables were a necessary Result from the Rule; and they do demonstratively prove it, and he admits they do: And then indeed, if the Premises are true, and of one of them he takes no Notice or offers to oppose, and the

the other he cannot contradict without contradicting himself; the Consequence, that the Rule is a wrong one, will follow very close at the Heels of them, for the Conclusion is a necessary and certain one, as necessary and certain as any in *Euclid*: For if the Rule does necessarily produce Values, which are certainly false, the Rule which produces them must necessarily be false likewise.

This Author however, notwithstanding this strong Proof, I may say Demonstration, of the Falsity of the Rule, seeming to offer something against it, I shall consider it. He says then, *that the Rule does not bid you deduct from the Value a Part proportionate to a Part deducted from a Term, but bids you deduct from the Value a Part proportionate to the Number of Chances of Mortality within that Year, which are Things widely different.* 'Tis in no sort material what are not the Directions of the Rule, nor whether they are Things widely different from the real Directions; however, I will admit this Allegation; but this Writer, I suppose, because he could say nothing to the real Question, had a Mind to raise an Imagination, and palm it upon the Reader, that the Question was, whether this or that was the Direction of the Rule, and how far the one might differ from the other. Be that as it will, in the *Essay* 'tis said, and here repeated by this Writer, that one of the Directions of the Rule is this, *viz.* from the Value of an absolute Annuity for a Year to deduct a Part of such Value proportionate to the Number of Chances of Mortality arising in that Year.

Year. Now this is so far from a Reason for not agreeing with me, that it shows, whether he sees it or not, or will acknowledge it, that he does or must agree with me, and proves for me that my Syllogistical Reasoning demonstrates the Falsity of the Rule. The Matter in Controversy is, whether this Rule for finding the Value of Annuities on a Life be a good one; and, having denied it, amongst other Exceptions I alleged this as one, that it directs some Deductions to be made in such a Manner as produces such a Value of these Annuities as make something to happen on the Life of Man, which is absolutely inconsistent with any Thing that ever can happen in the Life of Man. To this the Answer is no more or better than this: That the Rule is a good one, for that it directs these Deductions to be made in the Manner I allege, that is, in a Manner which produces those absurd Values. If the Matter were here fresh, and no Objection had ever been made to the Rule, this would have been a poor Argument to prove its Goodness; for 'tis in Effect begging the Question, and saying that the Rule and the Directions are such and such, for it does no more than repeat one of the Directions of the Rule, and tell us that it differs widely from another that is no Direction of the Rule, without offering one Word in Justification of it. But now, when he is attempting to answer a Treatise, in which are many Objections to the Rule, and to that very Direction in particular, and in which 'tis alleged, that such and such absurd Values in certain

certain Tables were produced by following that Direction, and demonstrated that those Values so produced were a necessary Result from so following it: When he tells us that the Rule bids us make the Deductions in the Manner alleged in the Exception; this manifestly confirms the Exception, for it admits the Verity and Justice of the Allegations; and, if those are true, I'm very confident that the Conclusion is right, that the Rule is not a good one.

Another Objection to the Rule mentioned in the *Essay*, and which this Writer's Observations upon it make necessary here to repeat is this: That according to such Rule the Value of an Annuity for a Life of ten Years Age is 13-44 Centesimals, which is equal to a Term of 28-1-00; and the Value of an Annuity for a Life of 80 Years Age is 3-84 Centesimals, which is equal to a Term of 4-2-00; but from the *Breslaw* Table, from whence this Calculation of these Values was made, it appears that the even Chance of the Duration of the Life of 10 Years Age is 41 Years, and the even Chance of the Duration of the Life of 80 Years is not full 4 Years. From thence it was argued: That an Annuity for the Life of a Person aged 10 Years should be in Value 13-44, that is, should be equal to a Term of 28-1-00 only, when his Life on the even Chance of Duration is equal to 41 Years and above; and at the same Time that an Annuity for the Life of a Person aged 80 should be in Value 3-84, which is equal to a Term of 4-2-00,

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4-2-00, when his Life on the even Chance of Duration is equal only to a Term of 4 Years: This is there said to be so palpable a Contradiction to common Sense, that nothing can maintain the Rule by which those Values were produced. The Jet of the Argument lies in this, and on this the Contradiction is grounded, *viz.* That the same Rule should in the Operation have such different Effects on the younger Life and the elder Life as to depress the Chance of Vitality of one below the even Chance, and exalt the other above it; which, in Effect and in other Words, is as much as to say, that in the one Case the Annuity will not have a Chance to continue so long as the Life will have a Chance to continue, and in the other Case will have a Chance to continue longer than the Life does: And, notwithstanding the learned Labors of this Author to reconcile this Matter, I think, and I believe most others will think, that it remains still in the same State of Inconsistency.

Let us now examin his Way of Reasoning here. He admits the Values of the Annuities, and the Chances of the Duration of the Lives, respectively, to be such as I have stated them; but a Repetition of those Computations, merely and by itself, sure enough does not prove either that the Values produced are the true Values of the Annuities, or that the Rule by which they were computed is a right one. This is neither better nor worse, than first to suppose and take for granted, that these Values thus produced are the true

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true ones, and then to infer that the Rule by which they were computed is a right one; or *vice versa*, to suppose and take for granted that the Rule by which they were computed is a good one, and from thence to infer that the Values are true ones; but this will not have much Weight with those who deny both that the Rule is a good one, and that these Values are the true ones.

However, to justify the Value given on the younger Life he urges, *p.* the 33<sup>d</sup>, that, when all the possible Chances are considerd, they will reduce it to an Equality in Value with a Term of 28-1-00. I observe here, that he speaks only of the possible Chances of Vitality, and not of the Value of those possible Chances; and I say, that all the possible Chances of Vitality have been considerd, and upon a Computation of them, it appears that 'tis an even Chance, that the Life will have Continuance for 41 Years, and this he will and does admit; therefore 'tis incumbent on him to prove that 'tis not an even Chance or Probability that the Annuity, which is to continue for the Life, will have a Duration for the same Term, as 'tis an even Chance or Probability that the Life will have a Duration, that is, for 41 Years: And until he does this, which I despair of ever seeing done, for indeed 'tis impossible to be done, the Difficulty will remain.

To justify the Value given on the elder Life, which seems to be the harder Piece of Work, he gives us, *p.* the 33<sup>d</sup>, a Computation of som of the possible Chances on such a Life, and refers us to *p.* the 18<sup>th</sup> for a like Computation on som of

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the like possible Chances on the younger Life; and then concludes: *That it will be no Contradiction to common Sense to say, that the Values of these two particular Annuities, produced by the Dr's Rule are the true Values of the Annuities on these Lives respectively: Because in estimating the Value of an Annuity on a given Life all the possible Chances must be computed; and the Chances in the Case of the elder Life have a different Operation from what they have in the Case of the younger Life.* If he means here no more than that all the Chances of possible Life must be computed, as his Words import, I shall readily agree with him, because we cannot compute the Value of an Annuity for a Life without estimating first the Chance of the Duration of that Life, and we cannot rightly estimate that Chance without computing all the possible Chances of that Life. When that is done according to the Rule laid down in the *Essay* for estimating the Value of Life Annuities, a Moiety of all the possible Chances gives us the even Chance of the Duration of the Life, the even Chance of the Duration of the Life gives us the Term to which the Life is equal, and that gives us the Value of the Annuity. But if he means here that the VALUE of all the possible Chances on the Life are to be computed, and the possible Chances of a Life and the Value of those possible Chances are quite different Things, and will produce Annuities of very unequal Value; and this latter he must mean, if he intends to support the Dr's Rule, for the Direction of that Rule is to take the VALUE of all the possible Chances

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Chances of Life; then he begs the Question, according to Custom, instead of proving it. The Objection to the Rule was this; that it produces such Values of Annuities, as that in the one Case they necessarily suppose the Chance of the Duration of the Annuity to fall short of the even Chance of the Duration of the Life, and in the other to exceed it; and these Values of Annuities are in the *Essay* asserted, and most People I believe will agree, to be absurd ones. 'Tis there demonstrated, that these Values of Annuities are produced by the Rule, and particularly by its directing, or because it directs, that the VALUE of all the possible Chances on Life be computed; and he concludes here that the Rule is a good one, without more saying, than that 'tis a good one, because it directs the Computation of the Value of such Chances to be made in a particular Manner, which Manner necessarily produces such a Value of these Annuities, as are manifestly absurd ones.

I have said that the possible Chances of a Life and the VALUE of those Chances are different Things, and so most certainly they are; and tho' this Author does not see it, or may not be ready to acknowledge it, that there is in Truth the Matter in Controversy, whether the Computation is to be made by the possible Chances of a Life, or by the Value of those possible Chances. I say 'tis to be made in the first Manner, and that such Computation will give us the Duration or Chance of the Duration of the Life; that the Duration of the Life is the Measure of the Duration of the Annuity depend-

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ing on that Life, and the Duration of the Annuity the Measure of the Value of it. On the other Side, if a Computation be made of the VALUE of such possible Chances, that is done, and can be done no otherwise, than by computing the Value of the several Annuities attending on such possible Chances in every Year of Life, and through the whole Life, and then assuming that those several Annuities put together constitute the Value of the Annuity for the the Life. Now this does directly and obviously make the Chance of the Duration of the Life to depend on the Value of the Annuity; for, in every Computation which has been made of the Value of Annuities on Lives constructed in this Manner, we have seen that the Fact is such, that in all Instances where the Interest is varied, and consequently the Value of the Annuity varied, there the Chance of the Duration of one and the same Life is varied likewise, and that the one increases or decreases when and in Proportion as the other does; and yet 'tis most undoubtedly true, and this Author asserts it, over and over again, that the Value of the Annuity is not to increase or decrease, or in any Shape to have an Influence on the Chance of the Duration of the Life, and yet in Practice he generally contradicts it.

He adds farther, that the possible Chances in the Case of the elder Life have a quite different Operation from what they have in the other Case of a younger Life, and from thence woud infer  
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that the Values of the Annuity on these two Lives are or may each be right; but this surely is a strange Inference, for he gives that as a Reason for the Rectitude of the Rule and for the Truth of the Values, which I have given, and most certainly is a good one, for Falsity of both. When I considerd the Value of the Annuity on these two Lives as produced by one and the same common Rule, I said that it could not be a good one for estimating the Values, and gave this Reason for it, *viz.* because in the Operation upon it the Effects in the Case of the younger Life, and in the elder were so different, as to depress the Chance of Vitality on one Life below the even Chance, and to exalt the other above it. The Answer he gives to this is no more than a bare Repetition of the Grounds of my Objection to the Rule and the Values in these Instances produced by it, with some small Difference in the Expression, *viz.* the Rule is a good one, and the Values of these Annuities are right; because the Value of the possible Chances, which the Rule directs to be computed, have a different Operation in the younger and in the elder Life. This, I say, is and will remain an unanswerable Objection to the Rule, until it can be shown, and I may be confident it can never be shown, that the Value of these possible Chances, or rather the Value of the Annuities attending those Chances, can in some Instances abridge the Chance of Vitality so as to make it less, and in other Instances enlarge it so as to make it greater than the even Chance: Or, as this Author expresses



presses himself, that the Manner of granting these Annuities, whether on a younger or on an elder Life, can increase or decrease the Chance of Vitality. That the Operations on the Value of the possible Chances on the younger and the elder Life, in their Way of computing them, have different Produces is a Thing visible, and is the Matter of the Objection or the Grounds of it; and to answer this the Author should have shown such a Difference between the Nature of the Life of the young Person, and the old one, as should reconcile this Difference in the Produces; but that being impossible to be done, as most certainly it is, for the Chance of Vitality of one to 41 Years is the very same Thing, and collected in the same Manner, as the Chance of Vitality of the other to 4 Years, he contents himself with repeating the Substance of the Reason and Grounds of the Objection, and there leaves the Matter.

However, to be more particular, he attempts to show that 13-44 may be the Value of an Annuity for a Life of ten Years Age, and 3-84 the Value of one for a Life of 80, tho' neither of them be consistent with the even Chance of Vitality of the respective Lives, which he must and does admit. For this Purpose he makes a Computation of the Value of the Chance which *A*, for Instance, supposed to be 10 Years old, has whether he shall die in the first Year, or live to be 90 Years old, and the Sum arising is 1-59. He makes the like Computation of the Value of the like Chance on the Life of *B*, for Instance, supposed

posed to be 80 Years old, and the Sum arising there is 4-5. If we inquire how the Computations of the Values of these particular possible Chances on these two Lives made by this Author, and his Reasoning upon them, are to be applied; I can make nothing other or farther of them than this.

