

# Inflation Persistence Differentials among Euro Area Countries\*

Masao Kumamoto<sup>#</sup>

## Abstract

The inflation differentials problem has been prominent in European economic debates. Since the ECB's monetary policy is directed at inflation rates in the euro area as a whole, persistent inflation differentials among euro area countries would cause the persistent real interest rate differentials. A popular argument claims that a unified monetary policy may be excessively tight for lower inflation countries and loose for higher inflation countries, and then, which would increase economic divergence among euro area countries. One of the sources of the persistent inflation differentials is the differences in degree of inflation persistence. In this paper, we will focus on the inflation persistence as the source of inflation differentials and we will estimate the degree of inflation persistence in lower inflation countries and higher inflation countries.

We introduce a closed economy DSGE model and estimate it using Bayesian estimation methods. We will show that intrinsic persistence is moderate and the differences in degree of intrinsic persistence are small between both groups. The degree of differences in expectation-based persistence is also small between both groups, which means that the frequency of firm's price adjustment is uniform across the euro area countries. On the other hand, extrinsic persistence is higher in lower inflation countries, while lower in higher inflation countries. It means that that inflation rates would rise higher in lower inflation countries, where GDP gap would be small due to the tight monetary policy, while inflation rates would rise lower in higher inflation countries, where GDP gap would be large due to the loose monetary policy. These adjustment mechanisms would eventually drive inflation differentials back to equilibrium. The impulse response functions analysis shows that the unified monetary policy would have similar effects among the euro area countries.

From these results, we can conclude that intrinsic and extrinsic persistence in

inflation rates are not the source of inflation differentials and it would not hamper the symmetric transmission of single monetary policy.

**JEL classification:** E31, E32, E52

**Keywords:** Euro Area, Inflation Differentials, Inflation Persistence, Small Open Economy DSGE Model, Bayesian Estimation

## 1. Introduction

The inflation differentials problem has been prominent in European economic debates. A unified monetary policy has been adopted among the euro area countries since the third stage of Economic and Monetary Union (EMU) started on 1 January 1999. A single monetary policy is conducted in the Eurosystem, which composes of the European Central Bank (ECB) and the 17 national central banks of the euro area countries. A primary objective of the ECB's monetary policy is to maintain inflation rates in the euro area as a whole below, but close to, 2 percent over the medium term. Since the ECB's monetary policy is directed at inflation rates in the euro area as a whole, persistent inflation differentials among euro area countries would cause the persistent real interest rate differentials in the situation where all countries face the same nominal interest rate. A popular argument claims that a unified monetary policy may be excessively tight for lower inflation countries and loose for higher inflation countries, and then, which would increase economic divergence among euro area countries.

There are many sources of the persistent inflation differentials. First, differences in cyclical positions and/or economic structures, which create the asymmetric shocks, would contribute to the persistent inflation differentials. Second, differences in productivity trend would also contribute to the persistent inflation differentials. It is known as Balassa-Samuelson effects. Countries with higher productivity growth in the tradable goods sector relative to non-tradable goods sector will have higher inflation rates. Third, the differences in degree of inflation persistence would contribute to the persistent inflation differentials. In this paper, we will focus on the inflation persistence as the source of inflation differentials.

According to Inflation Persistent Network (Altissimo, *et al.* (2006), henceforce IPN), inflation persistence refers to the tendency of inflation to converge slowly towards its long-run value, following a shock which has led inflation away from its long-run value. IPN discusses that there are three factors which contribute to the inflation persistence and these three factors can be captured well by hybrid New Keynesian Phillips Curve (henceforce, NKPC).

$$\pi_t = \gamma\pi_{t-1} + (1 - \gamma)E_t\pi_{t+1} + \kappa y_t$$

First factor is called intrinsic persistence, which means the dependence of inflation on its own past. It can be captured well by the first term of the NKPC. In theoretical framework, such dependence can be explained in two ways. The first explanation is the rule of thumb behavior of price setters as pointed out by Gali and Gertler (1999). Second explanation is backward-looking indexation proposed by Christiano, Eichenbaum and Evans (2005). In their model, firms can set new optimal price with certain probability in each period, and firms which cannot set new optimal price simply index their price to lagged inflation rates. An increase in the fraction of firms that set prices in the backward-looking fashion raises inflation persistence. Therefore, larger value of the coefficient on  $\pi_{t-1}$  means the higher intrinsic persistence.

Second factor is called expectation-based persistence, which means the persistence due to the formation of inflation expectation. It can be captured well by the second term of NKPC. In theoretical framework, it can be explained in two ways. First explanation is adaptive learning proposed by Milani (2005). When price setters do not know the precise structure of the economy, especially the precise structure of inflation process, they would use simple reduced form auto-regressions to forecast inflation. This type of learning makes the inflation persistence. The second explanation is sticky information proposed by Mankiw and Reis (2002). The essence of their model is that information about macroeconomic conditions diffuses slowly. This slow diffusion could arise because of either costs of acquiring information or costs of re-optimization. In either case, although prices are always changing, pricing decisions are not always based on current information. However, under the assumption of rational expectations, inflation expectations would not contribute to the persistence of the inflation process.

Third factor is called extrinsic persistence, which is derived from persistent changes in the determinants of inflation such as the output gap or marginal costs. The more rigid prices are, the less responsive inflation becomes to change in these determinants, which means the lower value of the coefficient on  $y_t$ .

The purpose of this paper is to estimate the degree of inflation persistence among euro area countries. We will divide our sample countries into two groups; lower inflation countries and higher inflation countries. Estimating the differences in degree of inflation persistence between both groups is useful to investigate the sources of the inflation rates differentials and to investigate whether inflation differentials among euro area countries would decrease or not in the future.

The remainder of this paper is organized as follows. In Section 2, we will introduce the closed economy DSGE model. In Section 3, we will estimate our dynamic stochastic general

equilibrium (hence force, DSGE) model using Bayesian estimation methods. Section 4 is a conclusion.

## 2. Model

Our model is based on recent DSGE models, which have a standard New Keynesian framework with monopolistically competitive market structure and staggered price setting<sup>1)</sup>. They were developed by Rotemberg and Woodford (1998), Christiano, Eichenbaum and Evans (1999), Erceg, Henderson and Levin (2000), Smets and Wouters (2002) and Steinsson (2003).