The Dr's Rule for estimating the Value of Annuities a Life may be a right one, altho, in computing the Value of the possible Chances of these two Lives, the Produces are so different as not to coincide with the even Chance of Vitality, which is the Point he undertakes to prove; for here is a Rule, he says, of his own, for computing som of the particular possible Chances on these two Lives, and on the Computations made thereby it appears that one is in Value 1-59, and the other 4-5. Very true; but these two Values do in Fact coincide with the even Chance of Vitality, as we shall see presently, and he himself says they do, and takes som Pains to prove it. The Argument then will stand thus: This Observer's Rule for computing the partiular Chances on these two Lives is a good one, as I do admit it is, because the Values produced by the Rule do coincide with, and are computed by the even Chance of Vitality; therefore Dr. *Halley's* Rule for computing the Value of the Chances for the whole Life (or of an Annuity for the whole Life) of these two Persons is a good one, which produces such Values as do not in either of them coincide with the even Chance of Vitality. I should speak too favourably

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of these two Computations, and the Application of them to the present Purpose, if I should say of them only that they prove nothing towards maintaining the Doctor's Rule; for some People will be apt to surmise that they prove rather the contrary, that no Computations on the possible Chances of Life, or the Values of them, are right but what do coincide, or at least are consistent, with the even Chance of Vitality of Life.—

To support such Surmise of theirs, they make Use of the two Computations made by this Writer to show that the Sums 1-59 is the Value of one of the Chances, and 4-5 the Value of the other; but, since they are both formed by the same Pattern, I will mention one only—He says, then; *That the Number of Persons dying in the first Year of the Life of A is 8, the Number of Persons living of the Age of 90 is 8; consequently it is an even Chance whether A is one of the first 8, or one of the last 8; that is, the Chances of A's dying the first Year, or living to 90, are equal. If he dies the first Year, he loses the Value of an Annuity for 41 Years, which is worth 15-12 Centesimals; whereas, if he lives to 90, he wins the Value of an Annuity for 39 Years, in Reversion after a Term for 41 Years, which is worth no more than 1-59 Centesimals.* I observe here that this Author, as great a Master as he is of Numbers and Figures, is guilty of a Mistake in computing the Value of the reversionary Interest of the 39 Years, and stating it at 1-59, when 'tis only 1-38; for the whole

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whole Term of 80 Years is, in the common Tables, set at 16-50 Value; and, since the Interest of 41 Years in Possession is 15-12, the Interest of the 39 Years in Reversion can be no more than 1-38. But to let that pass, for in the present Argument 'tis not at all material which is the truer Value; and we will suppose 1-59 to be the right one; I admit then that the Number of Chances are equal; that the Chance here is an even one; and that 1-59, the Sum produced, is the right Value: But it happens very unluckily that this Argumentation, when applied to the present Purpose, does directly and evidently prove my Position, and destroy the other. Mine is, that the even Chance of the Duration of the Life, computing one Chance only in the whole Life, is the Measure of the Value of the Annuity for that Life; and Dr. Halley's and this Author's is, that the Value of the Chance for each Year of the possible Life, computing as many Chances as there are Years in the possible Life, is such Measure. Now in the Case here stated 'tis supposed, as the Foundation of the Truth of the Value produced, that the Chance is an even one; and in the Computation itself 'tis taken for granted; that the Chance of the Duration of the Life is the real Measure of the Value of the Annuity for the Life. 'Tis here laid down that *A. by his Death in the first Year will lose an Annuity for 41 Years which is in Value 15-12; but by his Death at That Time he can lose nothing but an Annuity for his Life, for he can*

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lose nothing but what he has a Right or a Chance to have, therefore such Annuity must be equal to a Term for 41 Years, and be in Value 15-12. And since the even Chance of the Duration of the Life is a for a Term of 41 Years, and a Term for 41 Years is in Value 15-12, and the Loss by his Death in the first Year is such a Term and such a Value: It follows from thence, that the Chance of the Duration of the Life is here made the Measure of the Value of the Annuity for the Life, according to my Position.

I cannot forbear observing here, how naturally and easily this Author falls into the Notion, that the Chance of the Duration of a Life is the Measure of the Value of an Annuity on such Life; for his Reasoning is formed on Purpose to prove that such Chance is not the Measure, and that 15-12, the Sum produced by such Rule, is not the Value; yet even here he supposes that Sum to be the true Value of the Annuity, tho' at the same Time he makes Use of it as the Medium to prove that 'tis not the true Value. If then 1-59 be the Value of this particular Chance on such Life, and for that Reason, *viz.* because 'tis an even Chance; I don't see why I may not conclude that 15-12 is the Value of an Annuity for the younger Life, and 3-46 of an Annuity for the elder Life, beforementioned: For the even Chance is that the younger Person does not die before the End of 41 Years, and a Term for 41 Years is in Value 15-12; and the even Chance is that the elder Person dies in 4 Years, and a Term

Term for 4 Years is in Value 3-46. On the other Side, and from hence, I think I may infer that the Values of the Annuities on these Lives produced by Dr. *Halley's* Rule must be false, since one falls greatly short of the Value of the even Chance of the Duration of the Life, and the other exceeds it. Therefore, upon the Whole, I recommend it to the Author of these Observations, and leave it with him, to answer his own Arguments, as well as mine, for my fundamental Position; that the single Chance of the Duration of a Life, computed once for all his Life, is a good Rule, and the only Rule, to compute the Value of an Annuity depending on that Life.

I must, in the next Place, take Notice of the Remarks made on the Essay made by Mr. *Richards*, often therein mentioned, were it only to let him know that I have read them; but that I despise the foul-mouthed and scurrilous Language which he has so plentifully bestowed on me and my Performance, and for his Arguments and Objections against it, as he calls them, that I think they are already answered or obviated, as far as the Matter of them deserves and requires. All I shall add is, that I would recommended it to him to look into his own first Treatise on this Subject, if he thinks it worth his Trouble, which few perhaps besides himself will do, and compare what he says in P. 27 and 28. with what he says in P. 83, &c. In P. 27, having made a Supposition that three Persons are to enjoy an Estate by equal Portions during their joint Lives, Re-



mainder to the Survivors and Survivor, he makes a Quære what is the Value of each Man's Right ; and, in the next Page, solves it thus—*That each Person has a Right to one Third of the Value of their joint Lives, and to Half the Value of the Expectation after the Decease of either of the other two, and also to the whole Value of the Expectation of both the others ; which three Sums added together give the Value for each.*—In P. 83, &c. he supposes a Man to give his three Daughters an Annuity of 20*l. per Annum* each for their Lives ; and to order, that, when any of them dies, the Annuity of the Sister dying should fall to the other two in equal Shares ; and, at the Death of another, that the whole should belong to the Survivor for her Life. He goes on then to compute the Value of this Gift to each of these Women ; and gives us the Value of the Share of the eldest Sister thus :—First, 20*l. per Annum* during her own Life. Secondly, 10*l. per Annum* in Reversion of the youngest, to continue from thence during the joint Lives of the other two. Thirdly, 10*l. per Annum* in Reversion of the Second, to continue from thence during the joint Lives of the First and Third. Fourthly, 40*l. per Annum* in Reversion of the two youngest Lives during her own Life. Now, unless this Writer does, and until he has reconciled his Solution in the first Passage, in which he allots to each Person three Sums only, with his Computation in the second Passage, where he brings to Account four Sums : Which I suppose he will be in no great Haste to attempt, and,

and, if he does, will never be able to effect : Some People will, perhaps, call in Question his Faculties, Algebraical, Mathematical, or Intellectual, and, for my own Part, I shall never say a Word more to such a ———.

To Mr. *Hodgson*, who styles himself *F. R. S.* and Master of the Royal Mathematical School, who has lately published some Operations and Tables on the Value of Annuities for Lives, I have something more to say, because he builds, in Part at least, on the same Foundation on which *Dr. Halley* did, and has revived, seemingly, and unnecessarily I think at this Time of Day, the Controversy on the Rule for making the Valuation of these Annuities. In his Preface, by Way of Apology for framing a new Table of Mortality for *London*, to serve the Place of the old one of *Breslaw* ; he tells us, that at *Breslaw* one Half of the People that are born there live 'till they are about 41 Years of Age, in *London* that one Half die before they are of the Age of 10 Years. Where and whence this Writer pickd up this Notion about the Age to which the People who are born at *Breslaw* do live, or who should tell him so, I cannot devise ; for *Dr. Halley*, in his Treatise on this Subject, tells us, in so many Words, that it appears from the *Breslaw* Table, and in Fact it does so appear, that one Half of those who are born there are dead in 17 Years : But, this not being material to the main Design, let it pass. In his Book, after giving us, for four or five Pages, a Calculation of the Chance of the Duration of  
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 Lives for *London*; tho' by the Way, this long Detail, even were it a good one, is very little to the Purpose, unless such Chance be the Measure of the Value of the Annuities depending on those Lives; he says, that from thence the Value of an Annuity for a single Life of a given Age, or the Value of any limited Time the Person has a Probability of living, at any Rate of Interest, may be found. He proceeds, by Way of Instance of a Person of 10 Years Age, and at *3l. per Cent.* Interest, to compute the Value of an Annuity for his Life of *1l. per Annum*; and having told us the present Value of *1l.* payable at the End of one Year, when 'tis an absolute Certainty, and computed what must be deducted thereout on the Life Annuity for that Year, on Account of the Chance of Mortality arising in that Year; and having gone through the like Operation on the second and the third Year of the Life of such Person; he concludes, that, *by repeating these Operations to the utmost Extent of possible Life, the Value of 1l. per Annum, for a Life of 10 Years old, will be found to be 20-16-46, or 20-1646 Years Purchase:* And then adds, that *the Tables there following were constructed after this Manner.* I observe here, that this Method of computing the Value of Annuities on a single Life, tho' it may vary in the Expression and Shape of it, yet in Substance is the same as Dr. *Halley* prescribed for that Purpose; and therefore, that all the Exceptions taken to this Rule, as Dr. *Halley's*, will equally lie against it now adopted and abetted  
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 by this Author. This being premised, I say, that this Writer has either himself been imposed on, or attempts to impose upon his Reader, when he tells him that the Tables there following were constructed in the Manner mentioned; for, whatever was the Manner in which they were constructed, 'tis certain it was not this; and he himself shows it demonstratively, and so plainly and palpably, that one may read it as he runs. He says, on computing the Value of an Annuity for a Life of 10 Years Age, the Value or Years Purchase appears to be, when computed by his Method, 20-1646; which I believe to be right, because I have made the Computation on the same Life, at the like Interest, and by his Rule, and I make the Value 20-1623; and this very small Difference may possibly arise by his inserting an Account of the Fractions at the End of each Year, and my omitting them. But in the Table, which follows in the very next Page, he gives 19-5761 as the Value of an Annuity for a Life of 10 Years of Age, Interest computed at the Rate of *3l. per Cent.* when, in the Paragraph just preceding, he had computed and stated it, and at that Rate of Interest, at 20-1646. Now the Sum 20-1646 is equal to a Term of  $31\frac{2}{3}$ , and the Sum 19-5761 only to a Term of  $29\frac{2}{3}$ ; so that there is the Difference of a Year and an Half in the Term to which one and the same Life is equal, as computed in the precedent Paragraph and as given in the subsequent Table. At the Rate of *5l. per Cent.* there is the like Variation, but on the  
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the contrary Part; for the Value, on computing by the Rule given by this Writer, is only 15-0430, which is equal to a Term of  $28\frac{1}{2}$ , but in the Tables is put down 15-3588, which is equal to a Term of  $29\frac{1}{8}$ . At the Rate of 4*l. per Cent.* indeed, the Value arising on a Computation by the Rule and that inserted in the Tables is one and the same, viz. 17-2738, which is equal to a Term of  $29\frac{1}{8}$ . From these Calculations and Facts, 'tis evident that the Value of this Annuity, at the Rate of 3*l.* and 5*l. per Cent.* was not collected and ascertained on a Computation by the Rule, for they are not agreeable to it; but, on the Contrary, it must have been first supposed that the Values at the Rates of 3*l.* and 5*l.* were equal to the same Term to which the Value at 4*l.* is equal, and then a Value corresponding to that Term is inserted as the Value of the Annuity at the Rates of 3*l.* and 5*l.* The Truth is, and in his Table, P. 40. and which he calls, *The Value of Lives upon Annuities certain*, 'tis manifest, that this Writer did mean and intend that the Value of Annuities for a Life of the Age of 10, and indeed of all other Ages of Life, at all Rates of Interest, should respectively correspond to one and the same Term in an Annuity certain; for this Table exhibits them all, at every Rate of Interest, as equal to one and the same Value in an Annuity certain, which is the same Thing as saying, that they are all equal to one and the same Term in an Annuity certain: And that is the very Point for which I contend, and ought

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to be, but is not preserved, nor can possibly be obtained by Dr. *Halley's* Rule, and in the *Essay* have demonstrated it, tho' by this Author's new-fangled Regulator it may.