We consider a closed economy composed of households and firms. Infinitely-lived households maximize a utility function which depends positively on consumption of goods relative to an external habit variable and negatively on labor supply. For simplicity, we assume homogeneity of labor service of each household. Firms set their prices in staggered contracts in Calvo's (1983) fashion. In addition, we assume the existence of backward-looking firms which use a simple rule-of-thumb when they set prices.

### 2.1 Households

There is a continuum of infinitely-lived households. Each household has a utility function which depends positively on consumption of goods relative to an external habit variable and negatively on labor supply. A representative household maximizes its utility function given by

$$\max_{\{C_t, N_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \quad (1)$$

where  $\beta$  is a discount factor,  $\sigma$  is the inverse elasticity of intertemporal substitution,  $\varphi$  is the inverse elasticity of labor supply, and  $N_t$  denotes total hours of labor supply.  $C_t$  is a consumption index defined by familiar the Dixit-Stiglitz form,

$$C_t = \left[ \int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (2)$$

where  $C_t(i)$  is household consumption of good  $i \in [0,1]$ , and  $\varepsilon > 1$  is the elasticity of substitution between types of differentiated goods.  $H_t$  is an external habit variables and it is assumed to be proportional to aggregate past consumption and it is not affected by any single household. Thus,

$$H_t = hC_{t-1} \quad (3)$$

A representative household faces the inter-temporal budget constraint given by

$$\int_0^1 P_t(i)C_t(i)di + E_t[Q_{t,t+1}D_{t+1}] \leq D_t + W_tN_t + T_t \quad (4)$$

where  $P_t(i)$  denotes the price of home good  $i$ ,  $W_t$  is the nominal wage rate,  $D_t$  is the nominal pay off held at the end of period  $t-1$ , and  $Q_{t,t+1}$  is the stochastic discount factor between period  $t$  and  $t+1$ .

In each period, a household faces two utility-maximizing problems, namely, (i) how much to consume the consumption index  $C_t$  and (ii) how much to consume each individual differentiated good  $C_t(i)$ .

As for the problem (ii), the optimal allocation of expenditure within each category of goods is given by,

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t \quad (5)$$

where  $P_t$  denotes the price index of home good and it is defined so as to equal to a minimum expenditure for which a unit of  $C_t$  can be purchased, and it can be derived as

$$P_t = \left[ \int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \quad (6)$$

Note that combining the optimality conditions in equation (5) with the consumption index in equation (2) and price index in equation (6) yields

$$\int_0^1 P_t(i)C_t(i)di = P_tC_t$$

From the above result, we can rewritten the inter-temporal budget constraint in equation (4) as

$$P_tC_t + E_t[Q_{t,t+1}D_{t+1}] \leq D_t + W_tN_t + T_t \quad (7)$$

Therefore, as for the problem (i), we form the following Lagrangian,

$$L_t = E_t \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{C_t - hC_{t-1}}{1-\sigma} \right)^{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] + \lambda_t (D_t + W_tN_t + T_t - P_tC_t - E_t[Q_{t,t+1}D_{t+1}])$$

and solve the household's optimizing problem to obtain the following first order conditions:

$$(C_t - hC_{t-1})^\sigma N_t^\varphi = \frac{W_t}{P_t} \quad (8)$$

$$\beta R_t E_t \left[ \left( \frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right] = 1 \quad (9)$$

where  $\lambda_t$  is the Lagrangian multiplier and  $R_t \equiv 1/E_t[Q_{t,t+1}]$  is the gross nominal return on a risk-free one period discount bond paying off one unit of domestic currency in  $t+1$ .

### Log-linear approximations

Log-linear approximation of (8) and (9) can be written as:

$$w_t - p_t = \frac{\sigma}{1-h} (c_t - hc_{t-1}) + \varphi n_t \quad (10)$$

$$c_t = \frac{h}{1+h} c_{t-1} + \frac{1}{1+h} E_t[c_{t+1}] - \frac{1}{\sigma(1+h)} (r_t - E_t[\pi_{t+1}] - \rho) \quad (11)$$

where lower case letter denotes the percentage deviation of the corresponding variable around its steady state,  $r_t \equiv \log(R_t)$  is the nominal interest rate,  $\pi_{t+1} \equiv p_{t+1} - p_t$  is the CPI inflation rate between  $t$  and  $t+1$ , and  $\rho \equiv -\log \beta$ .

### 2.2 Firms

We assume a continuum of monopolistically competitive firms indexed by  $i \in [0,1]$  and they produce differentiated goods. For simplicity, we ignore capital as inputs and assume the constant return to scale production function,

$$Y_t(i) = A_t N_t(i) \quad (12)$$

where  $A_t$  represents productivity shock.

Before analyzing the firm's pricing decision, we will consider its cost minimization problem, which is specified as the following Lagrangian equation:

$$L_t \equiv \frac{W_t}{P_t} N_t(i) + \Phi_t(i) (A_t N_t(i) - Y_t(i)) \quad (13)$$

From the first order condition, we can see that real marginal cost equals across firms and equals to Lagrangian multiplier,

$$\frac{W_t/P_t}{A_t} = \Phi_t(i) = \Phi_t \quad (14)$$

Following Calvo (1983), we assume that a fraction  $1-\theta$  of firms can set a new price in each period. In other words, each firm is able to set a new price with probability  $1-\theta$  in each period. The probability that it will be allowed to reset its price in any period does not depend on how long its existing contract has been in effect. With probability  $\theta$ , it cannot change price so that its price is remained at  $P_{t-1}$ . Moreover, following Galí and Gertler (1999), there exist two types of the firms in the economy when it comes to pricing decisions. A fraction  $1-\omega$  of firms, which we refer to "forward-looking" firms behave optimally, and the remaining firms of fraction  $\omega$ , which we refer to "backward-looking" firms, use a simple rule-of-thumb when they set their prices.

Therefore, the price index in equation (6) can be rewritten as

$$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\omega P_t(b)^{1-\varepsilon} + (1-\theta)(1-\omega)P_t(f)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \quad (15)$$

where  $P_t(b)$  and  $P_t(f)$  are the new prices set by backward-looking firms and forward-looking firms, respectively.

We assume that backward-looking firms set their prices according to the following rule,

$$P_t(b) = P_{t-1}^n \Pi_{t-1} \quad (16)$$

where  $\Pi_{t-1} = P_t/P_{t-1}$  and  $P_t^n$  is the index of newly set prices defined by

$$P_t^n = P_t(b)^\omega P_t(f)^{1-\omega} \quad (17)$$

Since production functions are homogeneous so that average cost equals marginal cost, the present discounted value of forward-looking firms' profit can be written as

$$\max_{\{P_t(f)\}} E_t \left[ \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \{ (P_t(f) - P_{t+k} \Phi_{t+k}) Y_{t+k}(f) \} \right] \quad (18)$$

where  $Q_{t,t+k} \equiv \beta^k (C_{t+k} - hC_{t+k-1}/C_t - hC_{t-1})^{-\sigma} (P_t/P_{t+k})$  is the stochastic discount factor and  $Y_t(f)$  is the output of representative forward-looking firms.

In the equilibrium,  $Y_t(i) = C_t(i)$  must hold, and  $C_t(i)$  is given by equation (5). Therefore, we can rewrite equation (18) as

$$\max_{\{P_t(f)\}} E_t \left[ \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k} P_{t+k} \left\{ \left( \frac{P_t(f)}{P_{t+k}} \right)^{1-\varepsilon} - \Phi_{t+k} \left( \frac{P_t(f)}{P_{t+k}} \right)^{-\varepsilon} \right\} \right] \quad (19)$$

Hence, the first order condition for profit maximization of forward-looking firms in each sector is given by

$$\frac{P_t(f)}{P_t} E_t \left[ \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left( \frac{P_{t+k}}{P_t} \right)^{\varepsilon-1} Y_{t+k} \right] = \frac{\varepsilon}{\varepsilon-1} E_t \left[ \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \Phi_{t+k} \left( \frac{P_{t+k}}{P_t} \right)^{\varepsilon} Y_{t+k} \right] \quad (20)$$

### Log-linear approximations

Log-linearizing equations (12) and (14) yields the following equations:

$$y_t = a_t + n_t \quad (21)$$

$$\phi_t = (w_t - p_t) - a_t \quad (22)$$

Next, we log-linearize equations (15), (16), (17) and (20) to obtain the following equations:

$$\pi_t = \frac{1-\theta}{\theta} \{ \omega \hat{p}_t(b) + (1-\omega) \hat{p}_t(f) \} \quad (23)$$

$$\hat{p}_t(b) = \hat{p}_{t-1}^n - \pi_t + \pi_{t-1} \quad (24)$$

$$\hat{p}_t^n = \omega \hat{p}_t(b) + (1-\omega) \hat{p}_t(f) \quad (25)$$

$$\hat{p}_t(f) = (1-\beta\theta)\phi_t + \beta\theta E_t[\hat{p}_{t+1}(f)] + \beta\theta E_t[\pi_{t+1}] \quad (26)$$

where  $\hat{p}_t(b)$ ,  $\hat{p}_t(f)$  and  $\hat{p}_t^n$  denote percent deviations of  $P_t(b)/P_t$ ,  $P_t(f)/P_t$  and  $P_t^n/P_t$  from their

steady state values, respectively. Next, we combine equations (23), (24), and (25) to eliminate  $\hat{p}_t(b)$  and  $\hat{p}_t^n$  to obtain the following equation:

$$\hat{p}_t(f) = \frac{\theta + (1-\theta)\omega}{(1-\theta)(1-\omega)}\pi_t - \frac{\omega}{(1-\theta)(1-\omega)}\pi_{t-1} \quad (27)$$

We insert equation (27) into (26) to obtain the following marginal cost-based NKPC:

$$\pi_t = \frac{\omega}{\omega(1-\theta+\beta\theta)+\theta}\pi_{t-1} + \frac{\beta\theta}{\omega(1-\theta+\beta\theta)+\theta}E_t[\pi_{t+1}] + \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\omega(1-\theta+\beta\theta)+\theta}\phi_t \quad (28)$$

### 2.3 Equilibrium

At first, to be consistent with the definition of consumption index in equation (2), we define the output index as

$$Y_t \equiv \left[ \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (29)$$

Similarly, we define the aggregate labor supply index so as to be consistent with the specification of production function in equation (12)<sup>2)</sup>:

$$N_t \equiv \left[ \int_0^1 N_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (30)$$

Goods market clearing condition requires the following equation: for all  $i$ .

$$Y_t(i) = C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t. \quad (31)$$

Substituting equation (31) into equation (29) yields

$$Y_t = \left[ \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\varepsilon} di \right]^{\frac{\varepsilon}{\varepsilon-1}} C_t = C_t \quad (32)$$

Note that  $\int_0^1 (P_t(i)/P_t)^{1-\varepsilon} di = 1$  from equation (6).

As for the labor market clearing condition, equations (8) and (14) yield following condition:

$$\frac{W_t}{P_t} = (C_t - hC_{t-1})^\sigma N_t^\varphi = \Phi_t A_t \quad (33)$$

### Log-linear approximation

Log-linearization of goods market clearing condition in equation (32) yields

$$y_t = c_t \quad (34)$$

We insert equation (34) into equation (11) to obtain the IS curve as shown in the following equation:



$$y_t = \frac{h}{1+h} y_{t-1} + \frac{1}{1+h} E_t[y_{t+1}] - \frac{1}{\sigma(1+h)} (r_t - E_t[\pi_{t+1}] - \rho) \quad (35)$$

Next, log-linearizing the labor market clearing condition in equation (33) and combining it with equation (21) and (34) leads to the following log-linearized marginal cost

$$\begin{aligned} \phi_t &= \frac{\sigma}{1-h} (c_t - h c_{t-1}) + \varphi n_t - a_t \\ &= \frac{\sigma}{1-h} (y_t - h y_{t-1}) + \varphi (y_t - a_t) - a_t \\ &= \left( \frac{\sigma}{1-h} + \varphi \right) y_t - \frac{\sigma h}{1-h} y_{t-1} - (1 + \varphi) a_t \end{aligned} \quad (36)$$

At last, we combine equation (28) and (36) to obtain the NKPC:

$$\begin{aligned} \pi_t &= \frac{\omega}{\omega(1-\theta+\beta\theta)+\theta} \pi_{t-1} + \frac{\beta\theta}{\omega(1-\theta+\beta\theta)+\theta} E_t[\pi_{t+1}] \\ &\quad + \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\omega(1-\theta+\beta\theta)+\theta} \left\{ \left( \frac{\sigma}{1-h} + \varphi \right) y_t - \frac{\sigma h}{1-h} y_{t-1} - (1 + \varphi) a_t \right\} \end{aligned} \quad (37)$$