Now this Method of constructing the Tables of these Values does not only contradict the Manner by which he says, in the precedent Page, he constructed those Tables, but 'tis likewise an absolute and total Departure from the Rule laid down by Dr. *Halley*, observed by others, and pretended to be assumed by this Author, except in the single Article of 4*l. per Cent.* and is conformable to the Rule laid down in the *Essay*, so far forth as it makes the Term in an Annuity on one and the same Life to be the same as the Term in an Annuity certain, at all Rates of Interest. He does indeed by this Means, in Part, obviate or evade the main Objection made to Dr. *Halley's* Rule, but we shall see presently that my Exception to this Method remains as strong, at the Rate of 4*l. per Cent.* as it does against the old Rule, where the Rate of 6*l. per Cent.* is used. The Exception I mean is, that, by Virtue and in Pursuance of the old Rule, such a Sum is produced as the Value of a Life Annuity, as that one and same Life shall be equal to a different Term as the Rate of Interest varies; and it is demonstrated that this must be the Case, and admitted so to be by Mr. *H. B.* whom I have beforemention'd; and consequently that the Duration of a Life or the Chance of it depends on the Rate of Interest, and must, and in Fact is found to vary as oft as that

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varies.

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 varies. Since then the Term in the Value of an Annuity at every other Rate of Interest different from it is by this Method to be affixed and brought to the Standard of the Term in the Value of an Annuity at the Rate of 4%. then the Chance of Duration of Lives depends on that particular Rate of Interest as the common Standard. Now if the Question were asked, on what Pretence or by what Authority this particular Rate of Interest is made the Regulator here, I think no tolerable Answer could be given but this, that at present this is the common Price of Money. Be it so: But what if it should so happen that in 20 or in 50 Years Time, more or less, the Price should alter, and sink to three, or rise again to five? Why, we must then change our Standard as the Case shall come out; and whereas  $29\frac{7}{8}$  on a Life of ten Years Age is now the Term to which an Annuity on such a Life is equal, then *in futuro*, if the Price should be three, an Annuity on such a Life would be equal to a Term of  $31\frac{3}{4}$ , and, if it should rise to five, would be equal to a Term of  $28\frac{1}{2}$ . If we look backwards, we find the Price of Money about 50 or 60 Years ago to have been six, and about an hundred Years ago eight; and in the first Case the Term in the Annuity for the like Life would be  $27\frac{1}{4}$ , and in the latter about 25, and then these Terms respectively must be looked upon as the Term to which an Annuity on such a Life, at the several Times, and at the different Rates of Interest then current, must be reduced as to a common Standard. Now this Alteration of the  
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 common Standard changes the Chance of the Duration of a Life of one and the same Age, in the same Manner and as effectually as the Change of the Rate of Interest, at any one Time, does change the Chance of the Duration of one and the same Life at that Time: And I believe no Body will be so hardy as to assert, that the Chance of the Duration of a Life of one and the same Age is not the same now as it was 50 or 100 Years ago, or that it will not continue the same for 50 or an 100 Years to come; so that the Difference between the Dr's Rule and this Regulator would be this and no more than this, that by the Use of the former the Chance of the Duration of one and the same individual Life is varied as the Interest varies, and the Life is made shorter or longer, as that is higher or lower; and by the Use of the latter, that such Chance would vary on a Life of one and the same Age, in the same Manner and Proportion; from whence it follows, that this new Regulator is liable to the very same Objection, as the old Rule.

This Writer can himself give the best Account, what induced him in his Book to recommend and make an Operation by the Rule, but in constructing his Tables to give it up and desert it, and to fall into this new invented Method; but 'tis respectable that he saw and felt the Weight of the Exception to the Rule, and not being able to answer it, as I may be confident he was not, he entered into this Practice to elude or to cover it. But, whatever his Reason might be for using these Ar-

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tifices, he has faild of Succes; as we shall see presently he has done likewise in putting in Practice the same little Arts, with Respect to another material Exception to the Rule mentiond in the *Essay*. The Exception I mean here is; that in using the Rule prescribed, in the younger Life, the Term, to which the Chance of the Duration of the Life is equal, is longer than the Term to which the Value of the Annuity for the Life is equal: And on the Reverse, in older Life, the Term, to which the Value of the Annuity for the Life is equal, is longer than the Term to which the Chance of the Duration of the Life is equal: And this in the *Essay* was said to be contradictory to common Sense and Reason; and notwithstanding the learned Endeavors of Mr. *H. B.* in his Observations before-mentiond, to reconcile these Inconsistencies, I still adhere to my former Opinion, and am confirmd in it, on seeing the great Labor he has taken, and the little Succes he has met with, in the Attempt. Let us examin now how this Matter stands on computing the Value of these Annuities by this new Method. In the Case of the Value of an Annuity for a younger Life, suppose of ten Years Age, 'tis equal to a Term of  $29\frac{7}{8}$ , and the even Chance of the Duration of such a Life is 35 Years, Computation in both Instances being made by this Writer's Tables, so that the Difference amounts to a seventh Part of the whole Term, and something over: But in the Case of an Annuity for the older Life, suppose of 80 Years of Age, 'tis set down as equal to a Term of

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of 4-9103, which is four Years and eleven Months, wanting about three Days; and if we inquire, to what Term such a Life on the even Chance is equal, this Writer *p.* the 9<sup>th</sup> says between 4 and 5 Years; in *p.* the 7<sup>th</sup> five Years; and, if we compute such Chance by his Bills of Vitality and Mortality for *London*, I find, and if any one else will give themselves the Trouble of computing it, as I have done, he will find, that the even Chance is just four Years and ten Months; so that the Duration of the Annuity exceeds the Chance of the Duration of the Life, as well in Computations by this new Method, as by the old Rule; and therefore both of them absurd and defenceless.

After all, what do these little Subterfuges signify? Even just nothing at all; for Mr. *H. B.* a strenuous Advocate for Dr. *Halley's* Rule, has admitted the Fact to be such as I have stated it on an Operation made by such his Rule, and, on the like Operation by this Writer's Rule, the Fact must come out nearly the same, and consequently my Exception to the old Rule and this new Method will be equally strong, tho' perhaps it may not appear in a Light so clear, even tho' the Chance of the Duration of this Life were supposed to be full five Years, and to exceed the Term in the Annuity, or that the Terms in both were equal. The Way of Reasoning here stands thus: Since by Virtue of the Rule the Term in an Annuity for a Life is diminished and made less than the Term to which the Life on an even Chance is equal, 'tis necessary that a certain and regular

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 Proportion of such Decrease should be preserved through all the Stages of Life; but here, where in younger Life the Term in the Life is 35 Years, and the Term in the Annuity is  $29\frac{7}{8}$ , and in the elder the Term in the Life is 5 Years (supposing it were so) and the Term in the Annuity four Years and eleven Months, or very near it; the Decrease in one Case is one Year in seven, and in the other one Month only in 55 Months.—And if we should suppose that in some Instance the Term in the Chance of the Life was the same as the Term in the Annuity, which must be the Case in some one Instance, tho' not here: And if it should be said that there the Value of the Annuity would be the right one; I will admit it to be so, and admit it for that Reason, because it has fortuitously happen'd in that Case, that the Term in the Life and the Term in the Value are the same and coincide. But let us look a little into the State of an Annuity on a Life still older, suppose of 90 Years Age, and see how Matters stand there. The even Chance of such Life, *p.* the 9<sup>th</sup>, is said to be a Term of three Years, and the Value of an Annuity for such Life given in his Tables is 1-8772; but this is not near equal to a Term even of two Years, for a Term of two Years is in Value 1-9135, so that the Difference between them will exceed a whole Year; and consequently the Decrease in this Case is one third at least of the Term, and this is a greater Decrease than in a Life of ten Years Age, for there the Decrease is only a seventh Part of the Term. We find then that to  
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 the old Difficulties on the Rule an Addition is here made of another; and, unless Mr. *Hodgson* can discover a better Solution of them than *H. B.* has done, these Irregularities in the Proportions of the Decreases, believe me, do effectually destroy the Justice of the Regulator.

In the *Essay* very little Notice is taken of Mr. *Hayes's* Method of Valuing Annuities on Lives, because his Tables drawn for that Purpose seem'd to me so absurd and wide of the Mark, that I could not think that any one, of tolerable Understanding in Things of this Nature, would be guided by them: But since his Treatise on this Subject has lately received a new Impression, in a very superb Dress, I must say something to it now. I observe he gives us the Age of 30 as the first Age of Life, on which he begins his Valuation of Annuities; but for what Reason he omits all the Stages of Life precedent to that of thirty I cannot imagin, unless he were of Opinion that the Age of thirty was the best Age of Life: But for this Surmise there's no sort of Foundation in Nature or Experience; and on the contrary the *Breslaw* Table, the Bills of Mortality for *London*, and those of the Nominees in the Government Annuities on Survivorship, which is the best Guide of all, do plainly show, that 'tis quite a vulgar Error. In *p.* the 1<sup>st</sup> of his Book, as the Foundation of his subsequent Calculations, he tells us, that the Annuities, calculated in the following Pages, show, at one View, the present Value upon the Life of any Age from 30 to 73, according to the  
 Chance



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him the Annuitant will have been reimbursed the Principal, or received the Value, in 16 Years: and, on the other Side, if the Annuitant has a Chance to live only 16 Years, then with Regard to the Grantor at 4l. per Cent. the Annuitant himself will have been cheated, because there will be nine Years wanting to make up his Principal, or to receive the Value of the Money given on the Purchase. In short, if this Writer's fundamental Position, viz. that the Chance or Probability of the Continuance of a Life be the Measure of the Value of an Annuity for the Life, be a right one, as I make no Doubt it is; or indeed if he intended to make Use of such Chance as his Rule for constructing his Tables; instead of his long Rows of Numbers and Figures, which are extended for 50 Pages together and upwards, he might even as well have set out so many broken Reeds, for he that makes Use of one as his Guide, in the Valuation of Life Annuities, will be led into gross and very mischievous Errors, as surely as he will be deceived in his walking who leans on such rotten Staffs for his Support. This Writer, I think, somewhere in his Treatise, insinuates, that he grounded his Calculations and Rules on the Practices of som of the great Companies in London, or, *vice versa*, that they grounded their Practice on his Rule; but, whichever of the two were the Case, I believe most People at present will be ready to pronounce, that som Events have sufficiently damned both the Rules and the Practice.