## 2.4 Monetary Policy Rule

Our model is closed after specifying the monetary policy rule. We specify the monetary policy rule as an instrumental rule but not as a targeting rule. We suppose the monetary policy rule within a family of Taylor rule with interest rates smoothing, where the central bank responds to inflation and output gap as shown in the following equation:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) (\psi_y y_t + \psi_\pi \pi_t) \quad (38)$$

## 3. Estimation

In this section, we estimate the DSGE model derived in Section 2. We re-parameterize our model to reduce the number of estimated parameters. Our estimated model is constituted by following four equations with four endogenous variables  $x_t = \{y_t, \pi_t, a_t, r_t\}$  and with three shocks or measurement errors  $\varepsilon_t = \{\varepsilon_{y,t}, \varepsilon_{a,t}, \varepsilon_{r,t}\}$ :

### IS curve

$$y_t = \alpha_1 y_{t-1} + (1 - \alpha_1) E_t[y_{t+1}] - \alpha_2 (r_t - E_t[\pi_{t+1}] - \rho) + \varepsilon_{y,t} \quad (39)$$

### New Keynesian Phillips curve

$$\pi_t = \gamma_1 \pi_{t-1} + \gamma_2 E_t[\pi_{t+1}] + \gamma_3 y_t - \gamma_4 y_{t-1} - \gamma_5 a_t \quad (40)$$

### Productivity

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \quad (41)$$

### Monetary policy rule

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\psi_y y_t + \psi_\pi \pi_t) + \varepsilon_{r,t} \quad (42)$$

where  $\varepsilon_{y,t}$  is demand shock,  $\varepsilon_{a,t}$  is productivity shock and  $\varepsilon_{r,t}$  is monetary policy shock, respectively.

### 3.1 Estimation Methodology

We employ the Bayesian estimation methodology to estimate our model. Bayesian estimation is a system based estimation that allows us to incorporate additional information on parameters through priors. This feature is very useful when only small samples of data are available as in the case with euro area time series. The Bayesian estimation methodology consists of five steps<sup>3)</sup>.

At the first step, the linear rational expectations model is solved. In general, the log-linearized DSGE model can be written as a rational expectations system of the form

$$\Gamma_0(\theta)x_t = \Gamma_1(\theta)x_{t-1} + \Gamma_\varepsilon(\theta)\varepsilon_t + \Gamma_\eta(\theta)\eta_t \quad (43)$$

where  $x_t$  denotes the vector of variables in the model. As shown above,  $x_t$  includes four variables in our simplified model. Vector  $\varepsilon_t$  is the innovation of exogenous process and  $\eta_t$  is the rational expectation forecast errors. The solution algorithm described by Sims is applied to (43) to compute the state transition equation:

$$x_t = \Phi_1(\theta)x_{t-1} + \Phi_\varepsilon(\theta)\varepsilon_t \quad (44)$$

At the second step, the model is written in state space form by adding measurement equation to state transition equation (44). The measurement equation relates the model variables  $x_t$  to a vector of observables  $z_t$ :

$$z_t = A(\theta) + Bx_t \quad (45)$$

where  $A(\theta)$  captures the mean of  $z_t$ .  $B$  does not depend on  $\theta$  because it is selected from elements of  $x_t$ .

At the third step, the likelihood function  $L(\theta|Z)$  associated with the linear state space is evaluated with the Kalman filter, where  $Z = \{z_t\}_{t=1}^T$ .

At the fourth step, the likelihood function and the prior distribution  $p(\theta)$  are combined to obtain the posterior density function,

$$p(\theta|Z) \propto L(\theta|Z)p(\theta)$$

and a numerical optimization routine is used to maximize  $\ln L(\theta|Z) + \ln p(\theta)$ . Then the posterior mode denoted by  $\tilde{\theta}$ , and the inverse of the Hessian computed at the posterior mode denoted by  $\tilde{\Sigma}$  are calculated.

At the final step, the posterior distribution of the parameters is derived numerically using

a Monte Carlo Markov chain (henceforce, MCMC) algorithm. The specific MCMC algorithm used in this paper is the Metropolis-Hastings algorithm. In this algorithm,  $\theta^{(0)}$  is drawn from  $N(\tilde{\theta}, c_0^2 \tilde{\Sigma})$  as the starting value. Then, for  $s=1, \dots, n$ ,  $\theta$  is drawn from the proposal distribution  $N(\theta^{(s-1)}, c^2 \tilde{\Sigma})$ . The jump from  $\theta^{(s-1)}$  is accepted with probability

$$\min \left( 1, \frac{L(\theta|Z)p(\theta)}{L(\theta^{(s-1)}|Z)p(\theta^{(s-1)})} \right)$$

and rejected ( $\theta^{(s)} = \theta^{(s-1)}$ ) otherwise<sup>4)</sup>. We generate 10,000 draws in this manner discarding the first 3,000 iterations.

### 3.2 Data and the Priors

At present, euro area countries are constituted by 17 countries. We exclude from our sample countries Slovenia, Cyprus, Malta, Slovakia and Estonia which have introduced euro in recent years.

Figure 1 shows the averaged annual inflation rates of euro area and 12 countries from 1999 to 2009. Data on inflation rates are calculated from Harmonized Index of Consume Price (hence force, HICP). From this figure, we can find that inflation rates vary around 2%, with ranges from about 1.62% in Germany to about 3.09 % in Greece. We divided our sample countries into two groups, namely, lower inflation countries: Austria, Germany, Finland and France, and

Figure 1. Averaged Inflation Rates

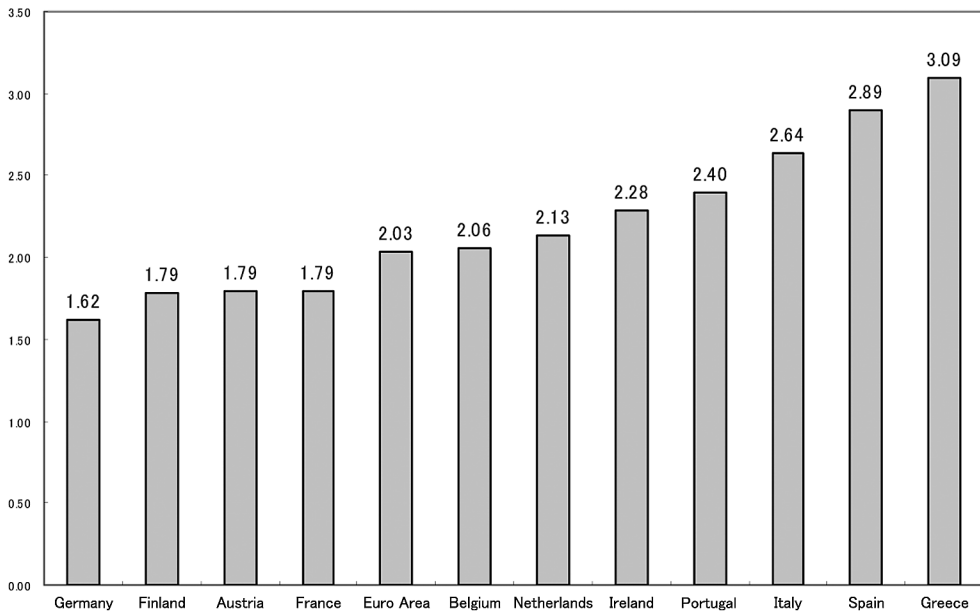


Table 1. Priors

	Distribution	Mean	Std.Dev.
$a_1$	Gamma	0.4	0.1
$a_2$	Gamma	0.6	0.2
$\gamma_1$	Beta	0.3	0.1
$\gamma_2$	Beta	0.6	0.1
$\gamma_3$	Gamma	0.2	0.05
$\gamma_4$	Gamma	0.1	0.05
$\gamma_5$	Gamma	0.1	0.05
$\rho_a$	Beta	0.9	0.1
$\rho_r$	Beta	0.7	0.1
$\psi_\pi$	Gamma	0.5	0.1
$\psi_y$	Gamma	1.5	0.3
$\sigma_y$	Inverse Gamma	1.2	0.5
$\sigma_a$	Inverse Gamma	1.2	0.5
$\sigma_r$	Inverse Gamma	1.2	0.5

Table 2. Calibrated Deep Parameters

$\beta$	0.99
$\sigma$	1
$\phi$	2
$\theta$	0.75
$\omega$	0.4
h	0.7

higher inflation countries: Belgium, Greece, Italy, Ireland, Luxemburg, Netherlands, Spain and Portugal.

Note that, there must be at least as many shocks or measurement errors as there are observed variables in order to avoid stochastic singularity in Bayesian estimation. Since our model includes three shocks or measurement errors, we can use at most three observable variables. We choose the three observable variables as  $z_t = \{y_t, \pi_t, r_t\}$ .

Variables in our model are specified as the log deviation from the steady state. In particular, data on real GDP are logged and smoothed by Hodrick-Prescott filter with smoothing parameter of 1,600. Data on nominal interest rates and on inflation rates are demeaned.

Data on real GDP in each country is calculated as nominal GDP deflated by HICP, which is aggregated to calculate data on real GDP  $y_t$  in each group. Data on inflation rates  $\pi_t$  is calculated as a weighted average of HICP, where each country's weight is based on its share of real GDP within each group. Data on domestic interest rate  $r_t$  is Euribor with three-month maturities.

The priors are specified in Table 1 which reports the distribution, the mean and standard deviations. We use Beta distribution for parameters between 0 to 1, Gamma distribution for parameters restricted to be positive and Inverse Gamma distribution for standard deviation of the shocks. The prior distributions are assumed to be independent across parameters. The priors are chosen to be consistent with the calibrated deep parameter values as shown in Table

Table 3. Empirical Results

	lower inflation countries	higher inflation countries
$a_1$	0.3404 (0.1860-0.4873)	0.3299 (0.1759-0.4800)
$a_2$	2.4074 (1.9842-2.8418)	2.3421 (1.9065-2.8388)
$\gamma_1$	0.1551 (0.0687-0.2462)	0.1511 (0.0533-0.2266)
$\gamma_2$	0.5095 (0.3647-0.6541)	0.5393 (0.3906-0.6898)
$\gamma_3$	0.1515 (0.0935-0.1985)	0.0964 (0.0521-0.1342)
$\gamma_4$	0.042 (0.0118-0.0702)	0.0301 (0.0092-0.0525)
$\gamma_5$	0.0055 (0.0021-0.0081)	0.008 (0.0039-0.0121)
$\rho_a$	0.3667 (0.1812-0.5715)	0.2712 (0.1180-0.4161)
$\rho_r$	0.3313 (0.2131-0.4562)	0.3323 (0.2033-0.4607)
$\psi_y$	1.9975 (1.2804-2.6115)	1.7761 (1.0773-2.3447)
$\psi_h$	0.974 (0.7316-1.2439)	1.0027 (0.7436-1.2777)
$\sigma_y$	0.3475 (0.3056-0.3899)	0.3453 (0.3052-0.3832)
$\sigma_a$	0.8664 (0.5077-1.1934)	0.8434 (0.4935-1.1921)
$\sigma_r$	0.3429 (0.3052-0.3802)	0.342 (0.3054-0.3807)