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I shall not need to add any Thing farther on this Subject, unless somewhat to inform the Reader on what Grounds I framed the Tables for the Value of an Annuity on a single Life, and in what Manner they were constructed; so that he may be able to judge for himself how far he may depend on them. I adhere then to the Position laid down in the *Essay*, that the Chance of the Duration of the Life is the only sure Measure of the Value of an Annuity depending on that Life; for no material Objection to it has occurred to me which has not been obviated or sufficiently removed. The Chance of Vitality being then the Ground-work, in order to investigate this, I reviewed the Bills of Mortality for London; and having an Opportunity of seeing and considering an Account of the Mortality of the Nominees in the Government Annuities on Survivorship, I had special Regard to such Account; since it contained a Bill of Mortality of Individuals without Variation, and therefore much the best Guide to discover and fix the Chance of Vitality.—

Now, since there is one certain Stage of Life which must have the greatest Chance of Vitality; for those Chances, from the Beginning of Life, do increase to a certain Time, and from thence decrease to the End of Life; my first Inquiry was what might be that Year or Years of Life wherein the Chance of Vitality was greatest.—To fix this Point, I consulted first the London Bills of Mortality, and those of the Nominees in the Government Annuities, and from these fixed the Age

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between



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between six and ten, including both, as the best Stage of Life, on the following Grounds. On a View of those *London Bills*, it appears that the Number of Persons dying between 10 and 20 Years Age, is 30 in a 1000; and in the Period between 5 and 10 Years age, tho' no more than Half the Length of the former Period, the Number is 38 in 1000; and in these two Periods the Proportion of Mortality is the lowest: But then, tho' the Proportion of Mortality in these two Periods is not the same, and in the Period of younger Life is much the greater, I considerd that the Children of the Ages between five and ten were respectively and gradually removed farther from the Extremity of Life, and that the one might nearly counterbalance the other, I stated all those five Years at one and the same Chance of 35 Years. I then examin'd how the Case stood in the Account of the Mortality of the Nominees in the Government Annuities, and found it to stand thus: The Persons were nominated some time before *Midsummer* 1693; and at *Christmas* 1693, the Account I have seen being drawn up from *Christmas* to *Christmas*, the Number of Persons then surviving was 1009, and at *Christmas* 1728 there were dead 505; so that in 35 Years one Half exactly were dead; and I suppose that all the Persons nominated, and when nominated, at an Average, might be something between the Age of six and ten. To satisfy myself farther in this Point, I made a Computation on the Bills of Mortality for *London*, and observed the

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the same Method prescribed by Dr. *Halley*, and made Use of by him and Mr. *Hodgson*, and another on the same Bills in a Method form'd by myself, and the Produce for the even Chance of Vitality for a Life of ten Years Age is, upon all the Operations, very nearly the same; and, in som of the Ages of Life younger than ten, this Chance runs rather higher.—

Having thus stated the Chances of Vitality for the best of Age of Life, viz. of six to ten, and supposed it to be equal to a Term of 35 Years; and since an Annuity for such Lives will be equal to the same Term; most People, I believe, will think the Estimate sufficiently high, even for that very best Age. From thence I proceeded to the Stages of Life precedent and subsequent to this, and propos'd to settle the Chances of these by inquiring into and fixing the Proportions of the Decreases of them from the said Term of 35 Years, for every single Year, or for every Period of ten Years, respectively. Here I was in Hopes of saving myself som Trouble, by using the Labors of som of my Predecessors who have wrote on this Subject, and particularly of Mr. *Hodgson*, who in his Treatise, P. 6, has given us a Computation of these Chances, from every Period of ten Years of Life, from ten Years Age to the Extremity of Life. For this Purpose, I threw such his Computation into a short Sketch, exhibited at the End, in Scheme No. 1. but, on doing this, I discovered immediately that such Scheme did not give us a right Partition of those

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 those 35 Years or Chances for all the several  
 Periods of ten Years there mentiond. The  
 Defect in it appears almost on the Face of the  
 Scheme; but, in the Instance which follows, 'tis  
 so gross, that it cannot fail of being seen, when  
 pointed out, by every Man of common Under-  
 standing. That the Chance of Vitality on a Life  
 for one Year does sink in that Proportion in  
 which a certain Number of Persons living at the  
 Beginning of such Year does die before the End  
 of it, is a plain and an indubitable Truth: And  
 yet the Decrease of these Chances given in the  
 Scheme, in some Periods of ten Years, where we  
 throw such single Years into a Period of ten  
 Years, runs counter to that Position, directly,  
 and to a very wide Degree. For Instance, in  
 the Period from ten to twenty Years Age, the  
 Chance of Vitality is sunk eight Years, and con-  
 sequently in Proportion for each single Year of these  
 ten Years, and in the succeeding Period, 'tis sunk  
 only five Years and an Half; and yet the Persons  
 dying in the first Period are only 31 out of 490  
 living at the Beginning of it, and in the second  
 the Persons dying are 94 out of 459 living; from  
 whence it appears that the Number of Persons  
 dying in the second Period is much more than  
 double the Number of those dying in the first,  
 and yet the Chances of Vitality sunk in such  
 second Period are not near three Quarters of those  
 sunk in the first: Therefore the Partition in this  
 Scheme, not giving the true Decreases of these  
 Chances of Vitality for some Periods and Years of  
 Life,

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Life, cannot give us a just Partition of these  
 Chances for the whole Life. From thence it will  
 follow, that the Tables for the Value of Annuities  
 for a Life, constructed by Mr. *Hodgson*, do not  
 give us the true Value of them, even tho' they  
 were in all other Instances faultless; for that the  
 Value must depend on such Chances, and he  
 himself supposes as much, and yet these Chances  
 themselves, in some Parts of Life, are not rightly  
 adjusted, by this Method; and this is the Case  
 in all the Stages of Life from the 10th to the 60th  
 Age of Life, or near it; and, believe me, this  
 is a very large, and, in all Respects, a consi-  
 derable Part of Life.—

I cannot forbear adding a few more Words  
 on this Subject, because I think that, from this  
 Writer's own Position, it may be demonstrated,  
 that his Repartition of the Chances of Vitality, at  
 least for a Life between 10 and 20, must be wide  
 of the Mark. In his making the Computation  
 of these Chances, I suppose that he made Use of  
 his own Tables of Vitality for *London*, by com-  
 puting in what Time the Number of Persons  
 living of the Age of 10, of 20, and 30, were  
 respectively reduced to an Half, and then taking  
 such Time as the Term to which Persons of those  
 respective Ages were equal; for that, on such  
 Computation, and on such Operation, they do  
 come out to be the same as we find them in his  
 Treatise, *viz.* 35, 27, and  $21\frac{1}{2}$ . Now as the  
 Decreases of the Chances between 10 and 20, and  
 between 20 and 30, appear to be unpropor-  
 tionate,

tionate, the first being eight Years by the Death of only 31 Persons out of 490, and the second being no more than five and an Half by the Death of 94 out of 459; I was so much surprized, as to mistrust this Method so far, as to examin it more accurately than I had done when I wrote my *Essay*, for there I gave into and made Use of it; but, on a closer Examination and frequent Trial of it, I am satisfied that the Rule will not hold in all Instances, tho' in som it does. I lookd then into the Position on which this Writer grounds and makes the Calculation of the Chances which immediately there follows; and, on a Calculation made by the Method prescribed by that preliminary Position, it appears, that 'tis 14, and much above, to 1, that a Person of 10 Years old lives to the End of the succeeding Period of 10 Years; and that 'tis only 5, and a little above, to 1, that a Person of 20 Years lives to the End of the succeeding Period of ten Years: From whence it follows, that the Chances of Vitality for the Lives in the Period between 10 and 20 Years, is very far from being rightly apportiond; —for he who is 10 Years of Age, and has the Odds of 14 to 1 to live 10 Years, has evidently a greater Chance of living to be 20 Years old, than he that is 20 Years of Age, and has only the Odds of 5 to 1 to live 10 Years, has of living to be 30 Years old: And yet this Author's Computation of the Chances of Vitality of a Person of 10 Years, and a Person of 20 Years Age, makes these Chances of the Person of the younger Life

be greatly less than the same Chances on the older Life. This being the Case, on the Method made Use of by this Author for adjusting the Chances of Vitality, I consulted Dr. *Halley* on this Subject, but I find that he prescribes and makes Use of the same Method as this Author does; but I must beg Leave, in this as well as in som other Matters of this Kind, to dissent from them both, and to affirm that this Rule does not in all Instances, tho' it may do in som, give us the even Chances of Vitality, and that for the Reason and on the Computations just now made; and those are built upon the Foundations laid down by Dr. *Halley* himself, where he gives the Rule to find the Odds, which a Person of a given Age has to live for a given Term. Being forced thus to resort to a different Method, I formd that which follows, both to ascertain the even Chance of Vitality for a Life, and for any Period of Years: And this differs very little from that given by the Dr. for calculating the Odds of Vitality for a certain Number of Years, in another Shape indeed, and with som Additions.

The Rule then is this: To take the Number of Persons living at the Beginning of any Year or Term of Years, and the Number living at the end of that Year or Term Years; and if we subtract the one from the other, the *Remanet* is the Number of Persons dying in that Year or Term of Years: Next, to take the Number of Years, which we may call Chances of Vitality, to which a Person of the Age named may live, if we compute to the Ex-

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 tremity of Life: and then to investigate what is the Proportion in such Years or Chances of Vitality, which the Number of Persons living at the Beginning of such Year or Term of Years bears to the Number of those dead at the End of the Year or Term of Years. To perform this, we must multiply the Number dead in the Year or Term of Years by the Number of Years or Chances of Vitality through the whole possible Life, and divide the Produce by the Number living at the Beginning of the Year or Term of Years: And the Quotient will give us the Chances of Vitality sunk in that Year or Term of Years: And, if we deduct the Number of Chances sunk in that Time out of the Number of Chances subsisting at the Beginning, the *Remanet* will be the Number of Chances due and belonging to the Life proposed, and the Term to which such Life will have an even Chance of Duration.

Or thus: In Case the Number of Years or Chances of Vitality for any Age of Life be stated and known; if we want to know what be the Number of Years or Chances of Vitality due and belonging to a Person of the next or any other subsequent Age of Life, we need only to take that Number of Years or Chances of Vitality, to which the Life of the Person of the Age next precedent to the Age of the Person after whose Chances of Vitality we are inquiring; and then to investigate what is the Proportion in such Years or Chances of Vitality, which the Number of Persons living at the Beginning of such Year or Term  
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 of Years bears to the Number of those dead at the End of the Year or Term of Years, and then to proceed as before directed.

I should observe here, whether we take a Number of Years or Chances, according to all the possible Years or Chances subsisting on the Life given, which in a whole Life may be supposed to be an 100, it being possible that a Person just born may live to be an 100 Years old; or whether we take such a Number of Years or Chances, as answers to the even Chance of Vitality on the Life of a Person of the Age next precedent to the Age of the Person of whom the Inquiry is made, where such even Chance is already stated and known, the Decrease of the Years or Chances in each Life or Term of Years must be and will be exactly in the same Proportion. So that either of those two Numbers may be used with equal Success.

As this Way of Reasoning and the Rule may seem a little abstruse, I will apply it in an Instance or two on the Persons named in the Government Annuities on Survivorship. The Nominees in these Annuities were named at or before *Midsummer* 1693, and at *Christmas* 1693, when my Account begins, the Number of them then living was 1009, and I did suppose that the Age of each Nominee, at an Average, might be about nine or ten. We want to know then to what Age any one of these Nominees might probably live, or what were the Chances of Vitality due and belonging to any one of them, supposing him

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to be ten Years old, and as of such Age to have 90 Years or Chances of possible Life. The Number of Persons living at the Beginning of this Year, viz. at *Christmas* 1693, was 1009: At the End of the Year 1728, there were living 504, and, if we deduct 504 out of 1009, the Number remaining will be 505, that is, in that Compass of Time died 505. The Age to which any one of these Nominees might possibly live, being all supposed to be ten Years old, is 90. Then under the Direction of the Rule we multiply 505, the Number of the dead, by 90, which is the Number of Years or Chances through the whole possible, and the Produce is 45450: And if we divide this Produce by 1009, which is the Number of Persons living at the Beginning of the Term, the Quotient comes out to be 45, a small Matter over. From hence, we find that between *Christmas* 1693 and *Christmas* 1728, that is, in 35 Years or very near it, one Half of these 90 possible Chances were lost, and consequently the other Half left: Therefore the Life of one of these Nominees, on the even Chance, was equal to a Term of 35 Years, or might probably continue in Being for such a Term; and on this Computation, and som others before mentioned, I stated the even Chance of a Life of ten Years Age in general at 35 Years, as I found it to be in this Particular.

To give another Instance: We will compute the Chances of a Life, where we will suppose that those Chances of Vitality for a Life of ten Years Age

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Age is fixed and known; and there will make Use of the second Part of the Direction of the Rule. The Number of Nominees in the Annuities on Survivorship living at *Christmas* 1693 was 1009: The Number dying between *Christmas* 1693 and *Christmas* 1703 was 117. The Age of each Nominee, at an Average, we have supposed to be ten Years or something near it: A Life of ten Years Age, on the even Chance, we have stated as equal to 35 Years: Now, if we multiply 117 by 35, the Produce will be 4095, and, if we divide that Number by a 1009, the Quotient will be 4, and a little over: So that the Decrease of the 35 Years or Chances of Vitality in those ten Years will be 4 Years and a small Matter over; and, if we deduct 4 out of 35, the Years or Chances of Vitality remaining will be 31, and this will be the Number of Years or Chances for a Life of 20 Years Age. If we proceed in like manner on a Life of 30 Years Age: The Number of Persons living at *Christmas* 1703, supposed then to be of the Age of 20 or near it, was 892: A Life of 20 Years Age, on the even Chances, has been computed and stated as equal to a Term of 31 Years: The Number of Persons dying between *Christmas* 1703 and *Christmas* 1713 was 169: And, on the like Operations as before, the Decrease of the Years or Chances in that Time will be 5 Years and  $\frac{2}{3}$ , or near six Years; and, if we deduct six Years out of 31, the Years or Chances then remaining will be 25, and



and will be the Number of Years or Chances for a Life of 30 Years Age.