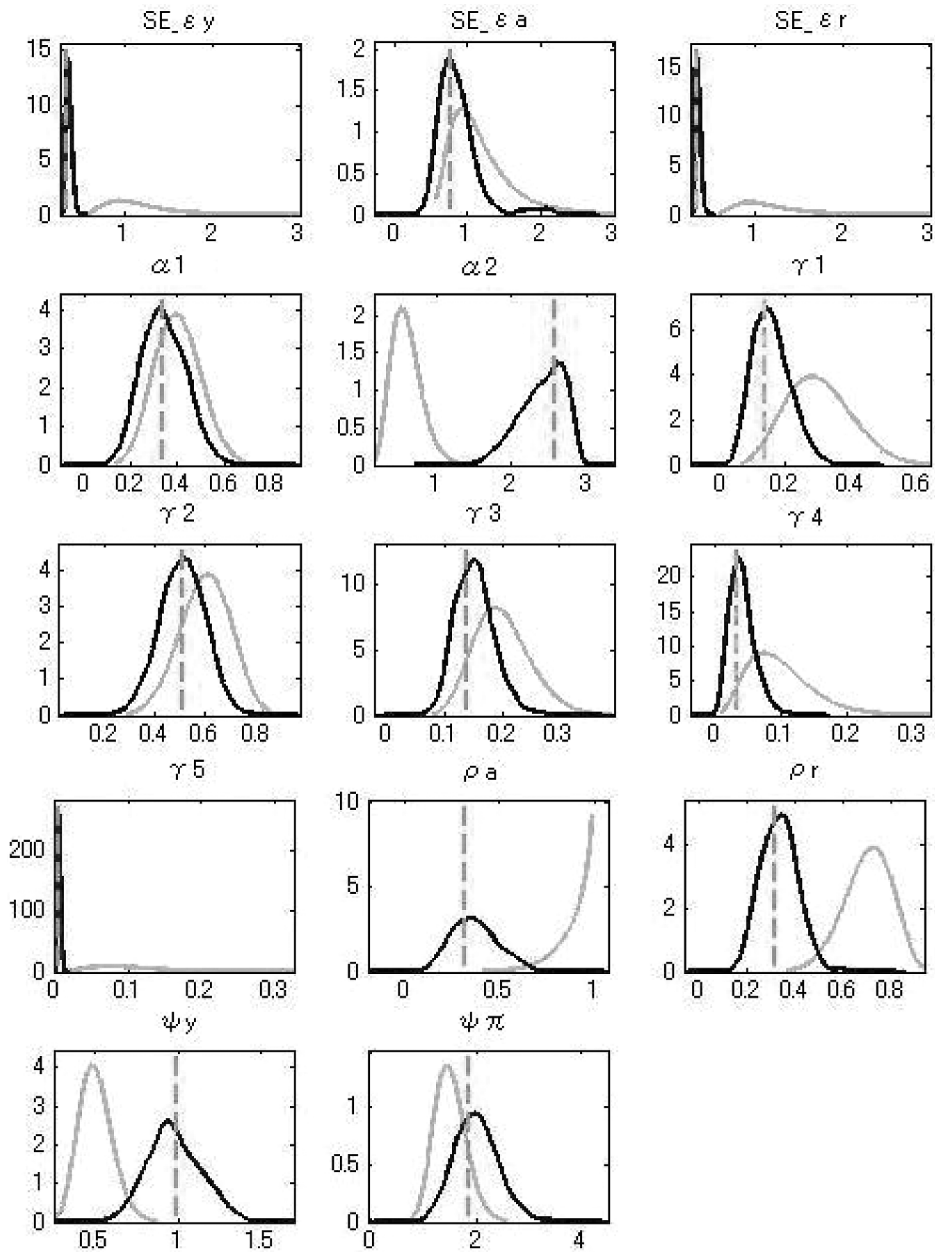
2. The discount factor  $\beta$  is considered fixed and is calibrated to be 0.99, which implies an annual steady state interest rate of 4 percent.

### 3.3 Empirical Results

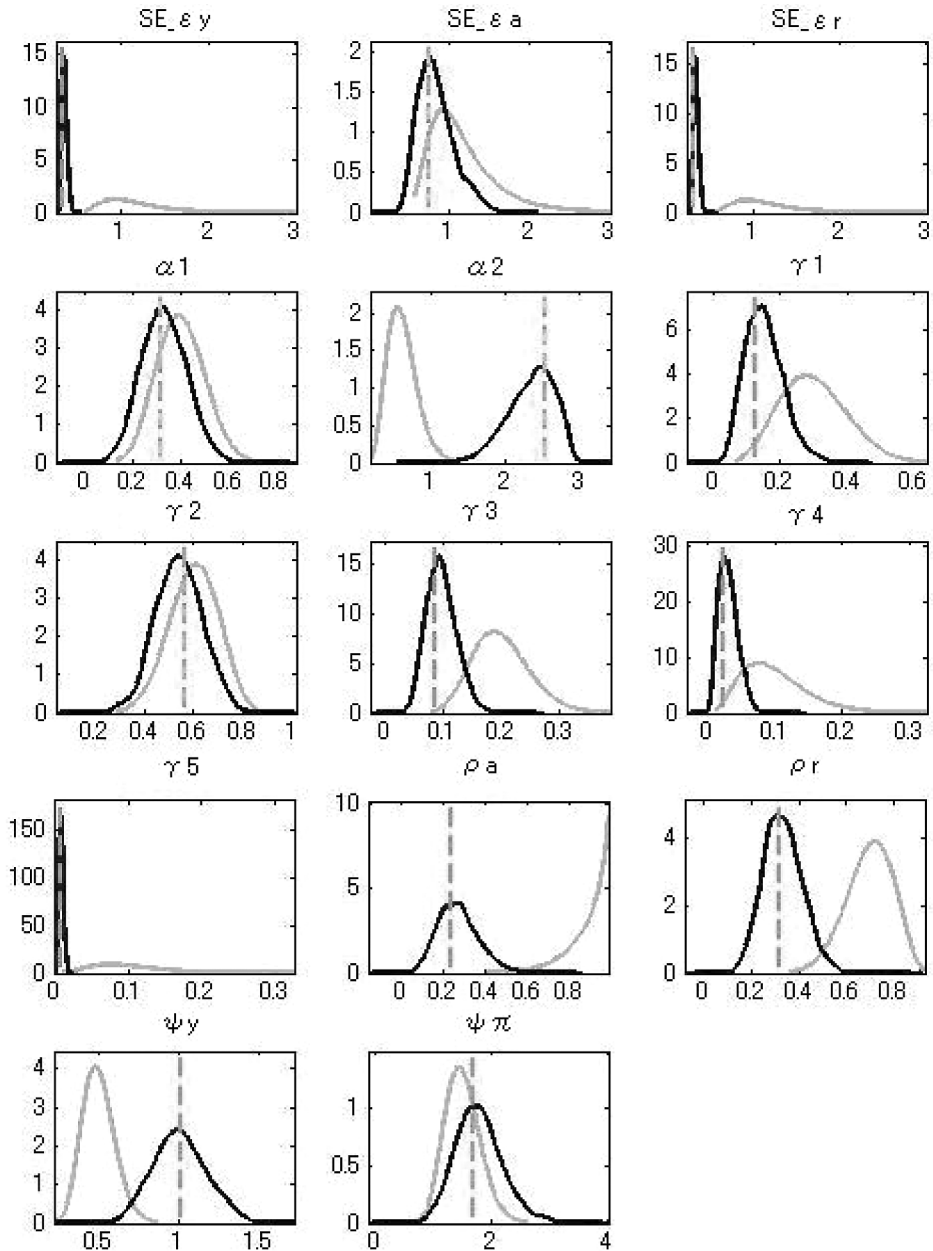
Figure 2 represents the prior and posterior distributions of the parameters. The gray solid lines represent the prior distributions, the solid black lines and dashed lines are the posterior distributions and their modes respectively. As can be seen, there are discrepancies between

Figure 2. Priors and Posteriors

(a) lower inflation countries



(b) higher inflation countries



the priors motivated by micro data and the posteriors that are influenced by macro data.