By this Rule, and in this Manner I computed the Decrease of these 35 Years or Chances, by Periods of ten Years, from the Age of 10 to an 100; and made Use of the Bills of Mortality, in the Annuities on Survivorship from the 10th to the 68th Year of Life, which was as far as those Bills could conduct me, and from thence, being destitute of this sure Guide, for the four Periods of remaining Life, I made Use of the *London* Bills of Mortality, observing still my own Rule: And by these Decreases I constructed the Value of Annuities exhibited in the Table annexd. I have before computed, on the Bills of Mortality for *London*, the Decrease of these 35 Years or Chances of Vitality, for each Period of ten Years, from 10 to an 100 Years Age, from the Account given of them by Mr. *Hodgson*, and have set out the Proportions of them for each Period of ten Years in Scheme N. I. — Here I have made a Computation likewise, in my own Method, of the Decreases of the same Chances on the same Bills of Mortality, for the same Periods of Life, and the Proportions of them, as stated by me, stand as in Scheme N. II; and I believe it may be safely left to the Reader, on a near Inspection on those two Schemes, to discern which is nearest a regular and proportionate Decrease, and deserves the Preference; especially if this Consideration be taken along with it, *viz.* that

that this Writer does in express Terms say, that, *after Persons arrive at ten Years, the Decrements of Life are little in Comparison with the former Years, and decrease regularly.*

I assert then, on computing by my Rule, and on the *London* Bills of Mortality, that the Years or Chances of Vitality lost in the ten Years, between the Age of ten and twenty, are no more than  $2\frac{7}{8}$ , as stated in the Scheme, N. II; and that a Life of 20 Years Age is equal to  $32\frac{7}{8}$ . Nay, I may go farther, and assert, that the Case is exactly the same, both with Regard to the Years or Chances lost in that Time, and the Years or Chances remaining at the End of it, even where we use Dr. *Halley's*, or this Author's Rule for computing the Odds or Chances that a Person of ten Years Age does or does not live to the Age of 20 Years, and this too, whether we make such Computation by the whole Period of ten Years collectively, or by each Year singly and separately. Consequently, if a Person of ten Years Age has at that Time an even Chance to a Term of 35 Years, which is a Point admitted on all Hands, and in living ten Years loses no more than  $2\frac{7}{8}$  of those Years, or Chances, the Years or Chances remaining must of Necessity be  $32\frac{7}{8}$ , and his Life at 20 Years Age be equal to that Term: And therefore the even Chance of Vitality for a Life of 20, computed by Dr. *Halley's* and this Author's Rule, and stated as equal to 27 Years only, is very far wide of the Mark. —

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The Upshot of the Matter is this; that in stating these Chances by Dr. *Halley's*, or this Writer's Rule, which is the same, the Decrease of them, in most Instances, especially in the younger Part of Life, is the greatest when the Number of Persons dying is proportionably lowest, and the Decrease the least, when the Number dying is highest; which is directly contrary to the Truth, for no Man, surely, can conceive that these Chances should sink 8 Years in the Period between 10 and 20, in which die only 31, out of 490, that is, at an Average, three only in each Year, and one over in the Whole; and yet, that they should sink no more than three Years in the Period between 40 and 50, in which there die 90 out of 294, that is, at an Average nine in each Year; and at the same Time admit, and it must be admitted, that the Number of Persons dying is the Measure of such Decrease.

In constructing the Tables for these Chances of Vitality, and the Value of Annuities on a single Life depending thereon, I made Use both of the Government Annuities, and *London* Bills of Mortality, as beforementioned but by the Method laid down by myself; and chose here to compute those Chances by Periods of ten Years first, and then divide each Period into single Years at an Average, rather than to compute them by single Years, in Regard that in some one Year or other the Number of Persons dying in that Year might much exceed or fall short of the Number dying in the precedent or subsequent Year, merely  
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## ANNUITIES for LIVES, &amp;c. 65

by Accident, from the greater or less Salubrity of the Air in one Year than another, and this appears to be frequently the Case in the Nominees in the Government Annuities. As for the Chances of the intermediate Years within each Period, I interpolated them by an Equation, as near as could be done, by keeping within the Compass of an eighth Part of a Year, and that there I suppose is as near the Mark as is necessary in these Matters, or can well be put in Practice. To this I have added, and inserted in Table N. IX. a Scheme of the Chances of Vitality for the City of *London*, constructed, according to my own Method, from the Bills of Mortality there; and, on comparing it with the Chances in the general Tables, the Difference does not seem so great, but that in Practice, in most Cases, a Man may safely enough make Use of either of them as he likes best: And, tho' I have not here added the Value of an Annuity on any of these Chances of Life for *London*, yet that Defect is easily supplied by one or other of the precedent Tables.—

The Value of Annuities on a Life given in the general Tables, and that which will arise on a Computation made by Table, N. IX, will at first View and to some Persons seem very high; but, for my own Part, I am fully persuaded, and others, perhaps, when they have well considered what has been here offered to prove it, may be induced to think so too; especially if they admit that the Chances or Probabilities of the Duration of Life are in the Tables rightly estimated. Where

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a great Number of Annuities is granted to Variety of Persons on their Lives, a Variety of Chances must in the Event necessarily come out, and I believe it will be found; that the Estimate here made, at an Average, is a fair and reasonable one, on the Whole: And on single Grants to single Persons there's no Way to form a Judgment of the Value of them, but by accumulating a great many Grants together, and taking a Medium. And in this my Opinion, that I have not overvalued these Annuities for a Life, I am very much confirmed by a late and recent Instance, and which will not soon be forgot by some People, since they have terribly suffered by not well considering or ill calculating the Value of such sort of Estates; and will be a Caution, at least, to Persons to be on their Guard in Transactions of this Nature: *Felix quem faciunt aliena Pericula cautum.*

## ADDENDA.

## ADDENDA.

AFTER writing the preceding Discourse, and indeed since it was prepared and ready for the Press, was put into my Hands Mr. De Moivre's Treatise on this Subject, the third Edition; and having no convenient Opportunity of inserting my Observations upon it in the Body of my Book, and as his Character challenges a special Attention, I hope I shall be excused, if I give my Thoughts upon it in this Place; and they are, in short, that the Rule which he proposes for investigating the Value of Annuities for a single Life is a wrong one, and all the Tables which he has constructed by that Rule, in every Instance, erroneous. My great Objection to this Rule, which I think quite unnecessary to be inserted here, is the same as I made to Dr. Halley's Rule, and is this; *viz.* that in Fact it does, and necessarily must, produce such a Sum as the Value of an Annuity for one and the same individual Life; as that, on the Variation of Interest, the Term to which such Value corresponds shall be a different one.

Now the Defect in the Doctor's Rule which produces these absurd Values arises from hence, that it directs, that out of the Value of an An-



nuity certain for a Year, in each Year of Life and through the whole Life, we are to deduct a Value corresponding to the Chance of Mortality arising in each Year respectively, or, which is much the same Thing, a Value in Proportion to the Degree of Probability that the Life may determine within the Year. Mr. *De Moivre*, in the first Edition of his Treatise on this Subject, P. 5, &c. gives the same Direction in Substance, tho' in Expression something variant; and in Pursuance and under this Direction, P. 11, &c. he computes the Value of an Annuity for the Life of a Person aged 50, and at an Interest of 5%. *per Cent.* and concludes that 'tis in Value 10-15. In this his third Edition, which he calls a more full one, he has thought fit to leave out the Whole of this Direction, and enters immediately upon the Solution of that Problem, *viz.* what may be the Value of an Annuity for the Life of a Person aged 50, and, after a Calculation in the same Method as in his first Edition, concludes that 'tis in Value 10-35. But 'tis impossible that 10-15 and 10-35 should both be the Value of this Annuity; and it happens very unluckily that neither of them are right, for the exact Value is 10-34. But to let this pass, for 'tis not material which of the three, or whether any of them, be the real Value; but it may be a proper Question, what might be the Reason of omitting, in this his fuller Edition, the Whole of this Direction. Som People, perhaps, will solve this Problem by saying, that, since his first Edition, he had discovered

covered that this Direction produced the preposterous Values of Annuities before-mentioned; but that, to be sure, must be an idle Surmise, because, even in this more correct Edition, and in his Tables inserted in it, he has pursued this very same Direction. Others will suggest, that on finding this Objection made to his Rule, and this Direction in particular, to which they are confident he was no Stranger; and that it was too clear and strong for him to grapple with, and most certainly it has never been answered, and probably never will be; he thought it the best Course to remove it out of Sight, as much and as far as he could.

But be that as it will; since I have already, in my *Essay*, treated of this Matter, *viz.* of the absurd Values produced by the Rule, and have farther reinforced my Objections to it in the preceding Treatise; I shall here content myself with asserting, which indeed is very little more than repeating, that 'tis in Nature impossible that one and the same Life can have one and the same Chance or Probability of Duration for different Terms of Years; for Instance, for 25, 26, or 28 Years, as the Rate of Interest, at which the Value of the Annuity is computed, is or may be varied; or indeed on any other Account, or by any other Means whatsoever. I need therefore only to show that the Rule does in Fact produce such Values; and that is seen by a mere Inspection into the Tables constructed by the Author himself, by the Rule, and there they stand thus.

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The Annuity for the Life of a Person of 10 Years Age, at 4*l.* per Cent. is in Value 16-88, which is equal to a Term of  $28\frac{2}{8}$ —of the same Age, at 5*l.* per Cent. is 14-60, which is equal to a Term of  $26\frac{2}{8}$ —and of the same Age, at 6*l.* per Cent. is 12-84, which is equal to a Term of  $25\frac{2}{4}$ . Now I say, and I believe most of Mankind will concur with me, if a Person of 10 Years Age has a Chance or Probability to live only 25 Years and  $\frac{1}{4}$ , that 'tis not possible he should have the same Chance or Probability to live to the Age of  $26\frac{2}{8}$ , or of  $28\frac{2}{8}$ .

If we carry this Matter a Step farther, and compute by this Author's Rule what is the Value and Term corresponding on a like Life, at 3*l.* per Cent. we shall find it to be 19-87, which is equal to a Term of  $30\frac{3}{4}$ ; and 'tis Matter of some Wonderment that this Author, in this his Edition of the Year 1750, has not inserted in his Tables the Value of Annuities for a Life at that Rate of Interest. Since the Government has reduced the Interest on public Securities, which in Stock and Annuities amount to more than fifty Millions, from 4 to  $3\frac{1}{2}$  for seven Years, and from thence to 3; and as the Prospect that the common Price of Money will be at that Rate seems to be no very distant one; I suppose, if private Persons, public Companies, or the Government, were disposed to grant Annuities for Life to any Number of Persons, that they woud hardly think it prudent to charge their Estates or Income with an Interest exceeding 3*l.* per Cent. especially on Securities

Securities not redeemable. On this Account I should have thought that such a Table, now at least, woud have been very proper, and so I have practised myself; but this Author, perhaps, might be apprehensive that such a Table woud have exposed the Error of his Rule too nakedly; for in Truth the Difference between the Term on a Life of 10 Years Age at an Interest of 3*l.* and the Term on the same Life Interest at 6*l.* is five Years and an Half: However, the Tables which he has exhibited have done it sufficiently.

To the Evidence from his Tables we must add, that the Author himself in the Body of the Book, in this last Edition, has inserted something which puts this Matter beyond all Doubt: And, since he has been so good as to have brought in so strong a Reinforcement to support my main Objection, I should be uncivil, if I did not acknowledge and take Notice of his Supply. Let us read them. In p. the 22*d.* he gives us this Problem. *Supposing three equal Lives of any given Age, for Instance 30, and that, upon the Failing of any one of them, that Life shall be immediately replaced (which by the Way has something of the impossible in it) and I then receive a Sum I agreed upon, and that to Perpetuity for me and my Heirs: What is the present Value of that Expectation, and at what Intervals of Time, one with another, may I expect to receive that Sum.*—I have set forth the Problem at large, because the subsequent Remark which he makes, on which I lay my Finger, becomes thereby more plain

plain and intelligible.—The first Question is, *viz.* what is the Value of such Expectancy, after having supposed that the Sum to be paid on these Contingencies was an 100 *l.* he resolves that the present Value is—162 *l.*—Now I very much doubt the Justice of this Solution; for the Sum produced differs considerably from the Sum produced by the Method which I use to compute the Value of such Expectancies, and I may be confident that my Method is a right one, because 'tis founded on the Reason of the Thing, and might be confirmed by Demonstration, and on a Computation in my Manner, if we supposed these Installments to incur once in ten Years for ever, and he supposes them to incur in  $9\frac{3}{4}$ , the Value of this Expectancy comes out to be 181-16-3-1, and near a Farthing more.