Table 3 shows the results of our Bayesian estimation, which shows that most of the estimated parameters are close to each other.

As for the IS curve, the estimated coefficients on real interest rates ( $\alpha_2$ ) are much larger when compared to priors and previous studies. As for the NKPC, the coefficients on backward-looking term ( $\gamma_1$ ) are estimated with similar magnitude in both groups, which means that inflation rates would remain lower in lower inflation countries, while they would remain higher in higher inflation countries. Therefore, the inflation differentials between both groups continue to remain. However, they are estimated with relatively lower value, which means that the influence of intrinsic persistence is moderate. The coefficients on forward-looking terms ( $\gamma_2$ ) are also estimated with similar magnitude between both groups. This result means that the frequency of firm's price adjustment is uniform across the euro area countries. It might be because, due to the unified ECB's monetary policy, the expected structure of inflation rates process is uniform across the euro area countries. The coefficients on domestic output gap ( $\gamma_3$ ) is higher in lower inflation countries and lower in higher inflation countries. As mentioned above, a unified monetary policy may be excessively tight for lower inflation countries and loose for higher inflation countries, and then, which would increase economic divergence. Higher extrinsic persistence in lower inflation countries means that inflation rates would rise higher in lower inflation countries, where GDP gap  $y_t$  would be small due to the tight monetary policy. On the other hand, lower extrinsic persistence in higher inflation countries means that inflation rates would lower in higher inflation countries, where GDP gap  $y_t$  would be large due to the loose monetary policy. Therefore, these adjustment mechanisms would eventually drive inflation differentials back to equilibrium.

Figure 3 shows the impulse response functions of real GDP  $y_t$ , inflation rates  $\pi_t$  and interest rate  $r_t$  to each shock. From these figures, we can see that almost of the impulse response functions show the quite similar patterns between both groups. Especially, unified monetary policy would have similar effects in both groups.

## 5. Conclusion

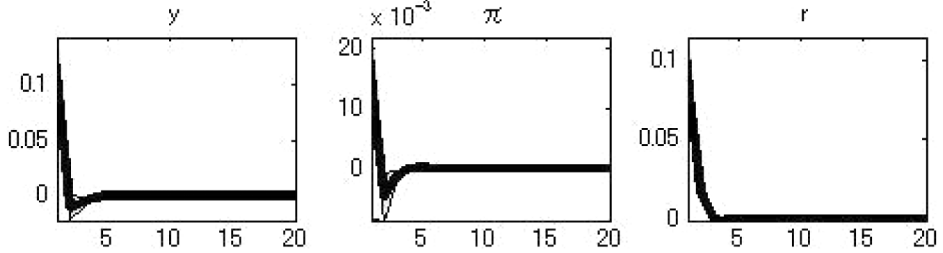
In this paper, we estimated the differences of inflation persistence between lower inflation countries and higher inflation countries in euro area. Estimating the differences in degree of inflation persistence is useful to investigate whether inflation differentials among euro area countries would decrease or not in the future.