But let the true Value of it be what it will: We will now read his Remark on the second Question, *viz.* at what Intervals of Time, one with another, he might expect to receive that Sum: And after solving that Question, in *p.* the 28th, he adds—Remark—*It is not to be expected, if the Interest of Money was higher than 5 per Cent. which was the Interest made made Use of in the preceding Calculations, the Intervals of Time, after which the Renewals are made, would be the same as now; for it will be found, that, as Interest is higher, the Intervals will be shorter, and, as it is lower, the Intervals will be longer: Yet one might make it an Objection to our Rules, that the Length of Life would thereby seem to depend upon the Rate*  
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of Interest. The Author admits then, that 'tis an Objection to his Rules, if they do in Fact make the Length of Life to depend on the Rate of Interest; and, whether he admits it or not, most certainly it is such—The sole Question then will be whether a Variation of the Rate of Interest in this Case does not likewise cause a Variation in the Length of the Lives, as it does in the Length of the Intervals. He lays it down, that the Intervals of Renewal, or the Space of Time between one Renewal and another, will and must vary as the Interest of Money varies, on the one Side or the other. From thence I argue thus: If the Intervals of Renewal are made shorter or longer (as the Case may happen to be) by a Variation of the Rate of Interest, then the Duration of the Lives, on which the Intervals depend, must be shorter or longer in the like Proportion, because the Duration of the Lives is made, and by this Author himself is taken for, the Measure of these Intervals. Or take the Reasoning thus: The Rate of Interest governs the Intervals, the greater or shorter Length of the Lives constitutes, or rather is one and the same Thing as, those Intervals: The Rate of Interest therefore governs the Length of the Lives. This Way of Reasoning is sure enough right and conclusive; but it may be further added, that this Writer's Tables, drawn by himself, and by his own Rule, in the Instances beforementioned, do demonstrate that, in the Case of Annuities for a single Life, the Rate of Interest governs the Length of Life,  
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for, as one is varied, the Length of the other is  
varied with it and by it.

However, since this Writer offers at something  
which he calls an Answer to this Difficulty, let us  
read; and I choose to give it in his own Words,  
because it has something of the extraordinary in  
it, *viz.* a Desire to seem to say something, a Study  
to involve that seeming something in obscure and  
perplex'd Language, and a Care at the same Time  
to say nothing to the Purpose; which last may  
indeed very easily be done in a Matter to which  
nothing can be said to the Purpose. The Words  
are these. *It must be observed, that the calculating  
Time imports no more, than that (considering the  
Circumstances of the Purchaser and Proprietor of the  
Lives, in Respect to the Rate of Interest agreed upon,  
and the Sum to be given upon the Renewal of a Life  
or Lives) the Proprietor makes the same Advantage  
of his Money, as if he had agreed with the Pur-  
chaser, that he should pay him a certain Sum of Mo-  
ney at equal Intervals of Time, for redeeming the  
Risque, which he the Purchaser (read rather the  
Proprietor) runs of paying that Sum when the Life  
or Lives drop; but the real Intervals of Time will  
be shown afterwards.* Part of this Sentence I have  
included in a Parenthesis, so that those Words  
being omitted in the reading, as they may  
very well be, it may more easily be understood;  
and for the Word *Purchaser* we must read *Pro-  
prietor*: And then the Substance of his Answer  
will be, that the Proprietor of the perpetual  
Lease, as I would call it, will make the same Advan-

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Advantage of his Money, if he continues to  
make his Payments on the Contingency of Lives  
dropping, as if he made an Agreement to pay  
the same Sum of Money at the End of a deter-  
minate Number of Years, equal to the computed  
Intervals. But is not the Calculation of the Time  
a just one, and the very same, whatever was the  
Import or Views in making it? Do not the  
Words at the Close of the Sentence, that the real  
Intervals of Time shall be shown afterwards,  
strongly intimate, and suppose that there is a  
real Difference in those Intervals, where there is  
an Alteration in the Interest, otherwise no Oc-  
casion to show them, nor could any be shown?  
Does not the Author himself assert as much, and  
is not the Fact notoriously true? And is it not  
evident, that these Intervals are created by, or are  
one and the same Thing as, the Duration of the  
Lives, and this Author himself measures and de-  
monstrates the Length of the one by the Length  
of the other? To these Questions if the Answer  
be in the Affirmative, and in the Affirmative the  
Answer must be; it will follow, as clearly and  
certainly as any Demonstration in this Writer's  
Book, that the Length of the Lives as well as  
of the Intervals is by his Rule made to vary  
and depend upon the Rate of Interest: And  
thus by one Breath of his Mouth has he blown  
in Pieces all his fine Piece of Machinery.—

If we carry our Inquiry farther and examine  
what Effect his Rule has on the Value of An-  
nuities for joint and surviving Lives, and whe-

ther it does not there likewise make the Duration of Lives depend on the Rate of Interest, we shall find the Case to be the very same: And indeed 'tis impossible but that it should have the same Influence there as in Annuities for a single Life, for the Values of Annuities for two or more Lives, either joint or surviving, are all of them, by an ill-concerted Jumble of Values, form'd from the Values of Annuities for a single Life. If this Author had been so good as to have given us Tables of the Values of Annuities for these Lives, as he has done of those for a single Life, I doubt not but he would have enabled us to have shown in like Manner, that is, out of his own Mouth, that his Values of Annuities for these Lives likewise had the very same Fault: But, since we are deprived of an Opportunity of doing it in that Way, I will try what may be done without that Assistance.

In p. the 54<sup>th</sup>, third Edition, he gives us this Problem, *viz.* to find the Expectation of two Joint Lives, that is, the Time which two Lives may expect to continue together in Being; and then having taken two Lives of the same Age with those given in the Solution of his second Problem, *viz.* one of 40 and the other of 50, he solves the Question, and says, that 13-31 is the Number of Years due to the two Joint Lives. Now if the Expectation or Probability be, that one or other of these two Lives will have a Being for that Term, I humbly apprehend that the Annuity too will subsist for that Term, and at 5 l. per Cent. be of the Value

Value, 9-54. No, by no Means says a Learned Advocate of Mr. De Moivre; for he has demonstrated, in his Solution of the second Problem, and the Demonstration which follows, that the Sum 7-62 is the true Value of the Annuity for these two Joint Lives. The Matter under Consideration here is, whether the Rate of Interest does not govern this Value, and this Value govern the Duration of the two Lives, and not whether the Value be the right one or there be a Demonstration of it, tho' if there were Room for it here, and it were pertinent to the present Purpose, I could show there was none; let us see to what Term of Years this Value corresponds. If we look into this Author's third Table, which is constructed for the Value of Terms for Years at 5 l. per Cent. and that is the Interest which is made Use of in computing the Value of the Annuity for these two Joint Lives; we find 7-72 to be the Value of an Annuity certain for a Term of ten Years; so that this Annuity is not equal to a Term of ten Years, but must determine before that Time, tho' both the two Lives will subsist and continue for thirteen Years and above.—

As I do not design, here, to deprive 'this Gentleman and his learned Friend of the Pleasure of their Conceit (and I can call it by no other Name) that there is Demonstration of this Value, I will admit it to be so for the present, on a Proviso nevertheless and Condition that they return me the like Civility, and admit that there is Demonstration

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tion likewise of the 18th Problem beforementioned, and then we shall have Demonstration fighting against Demonstration; for one says that the Annuity is in Value 7-62, and the other says that it is 9-54, as before computed. Now, if it may be demonstrated, nay, whether demonstrated or not, if it be a reasonable Expectation or Probability, that one or other of these two Lives will subsist for thirteen Years and above, I very much doubt, whether the Learning of Mr. *De Moivre* and his Friends, both put together, will ever convince any Man, unless it be a very special Friend or so, that 'tis probable that this Annuity, which determines, when one of the two Lives fails, and not before, will determine in ten Years Time or less, and at the same be equally probable, that both the Lives will continue in Being thirteen Years and above. *Credat Judæus appella non ego.*

To apply this. Let then the Value of this Annuity be 7-62, or any other, except 9-54, which is the Value when computed on the Probability of the Duration of the Lives; since the Rate of Interest creates this Value, and this Value determines the Duration of these Lives, if that same Rate of Interest does not govern the Length of the Lives, I should be glad to be inform'd what does, since 'tis evident that the Expectation or Probability of the Lives does not; for then the Value would be 9-54, as before computed. But that which will put the Matter beyond all Dispute will be to vary the Rate of Interest, and to calculate the Value of Annuities on two Joint Lives at the  
several

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several Rates of 4, 5, and 6 per Cent, and then examining to what Terms those Values do respectively correspond—On an Annuity for two Joint Lives, each of 45 Years, the Values, and the Terms to which they respectively correspond, as I compute them, stand thus—

At 4 per Cent. Value 8-21, equal to a Term of  $10\frac{1}{3}$   
At 5 per Cent. Value 7-42, equal to a Term of  $9\frac{1}{2}$   
At 6 per Cent. Value 6-76, equal to a Term of  $8\frac{7}{8}$

In Mr. *De Moivre's* Way of computing these Values, they differ a little, and may stand thus—

At 4 per Cent. Value 8-34, equal to a Term of  $10\frac{2}{3}$   
At 5 per Cent. Value 7-72, equal to a Term of 10  
At 6 per Cent. Value 7-06, equal to a Term of  $9\frac{1}{2}$

Now let us take which of the two Valuations we please, 'tis plain and obvious that the Difference in the Rate of Interest produces such a Difference in the Value as creates a Difference in the Term to which the Lives are equal, and this is no other than governing the Length of the Duration of the Lives. It will be said, perhaps, that my Valuation is a wrong one, but, if it should be so, yet the Valuation made in Mr. *De Moivre's* Method justifies the Conclusion; and, if Mr. *De Moivre* has demonstrated that these are the true Values of this Annuity at these three several Rates of Interest, he has then demonstrated the Point in Question, *viz.* that by his Rule the Length of the Lives is made to depend on the Rate of Interest—I say, however, that my Valuation, where 'tis  
made



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made by a Computation on the Values, is a right one, as founded on the Reason of the Thing, and confirm'd by this Author's Solution of his 18th Problem, where he says, that on two Lives of equal Age, the Probability is  $\frac{1}{3}$  of the Complement of any one of the two Lives. Now the Method by which I attain'd the Values before given by me is a very easy and a very obvious one, *viz.* by taking two thirds of the Value of one of the Lives singly taken, or, which is the same Thing, by taking one third of the Values of both put together. On this Account and in this View, som People will be apt to surmise that Mr. *De Moivre* had the same Thing in his View, *viz.* to take two thirds of the Value of one Life, or one third of the Value of both put together, because 'tis consonant to his Reasoning on this 18th Problem; but that he has enveloped the Matter in such Clouds of Multiplications, Subtractions, Divisions, and of Things not subject to the same Rules of increasing or decreasing, that he has made a shift to miss his own Mark: And this most certainly he has done, unless he disclaims his own Problem.

But som<sup>r</sup> will say that on Mr. *De Moivre*'s Side there is Demonstration of the Value, but I say there is no such Thing. His Solution of his second Problem, which produces this Value 7-72, contains no more than this. The Value of an Annuity for two Lives singly taken, being given; then follows a Multiplication of these two Values together, then a Multiplication of this Product by the supposed Rate of Interest, afterwards a Subtraction

## ANNUITIES for LIVES, &amp;c. 81

Subtraction, and lastly a Division; and the Quotient is said to be the Value of the Annuity. Now this Process does indeed demonstrate that the last remaining Quantity is 7-72: But does no more prove that 'tis the Value of an Annuity for these two Joint Lives, than it does, that 'tis the Value of a Mare or a Mule. The Thing which ought to be proved is, that this Process on the given Suppositions will produce such a Sum as is the Value of the Annuity, and not merely what is the Sum which the Process will produce, and the Process carries us no farther. Ay, but then follows a Demonstration; very true, but that still is no more than this, that Letters and Lines are substituted in the room of Numbers or Figures, and upon an Operation on them that the Value produced on such Operation is one and the same, as that produced by the Process, and it would be very strange if it were otherwise; but no Proof yet that this Value is the Value of the Annuity, or that the Process in one or the Operation in the other does produce such a Value. But that's not all: For there is, or seems to be, a Defect running both through the Solution and Demonstration, which must necessarily make them produce a Sum not exactly the true Value. It may indeed produce a Sum which shall come something near the Truth, and he must be a very bungling Operator who could not contrive Matters, so as not to be wide of the Mark; but 'tis a Rule in Mathematics, I think, that if a Man does not hit the Bird in the Eye, as the common saying is,

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he is as much in the wrong, as if he were hundred Miles off.

The Defect I meant is this. In computing and fixing the Value of these two Annuities, when singly taken, he lays it down that the Decrements of Life, or of the Probabilities of the Duration of Life, are in Arithmetic Progression: and then proceeds to multiply these two Values thus fixd, and then multiplies the Product by the given Rate of Interest, *viz.* by 5. The Years or Chances of Life he calls the Complement of Life, and on the whole possible Life states them at 86, asserting that Life cannot reasonably be extended beyond 86; and then, these two Lives being each of the Age of 45, the Complement or Chances of Life resting on each of them will be 41: And then says, that these 41 Chances will decrease in Arithmetic Progression, as the Life proceeds in Age. I will admit that the Chances of Life do decrease in that Proportion, under the same Limitations as he puts upon them; *viz.* that it holds for small Portions of Life, and when the Years are near one to another, but not absolutely for the whole Life, when taken intire. But then, as to the fixd Values of the Annuities on these two Lives, which are each stated at 11-14, there can be no Pretence that they likewise decrease in the like Arithmetic Progression; for, if they did, then at the End of 20 Years and an Half, for Instance, when the Complement or Chances of Life are reduced to Half, then the Value of the Annuity for the Life should be reduced to Half in like Manner; but this

this is notoriously untrue and visible by a meer Inspection into his Tables of these Values, and comparing them, by which it appears that one is 11-14, and the other 7-05; and the Truth of the Case is, that the Decrease in the Values is in geometric Progression. And then as to the Number 5, the supposed Rate of Interest, in that there's no Decrease at all, it being invariably the same through the whole possible Life; what shall we do with this? Why truly I was thinking, since we are upon the decreasing Pin, that the Show, perhaps, would have a more ostentatious Appearance, if, instead of 5, we should take 4-9524, which is the present Value of 5*l.* to be paid at the End of the Year; and we know Interest is not payable till the End of each Year, and in Fact is seldom paid so soon, nor do the Probabilities of Life decrease in their arithmetic Progression, nor the Values in their geometric Proportion, till the End of the Year; and therefore I hold it advisable, in the next Edition, by all Means to make them sink, one and all, and all at one Time. If then we substitute 4-9524 in the Place of 5, there will be an Opportunity of adding som new Numerical and Mathematical Decorations; and besides upon my Word, *aut video, aut vidisse puto*, I see, or think I have seen, and I must not say more, because I have not computed it, nor design ever to compute it, that the Sum forth coming will be nearer the Mark, *viz.* 7-42, for that most certainly is the Value when the Computation is made by Values, as is done by him in this Case.

Now to blend and mix together, as it were in Hotchpot, two Things which decrease in one Proportion with two other Things which decrease in another Proportion, and then to throw in a third Thing, which has no Decrease at all; and from such heterogenous Ingredients to hope that we may extract the Value of a third Thing, is surely an idle and vain Imagination. But if Mr. *De Moivre* thinks otherwise, and will adhere to his Demonstration, he may, for me, and, for the present at least, continue in the peaceable Enjoyment of his beloved Mathematical Proofs, and I will content myself with the humble Productions of plain common Sense and Reason. I have said at present, because the Question here is not so much, whether 7-72 or 7-42 be the true Value of the Annuity for these two Joint Lives, and for my own Part I am satisfied that neither of them be so, as whether on a Variation of the Rate of Interest, in this Method of computing their Values, there be not such Values produced as make a Variance likewise in the Term to which the Lives correspond, and in the Length of the Lives. And, since that Point is evident, I shall not pursue the other any farther here, because it tends rather to show that this Author's Method of investigating the Value of Annuities for Joint Lives is erroneous, and the preceding Discourse treats only of the Value of Annuities for a single Life: And one Time or other, if Health and Leisure will give me Leave, tho' at my Age of Life neither of them are much

to be expected, I may possibly resume the Discussion of the second Matter.

I must say a few Words on the Subject of Annuities for two or more Lives and the Survivor, and I need to say very few, because the Point here is clear, that the Length of the Lives is by the Rule made to depend on the Rate of Interest. On computing, by the Rule, the Value of an Annuity for three Lives and the Survivor, all of the Age of 12, and at different Rates of Interest, the Values, and the Term to which such Values correspond, stand thus—

- At 4 per Cent. Value 23-01, equal to a Term of 65
- At 5 per Cent. Value 19-05, equal to a Term of 63
- At 6 per Cent. Value 16-19, equal to a Term of 61
- At 7 per Cent. Value 13-92, equal to a Term of 58-2
- At 8 per Cent. Value 12-29, equal to a Term of 56

I do not affirm that these Values, all of them, are exactly true; however, they are near enough the Truth to answer the Purpose for which they are here inserted, viz. to show that, whenever we vary the Rate of Interest, we find a Variance likewise in the Term to which the Lives are equal; that is, that the Rate of Interest governs and determines the Length of the Lives; and it appears here that, between the highest and lowest Interest, the Difference in the Term is nine Years. This wants no farther Comment, so I shall add only a short Observation, tho', short as it is, it trenches deep into the Scheme, and all the Rules which Mr. *De Moivre* has contrived for investigating the Value

86     *A VALUATION of, &c.*

Value of Annuities for one or more Lives, of any Kind, or in any Shape; and my Remark is this. On computing, by his Scheme and his Rules, the Value of Annuities for a single Life, and for two or more Joint Lives, the Values come out to be such as correspond to a Term below the even Chance, or Probability of the Duration of the Life or Lives: And on the contrary, on computing the Value of Annuities for two or more Lives and the Survivor, the Values are always such as correspond to a Term above the even Chance, or Probability of the Duration of the Lives, or the Survivor of them. And I will say here, as I said in my *Essay*, on much the like Occurrence, that this is a Contradiction to common Sense and Reason, and leave it to Mr. *De Moivre* to reconcile it if he can; but I bar all Proofs from Mathematical Schemes, unless he take special Care that his Data and Postulate are all right and clear.



T A B L E S  
Of the VALUE of  
A N N U I T I E S

A N D

L E A S E S certain,

For Years, and for a single Life, at every  
Age of Life.

At the the Rates of 3, 4, 5, and 6 *per*  
*Cent.* Interest.



TABLE I.

Showing the Value of Annuities and Leases certain, for any Term of Years, in Years, and in the eighth Part, and hundredth Part of a Year. Interest, three per Cent.

Term of Years.	Value in th th. Y. 8 100	Term.	Value in th th. Y. 8 100	Term.	Value in th th. Y. 8 100
1	00-7-97	26	17-6-86	51	25-6-86
2	01-7-91	27	18-2-31	60	27-5-68
3	02-6-83	28	18-6-75	70	29-1-12
4	03-5-61	29	19-1-17	80	30-1-21
5	04-4-58	30	19-4-58	90	31---
6	05-3-42	31	19-7-98	100	31-4-60
7	06-2-24	32	20-3-37	Fee	33-2-33
8	07--03	33	20-6-75		
9	07-6-81	34	21-1-12		
10	08-4-56	35	21-3-48		
11	09-2-29	36	21-6-83		
12	10---	37	22-1-17		
13	10-5-67	38	22-4-19		
14	11-2-33	39	22-6-79		
15	11-7-97	40	23--11		
16	12-4-59	41	23-3-42		
17	13-1-19	42	23-5-71		
18	13-6-78	43	24---		
19	14-2-35	44	24-2-27		
20	14-7-89	45	24-4-53		
21	15-3-42	46	24-6-78		
22	15-7-94	47	25--02		
23	16-3-44	48	25-2-25		
24	16-7-92	49	25-3-47		
25	17-3-40	50	25-5-67		

TABLE II.

Showing the Term to which Annuities and Leases certain, for a single Life, are computed to be equal; and the Value of such Annuities and Leases, in Years, and the eighth Part of a Year. Interest, three per Cent.

Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.
1	25--	17-3	26	27-4	18-4	51	15-3	12-1
2	28--	18-6	27	26-7	18-2	52	15--	11-7
3	31--	19-7	28	26-2	17-7	53	14-4	11-5
4	33--	20-6	29	25-5	17-5	54	14--	11-2
5	34-4	21-2	30	25--	17-3	55	13-4	11--
6	35--	21-3	31	24-4	17-1	56	13--	10-5
7	35--	21-3	32	24--	16-7	57	12-4	10-3
8	35--	21-3	33	23-4	16-5	58	12--	10--
9	35--	21-3	34	23--	16-3	59	11-4	9-5
10	35--	21-3	35	22-4	16-1	60	11--	9-2
11	34-4	21-2	36	22--	15-7	61	10-4	8-7
12	34--	21-1	37	21-4	15-5	62	10--	8-4
13	33-5	20-7	38	21--	15-3	63	9-4	8-1
14	33-2	20-6	39	20-4	15-1	64	9--	7-6
15	32-7	20-5	40	20--	14-7	65	8-4	7-3
16	32-4	20-4	41	19-4	14-5	66	8--	7--
17	32-1	20-3	42	19--	14-3	67	7-3	6-4
18	31-6	20-2	43	18-4	14--	68	6-6	6--
19	31-3	20-1	44	18--	13-6	69	6-1	5-4
20	31--	19-7	45	17-5	13-4	70	5-4	5--
21	30-4	19-6	46	17-2	13-2	75	4-2	3-7
22	30--	19-5	47	16-7	13--	80	3--	2-6
23	29-3	19-3	48	16-4	12-7	85	2--	1-7
24	28-6	19--	49	16-1	12-5	90	1--	0-7
25	28-1	18-6	50	15-6	12-3	&c.		

T A B L E III.

Showing the Value of Annuities and Leases certain, for any Term of Years; in Years, and in the eighth Part, and hundredth Part of a Year. Interest, four per Cent.

Term of Years.	Value in th. 8. 100.	Term.	Value in th. 8. 100.	Term.	Value in th. 8. 100.
1	00-7-96	26	15-7-97	51	21-4-60
2	01-7-88	27	16-2-31	60	22-4-61
3	02-6-78	28	16-5-64	70	23-3-39
4	03-5-62	29	16-7-97	80	23-7-90
5	04-3-44	30	17-2-29	90	24-2-30
6	05-1-23	31	17-4-57	100	24-4-50
7	06-—	32	17-6-85	Fee	25-—
8	06-5-72	33	18-—11		
9	07-3-42	34	18-3-37		
10	08-—10	35	18-5-62		
11	08-6-75	36	18-6-86		
12	09-3-37	37	19-—10		
13	09-7-97	38	19-2-33		
14	10-4-55	39	19-4-56		
15	11-—10	40	19-6-78		
16	11-5-64	41	20-—		
17	12-1-15	42	20-1-20		
18	12-5-64	43	20-3-39		
19	13-1-12	44	20-4-57		
20	13-4-59	45	20-5-73		
21	14-—01	46	20-7-90		
22	14-3-46	47	21-—05		
23	14-6-84	48	21-1-20		
24	15-2-25	49	21-2-34		
25	15-4-61	50	21-3-47		

T A B L E IV.

Showing the Term to which Annuities and Leases certain, for a single Life, are computed to be equal; and the Value of such Annuities and Leases, in Years, and the eighth Part of a Year. Interest, four per Cent.

Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.
1	25-—	15-3	26	27-4	16-3	51	15-3	11-2
2	28-—	16-5	27	26-7	16-2	52	15-—	11-—
3	31-—	17-4	28	26-2	16-—	53	14-4	10-6
4	33-—	18-—	29	25-5	15-6	54	14-—	10-4
5	34-4	18-4	30	25-—	15-4	55	13-4	10-2
6	35-—	18-5	31	24-4	15-3	56	13-—	9-7
7	35-—	18-5	32	24-—	15-2	57	12-4	9-5
8	35-—	18-5	33	23-4	15-—	58	12-—	9-3
9	35-—	18-5	34	23-—	14-6	59	11-4	9-—
10	35-—	18-5	35	22-4	14-5	60	11-—	8-6
11	34-4	18-4	36	22-—	14-3	61	10-4	8-3
12	34-—	18-3	37	21-4	14-2	62	10-—	8-—
13	33-5	18-2	38	21-—	14-—	63	9-4	7-6
14	33-2	18-1	39	20-4	13-6	64	9-—	7-3
15	32-7	18-—	40	20-—	13-4	65	8-4	7-—
16	32-4	18-—	41	19-4	13-2	66	8-—	6-5
17	32-1	17-7	42	19-—	13-1	67	7-3	6-2
18	31-6	17-6	43	18-4	12-7	68	6-6	5-6
19	31-3	17-5	44	18-—	12-5	69	6-1	5-2
20	31-—	17-4	45	17-5	12-3	70	5-4	4-6
21	30-4	17-3	46	17-2	12-2	75	4-2	3-6
22	30-—	17-2	47	16-7	12-—	80	3-—	2-6
23	29-3	17-1	48	16-4	11-7	85	2-—	1-7
24	28-6	16-7	49	16-1	11-5	90	1-—	0-7
25	28-1	16-5	50	15-6	11-4	&c.		