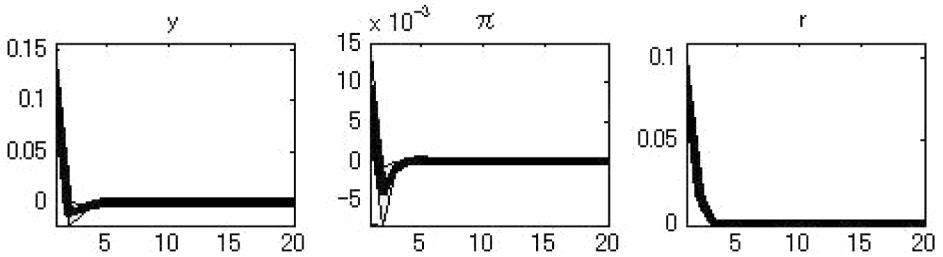
Figure 3. Impulse Response Functions

(i) Demand Shock

(a) lower inflation countries

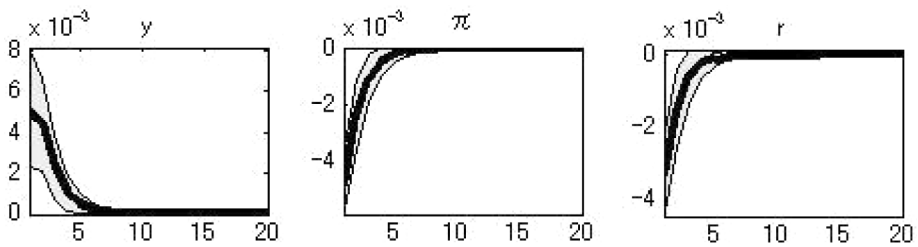


(b) higher inflation countries

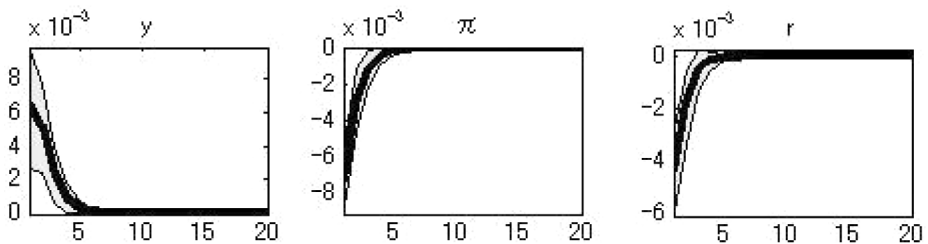


(ii) Productivity Shock

(a) lower inflation countries

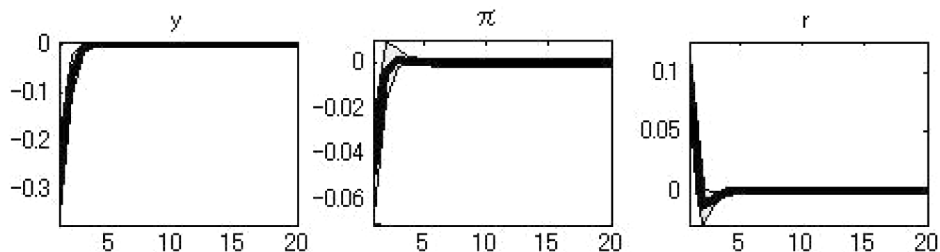


(b) higher inflation countries

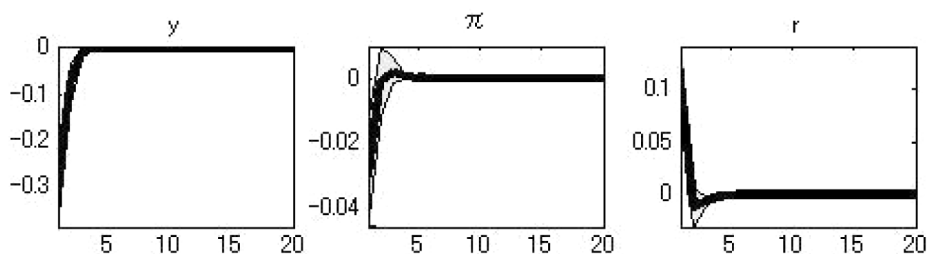


(iii) Monetary Policy Shock

(a) lower inflation countries



(b) higher inflation countries



persistence is higher in lower inflation countries while lower in higher inflation countries, which means that that inflation rates would rise higher in lower inflation countries, where GDP gap  $y_t$  would be small due to the tight monetary policy, while inflation rates would rise lower in higher inflation countries, where GDP gap  $y_t$  would be large due to the loose monetary policy. Therefore, these adjustment mechanisms would eventually drive inflation differentials back to equilibrium.

The impulse response functions analysis shows that impulse responses of each variable to each shock show the quite similar patterns between both groups. Especially, unified monetary policy would have similar effects among countries.

From these results, we can conclude that intrinsic and extrinsic persistence in inflation rates are not the source of inflation differentials and it would not magnify the inflation differentials furthermore. Then, the differences in cyclical positions and/or asymmetric shocks might contribute to the current inflation differentials, but it awaits future studies.

Moreover, von Hagen and Hofmann (2004) pointed out that the international adjustment mechanism through real exchange rates is important to investigate the inflation differentials among the euro area countries. Excess inflation in a given country generates appreciation of real exchange rates, which decreases the output gap and hence creates the deflationary

pressures through Phillips curve, therefore, which eventually drives inflation differentials back to equilibrium. Therefore, it is useful to extend our closed economy model to open economy model.

\* This research was supported by Tokyo Keizai University, Research Grant 2008, and is a revised version of a paper presented at the International Conference: Economic and Monetary Union: 10 Years of Success? on November 27-28, 2008 at Mendel University (see Ogawa and Kumamoto (2010)).

# 坪沼秀昌先生には、着任時より、いつも温かく見守って戴き、身に余るご厚誼を賜りました。先生の早すぎのご逝去に接し、悲しみに耐えられません。先生に改めて感謝の意を表しますとともに、先生のご冥福を心よりお祈り申し上げます。

#### Notes

- 1) See Galí (2008) for basic New Keynesian models.
- 2) With this definition and production function in equation (12),  

$$Y_t = \left[ \int_0^1 (A_t N_t(i))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} = A_t \left[ \int_0^1 N_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \equiv A_t N_t$$
 would hold.
- 3) See Justiano and Preston (2004), An and Schorfheide (2005) and Lubik and Schorfheide (2005) for a review of Bayesian methods for estimation of DSGE model.
- 4) The parameter draws  $\theta$  are converted into the impulse response functions and variance decompositions.

#### References

- Altissimo, F., M. Ehrmann, and F. Smets (2006), "Inflation Persistence and Price-Setting Behavior in the Euro Area: A Summary of the IPN Evidence" *European Central Bank Occasional Paper Series*, No.46.
- An, S. and F. Schorfheide (2007), "Bayesian Analysis of DSGE Models", *Econometric Review*, Vol. 26, pp. 113-172.
- Beningo, P. and D. López-Salido (2002) "Inflation Persistence and Optimal Monetary Policy in the Euro Area" *European Central Bank Working Paper* No. 178.
- Calvo, G.A. (1983) "Staggered Prices in a Utility Maximization Framework", *Journal of Monetary Economics* Vol. 12, pp. 383-393.
- Christiano, L. J., M. Eichenbaum, and C. L. Evans (1999), "Monetary Policy Shocks: What Have We Learned and to What End?", in Taylor, J. B. and M. Woodford (eds.), *Handbook of Macroeconomics* Vol.1A, Amsterdam: Elsevier Science, pp. 65-178.
- Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, Vol. 113 (1), pp. 1-45.
- Erceg, C. J., D. W. Henderson and A.T. Levin (2000), "Optimal Monetary Policy with Staggered Wage

## Inflation Persistence Differentials among Euro Area Countries\*

- and Price Contracts”, *Journal of Monetary Economics* Vol. 46, pp.281-313.
- Galí, J. (2008), *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press
- Galí, J. and M. Gertler (1999), “Inflation Dynamics: A Structural Econometric Analysis”, *Journal of Monetary Economics* Vol. 44, pp.195-222.
- Justiano, A. and B. Preston (2004), “Small Open Economy DSGE Models: Specification, Estimation and Model Fit”, mimeo.
- Lubik ,T. and F. Schorfheide (2005), “A Bayesian Look at New Open Economy Macroeconomics”, mimeo.
- Mankiw, G. and R. Reis (2002) “Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve”, *Quarterly Journal of Econometrics*, Vol. 117 (4), pp. 1295-1328.
- Milani, F. (2005) “Adaptive Learning and Inflation Persistence”, mimeo.
- Ogawa, E. and M. Kumamoto (2010) “Inflation Differentials and the Differences of Monetary Policy Effects among Euro Area Countries”, in L.Lacina, P.Rozmahel and A.Rusek (eds.) *10 Years of Euro: Success ?*, Ales Cenek Publishing, Chapter 8, pp. 205-238.
- Rotemberg, J.J. and M. Woodford (1998), “An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version” *NBER Technical Working Paper* 233.
- Smets, F and R. Wouters (2002), “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area”, *National Bank of Belgium Working Papers-Reserch Series*
- Steinsson, J. (2003) “Optimal Monetary Policy in an Economy with Inflation Persistence”, *Journal of Monetary Economics* Vol. 50, pp. 1425-1456.
- von Hagen, J. and Hofmann, B. (2004), “Macroeconomic Implications of Low Inflation in the Euro Area”, *North American Journal of Economics and Finance*, Vol. 15, pp.5-23.