T A B L E V.

Showing the Value of Annuities and Leafes certain, for any Term of Years; in Years, and in the eighth Part, and hundredth Part of a Year. Interest, five *per Cent.*

Term of Years.	Value in th th. Y. 8 100	Term	Value in th th. Y. 8 100	Term	Value in th th. Y. 8 100
1	00-7-95	26	14-3-37	51	18-2-33
2	01-6-86	27	14-5-64	60	18-7-93
3	02-5-72	28	14-7-89	70	19-2-34
4	03-4-54	29	15-1-13	80	19-4-59
5	04-2-33	30	15-2-36	90	19-6-75
6	05-07	31	15-4-59	100	19-6-85
7	05-6-79	32	15-6-80	Fee	20
8	06-3-46	33	16		
9	07-11	34	16-1-19		
10	07-5-71	35	16-3-37		
11	08-2-31	36	16-4-54		
12	08-7-87	37	16-5-70		
13	09-3-39	38	16-6-86		
14	09-7-89	39	17-0-01		
15	10-3-38	40	17-1-15		
16	10-6-84	41	17-2-30		
17	11-2-27	42	17-3-43		
18	11-5-68	43	17-4-53		
19	12	44	17-5-60		
20	12-3-46	45	17-6-77		
21	12-6-82	46	17-7-88		
22	13-1-16	47	17-7-98		
23	13-3-48	48	18-0-08		
24	13-6-79	49	18-1-17		
25	14-0-09	50	18-2-25		

T A B L E VI.

Showing the Term to which Annuities and Leafes certain, for a single Life, are computed to be equal; and the Value of such Annuities and Leafes, in Years, and the eighth Part of a Year. Interest, five *per Cent.*

Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.
1	25--	14--	20	27-4	14-6	51	15-3	10-4
2	28--	14-7	27	26-7	14-5	52	15--	10-3
3	31--	15-4	28	26-2	14-3	53	14-4	10-1
4	33--	16--	29	25-5	14-2	54	14--	9-7
5	34-4	16-2	30	25--	14--	55	13-4	9-5
6	35--	16-3	31	24-4	13-7	56	13--	9-3
7	35--	16-3	32	24--	13-6	57	12-4	9-1
8	35--	16-3	33	23-4	13-5	58	12--	8-7
9	35--	16-3	34	23--	13-4	59	11-4	8-4
10	35--	16-3	35	22-4	13-3	60	11--	8-2
11	34-4	16-2	36	22--	13-1	61	10-4	8--
12	34--	16-1	37	21-4	12-7	62	10--	7-5
13	33-5	16--	38	21--	12-6	63	9-4	7-3
14	33-2	16--	39	20-4	12-5	64	9--	7--
15	34-7	15-7	40	20--	12-3	65	8-4	6-6
16	32-4	15-7	41	19-4	12-2	66	8--	6-3
17	32-1	15-6	42	19--	12--	67	7-3	6--
18	31-6	15-5	43	18-4	11-7	68	6-6	5-4
19	31-3	15-5	44	18--	11-5	69	6-1	5-1
20	31--	15-4	45	17-5	11-4	70	5-4	4-5
21	30-4	15-3	46	17-2	11-3	75	4-2	3-5
22	30--	15-2	47	16-7	11-1	80	3--	2-5
23	29-3	15-1	48	16-4	11--	85	2--	1-6
24	28-6	15--	49	16-1	10-7	90	1--	0-7
25	28-1	14-7	50	15-6	10-5	&c.		

T A B L E VII.

Showing the Value of Annuities and Leases certain, for any Term of Years, in Years, and in the eighth Part, and hundredth Part of a Year. Interest, six per Cent.

Term of Years.	Value in th. 8 100	Term.	Value in th. 8 100	Term.	Value in th. 8 100
1	00-7-94	26	13—	51	15-6-79
2	01-6-83	27	13-1-21	60	16-1-17
3	02-5-67	28	13-3-40	70	16-3-39
4	03-3-46	29	13-4-58	80	16-4-50
5	04-1-21	30	13-6-76	90	16-4-58
6	04-7-92	31	13-7-93	100	16-4-61
7	05-4-58	32	14—09	Fee	16-5-67
8	06-1-21	33	14-1-23		
9	06-6-80	34	14-3-38		
10	07-2-36	35	14-4-51		
11	07-7-88	36	14-5-63		
12	08-3-38	37	14-6-74		
13	08-6-85	38	14-6-84		
14	09-2-29	39	14-7-94		
15	09-5-71	40	15—03		
16	10—10	41	15-1-12		
17	10-3-47	42	15-1-20		
18	10-6-83	43	15-2-28		
19	11-1-16	44	15-2-36		
20	11-3-47	45	15-3-43		
21	11-6-76	46	15-4-50		
22	12—03	47	15-4-56		
23	12-2-30	48	15-5-62		
24	12-4-55	49	15-5-68		
25	12-6-78	50	15-6-74		

T A B L E VIII.

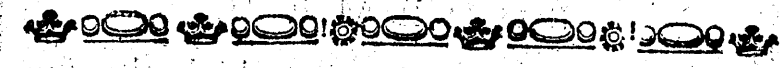
Showing the Term to which Annuities and Leases certain, for a single Life, are computed to be equal; and the Value of such Annuities and Leases, in Years, and the eighth Part of a Year. Interest, six per Cent.

Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.
1	25—	12-6	26	27-4	13-2	51	15-3	9-6
2	28—	13-3	27	26-7	13-1	52	15—	9-5
3	31—	13-7	28	26-2	13—	53	14-4	9-4
4	33—	14-1	29	25-5	12-7	54	14—	9-2
5	34-4	14-3	30	25—	12-6	55	13-4	9—
6	35—	14-4	31	24-4	12-5	56	13—	8-6
7	35—	14-4	32	24—	12-4	57	12-4	8-4
8	35—	14-4	33	23-4	12-3	58	12—	8-3
9	35—	14-4	34	23—	12-2	59	11-4	8-1
10	35—	14-4	35	22-4	12-1	60	11—	7-7
11	34-4	14-3	36	22—	12—	61	10-4	7-5
12	34—	14-3	37	21-4	11-7	62	10—	7-2
13	33-5	14-2	38	21—	11-6	63	9-4	7—
14	33-2	14-2	39	20-4	11-5	64	9—	6-6
15	32-7	14-1	40	20—	11-3	65	8-4	6-4
16	32-4	14-1	41	19-4	11-2	66	8—	6-1
17	32-1	14—	42	19—	11—	67	7-3	5-6
18	31-6	14—	43	18-4	10-7	68	6-6	5-3
19	31-3	13-7	44	18—	10-6	69	6-1	5—
20	31—	13-7	45	17-5	10-5	70	5-4	4-5
21	30-4	13-6	46	17-2	10-4	75	4-2	3-4
22	30—	13-6	47	16-7	10-3	80	3—	2-5
23	29-3	13-5	48	16-4	10-2	85	2—	1-6
24	28-6	13-4	49	16-1	10-1	90	1—	0-7
25	28-1	13-3	50	15-6	10—	£c.		

T A B L E IX.

Showing the Term in Years, and the eighth Part of a Year, to which Lives of all Ages in the City of London are computed to be equal.

Ages.	Y.	8th.	Ages.	Y.	8th.	Ages.	Y.	8th.
1	25	-	26	30	-	51	14	-2
2	28	-	27	29	-4	52	13	-6
3	31	-	28	28	-7	53	13	-2
4	33	-	29	28	-2	54	12	-6
5	34	-4	30	27	-5	55	12	-2
6	35	-	31	27	-	56	11	-6
7	35	-	32	26	-3	57	11	-2
8	35	-	33	25	-6	58	10	-6
9	35	-	34	25	-1	59	10	-2
10	35	-	35	24	-4	60	9	-6
11	34	-5	36	23	-7	61	9	-2
12	34	-2	37	23	-2	62	8	-6
13	34	-	38	22	-5	63	8	-2
14	33	-6	39	21	-7	64	7	-6
15	33	-5	40	21	-1	65	7	-3
16	33	-4	41	20	-3	66	7	-
17	33	-3	42	19	-6	67	6	-5
18	33	-2	43	19	-1	68	6	-2
19	33	-1	44	18	-4	69	5	-7
20	32	-7	45	17	-7	70	5	-4
21	32	-4	46	17	-2	80	2	-7
22	32	-	47	16	-5	90	1	-1
23	31	-4	48	16	-	&c.		
24	31	-	49	15	-3			
25	30	-4	50	14	-6			



T W O  
S C H E M E S:

The F I R S T,

Showing the Proportion in which the supposed 35 Years or Chances of Vitality, on the Life of a Person of ten Years Age, decrease in each Period of ten Years; as stated by Mr. Hodgson on the London Bills of Mortality, p. 9, &c.

The S E C O N D,

As computed by Mr. Lee on the same London Bills of Mortality.



S C H E M E I.

Showing the Proportion in which the supposed 35 Years or Chances of Vitality, on the Life of a Person of ten Years Age, decrease in each Period of ten Years; as stated by Mr. Hodgson on the London Bills of Mortality, p. 9, &c.

From	to	Y.	8th.
10	to 20	8	-
20	to 30	5	4
30	to 40	4	-
40	to 50	3	-
50	to 60	3	4
60	to 70	2	4
70	to 80	3	4
80	to 90	2	-
90	and upwards	3	-
		35	-

S C H E M E II.

Showing the Proportion in which the supposed 35 Years or Chances of Vitality, on the Life of a Person of ten Years Age, decrease in each Period of ten Years; as computed by Mr. Lee on the same London Bills of Mortality.

From	to	Y.	8th.
10	to 20	2	1
20	to 30	5	2
30	to 40	6	4
40	to 50	6	3
50	to 60	5	-
60	to 70	4	3
70	to 80	2	6
80	to 90	1	4
90	and upwards	1	1
		35	-

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TABLE

Showing the proportion in which the total amount of ... on the ... of ... in each ... of ... computed by ... on the ...

1	10	10	10
2	10	10	10
3	10	10	10
4	10	10	10
5	10	10	10
6	10	10	10
7	10	10	10
8	10	10	10
9	10	10	10
10	10	10	10
11	10	10	10
12	10	10	10
13	10	10	10
14	10	10	10
15	10	10	10
16	10	10	10
17	10	10	10
18	10	10	10
19	10	10	10
20	10	10	10
21	10	10	10
22	10	10	10
23	10	10	10
24	10	10	10
25	10	10	10
26	10	10	10
27	10	10	10
28	10	10	10
29	10	10	10
30	10	10	10
31	10	10	10
32	10	10	10
33	10	10	10
34	10	10	10
35	10	10	10
36	10	10	10
37	10	10	10
38	10	10	10
39	10	10	10
40	10	10	10
41	10	10	10
42	10	10	10
43	10	10	10
44	10	10	10
45	10	10	10
46	10	10	10
47	10	10	10
48	10	10	10
49	10	10	10
50	10	10	10
51	10	10	10
52	10	10	10
53	10	10	10
54	10	10	10
55	10	10	10
56	10	10	10
57	10	10	10
58	10	10	10
59	10	10	10
60	10	10	10
61	10	10	10
62	10	10	10
63	10	10	10
64	10	10	10
65	10	10	10
66	10	10	10
67	10	10	10
68	10	10	10
69	10	10	10
70	10	10	10
71	10	10	10
72	10	10	10
73	10	10	10
74	10	10	10
75	10	10	10
76	10	10	10
77	10	10	10
78	10	10	10
79	10	10	10
80	10	10	10
81	10	10	10
82	10	10	10
83	10	10	10
84	10	10	10
85	10	10	10
86	10	10	10
87	10	10	10
88	10	10	10
89	10	10	10
90	10	10	10
91	10	10	10
92	10	10	10
93	10	10	10
94	10	10	10
95	10	10	10
96	10	10	10
97	10	10	10
98	10	10	10
99	10	10	10
100	10	10	10